Workbook Cum Question Bank with Answers

Mathematics

Class-XII (CHSE)

SCHEDULED CASTES & SCHEDULED TRIBES
RESEARCH & TRAINING INSTITUTE (SCSTRTI)
ST & SC DEVELOPMENT DEPARTMENT
BHUBANESWAR

MATHEMATICS

WORKBOOK-CUM-QUESTION BANK
WITH ANSWERS

CLASS - XII (CHSE)

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CHAPTER - 1

RELATIONS AND FUNCTIONS

Group - A

A. Choose the correct answer from the given choices:

1. If $A=\{1,2,3\}$ is a given set and the relation R on the set A is defined as : $R=\{(1,1),(1,2),(2,1)\}$ then R is

   a) only reflexive

   b) only symmetric

   c) only transitive

   d) transitiue and symmetric

2. If the relation R on the set $A=\{1,2,3\}$ is defined by $R=\{(1,1),(2,2),(3,3)\}$ then R is

   a) reflexive but not symmetric

   b) reflexive but not transitive

   c) transitive but not reflexive

   d) an equivalence relation

3. The relation R on the set $A=\{1,2,3\}$ defined by $R=\{(1,2),(2,3),(1,3)\}$ then R is

   a) only reflexive b) only symmetric

   c) only transitive d) none of these

4. The relation R on the set $A=\{1,2,3\}$ defined by $R=\{(1,2),(2,1),(2,3),(3,2)\}$ then R is

   a) only reflexive b) only symmetric

   c) only transitive d) none of these

5. If A be a non-void set of children in a family then the relation "a is a brother of b" on A is

   a) reflexive b) symmetric

   c) transitive d) none of these

6. Let $A=\{1,2,3,4,5\}$ then the relation $R=\{(1,1),(2,2),(3,3),(4,4),(1,2),(2,3)\}$ is

   a) reflexive b) symmetric

   c) transitive d) none of these

7. Let R be a relation on a finite set A having 'n' elements, then the number of relations on A is

   a) 2^n b) 2^{n^2}

   c) n^2 d) n^n

8. If R is a relation from a finite set A having m elements to a finite set B having n elements, then the number of relations from A to B is

   a) 2^{mn} b) $2^{mn}-1$

   c) $2m^n$ d) m^n

9. Let R be a relation on the set A such that $R=R^{-1}$, then R is

   a) reflexive b) symmetric

   c) transitive d) none of these

10. Let $A=\{1,2,3\}$ and let the relation $R=\{(1,2),(2,3)\}$.

    Then the minimum number of order pairs when introduced to R to make of an equivalence relation is

    a) 10 b) 8

    c) 7 d) 4

11. The relation $R=\{(x,y): x^2+y^2=1\, when\, x,y\in R\}$

    a) reflexive b) symmetric

    c) transitive d) anti symmetric

12. The relation "is a subset of" on the power set P(A) of a set A is

a) symmetric

b) anti symmetric

c) equivalence relation

d) none of these

13. The relation on R defined on N an $aRb \Rightarrow$ a divides b is

a) reflexive but not symmetric

b) symmetric but not transitive

c) symmetric and transitive

d) none of these

14. If R be a relation on the set $A = \{1,2,3\}$ is given by $R = \{(1,1),(2,2),(3,3)\}$ then R is

a) only reflexive

b) only symmetric

c) only transitive

d) all the three above

15. If $A = \{1,2,3\}, B = \{1,3,5\}$ and if R be relation from A to B given by $R = \{(1,3),(2,5),(3,3,\}$ then R^{-1} is

a) $\{(1,3),(2,5),(5,3)\}$

b) $\{(3,3),(3,1),(5,2)\}$

c) $\{(1,5),(2,3),(5,2)\}$

d) None of these

16. If $A = \{a,b,c,d\}$ then the relation $R = \{(a,b),(b,a),(a,a)\}$ on A is

a) symmetric and transitive only

b) reflexive and transitive only

c) symmetric only

d) transitive only

17. If $A = \{a,b,c,d\}$ and $R = \{(a,a),(a,b),(a,c),(b,c),(b,d),(c,d),(d,a)\}$ be a relation on A, then R is

a) reflexive b) symmetric

c) transitive d) none of these

18. If $A = \{1,2,3\}$ then the relation R on A defined by $R = \{(2,3),(3,1),(2,1)$ is

a) symmetric only

b) transitive only

c) symmetric and transitive only

d) none of these

19. If R be the largest equivalence relation on a set A and S is any relation on A then

a) $R \subset S$ b) $S \subset R$

c) $R = S$ d) none of these

20. If R is a relation on the set $A = \{1,2,3,4,5,6,7,8,9\}$ given by $xRy \Leftrightarrow y = 3x$ then $R =$

a) $\{(3,1),(6,2),(8,2),(9,3)\}$

b) $\{(3,1),(6,2),(9,3)\}$

c) $\{(3,1),(2,6),(3,9)\}$

d) $\{(1,3),(2,6),(3,9)\}$

21. A relation R from $A = \{1,2,3\}$ to $B = \{1,3,5\}$ is given by $R = \{(1,3),(2,5),(3,3)\}$. Then R^{-1} is

(a) $\{(1,3),(2,3),(3,5)\}$

(b) $\{(3,3),(3,1),(5,2)\}$

(c) $\{(2,3),(2,5),(2,1)\}$

(d) none of these

22. If R be a relation on N defined by $xRy \Leftrightarrow x+2y=8$, then the domain of R is

(a) $\{2,4,8\}$ (b) $\{2,4,6,8\}$

(c) $\{2,4,6\}$ (d) $\{1,3,4,5\}$

23. A relation R from $\{11,12,13\}$ to $\{8,10,12\}$ defined by $R=\{(11,8),(13,10)\}$ then R^{-1} is

(a) $\{(10,13),(8,11),(12,10)\}$

(b) $\{(11,8),(13,10)\}$

(c) $\{(8,11),(10,13)\}$

(d) none of these

24. If R be a relation from a set A of another set B, then

(a) $R=A\cup B$

(b) $R=A\cap B$

(c) $R\subseteq A\times B$

(d) none of these

25. The relation R define on N as $aRb \Rightarrow$ "a divides b" is

(a) reflexive but not symmetric

(b) symmetric but not transitive

(c) symmetric and transitive

(d) none of these

26. The relation $R=\{(m,n):\frac{m}{n}\ \textit{is a power of}\ 5\}$ defined on $Z-\{0\}$ is

(a) Reflexive and not symmetric

(b) Reflexive and symmetric

(c) Reflexive, symmetric and transitive

(d) none of these

27. Let $A=\{1,2,3\}, B=\{3,5,7,9\}$ Let R be a relation defined from A to B by $R=\{(x,y):y=2x+1, x\in A\}$ then $\overrightarrow{R}$ is

(a) $\{(1,3),(2,5),(3,7)\}$

(b) $\{(3,1),(5,2),(7,3)\}$

(c) $\{(3,1),((5,2),(7,3),(9,1)\}$

(d) None of these

28. If $A=\{1,2,3\}$ and $B=\{2,3,4\}$ then which of the following is a function from A to B

(a) $\{(1,3),(1,4),(2,3),(2,4)\}$

(b) $\{(1,2),(2,3)\}$

(c) $\{(1,3),(2,2),(3,4)\}$

(d) $\{(2,3),(2,4),(2,2)\}$

29. If $A=\{1,2,3\}, B=\{8,9\}$ then the number of functions that can be defined from A to B is

(a) 4 (b) 6

(c) 8 (d) 10

30. If $f(x)=\cos(\log x)$, then $f(x)f(y)-\frac{1}{2}\left\{f(xy)+f\left(\frac{x}{y}\right)\right\}$ has the value

(a) 0 (b) 1

(c) -1 (d) 2

31. If $f(x)=64x^3+\frac{1}{x^3}$ and α,β are the roots of $4x+\frac{1}{x}=3$, then

(a) $f(\alpha)=f(\beta)=-9$

(b) $f(\alpha)=f(\beta)=63$

(c) $f(\alpha)\neq f(\beta)$

(d) none of these

32. If $f(x)=x+\frac{1}{x}$ then $f\left(x^3\right)+3f\left(\frac{1}{x}\right)=$

(a) $\left[f(x)\right]^2$ (b) $\left[f(x)\right]^3$

(c) $\left[f(x)\right]^4$ (d) none of these

33. If $f(x)=\frac{x-1}{x+1}, x\neq -1$ then $f\left[f(x)\right]$ is

(a) $\frac{1}{x}$ (b) $-\frac{1}{x}$

(c) $\frac{1}{x^2}$ (d) $-\frac{1}{x^2}$

34. If f be a real function satisfying $f\left(x+\frac{1}{x}\right)=x^2+\frac{1}{x^2}$ for all $x\in R-\{0\}$, then $f(x)$ is

(a) x^2-1 (b) x^2-2

(c) x^2-3 (d) x^2-4

35. The range of the function $f(x)=\frac{x}{|x|}$ is

(a) $R-\{0\}$ (b) $R-\{-1,1\}$

(c) $\{-1,1\}$ (d) none of these

36. If $f:A\rightarrow B$ and $g:B\rightarrow C$ be bijective, then $(fog)^{-1}$ is

(a) $f^{-1}og^{-1}$ (b) fog

(c) $g^{-1}of^{-1}$ (d) gof

37. If $A=\{x,y,z\}, B=\{u,v,w\}$ and $f:A\rightarrow B$ be defined by $f(x)=u$, $f(y)=v$, $f(z)=w$, then f is

(a) surjective but not injective

(b) injective but not surjective

(c) bijective

(d) none of these

38. The composite mapping fog of the map $f:R\rightarrow R$ defined by $f(x)=\sin x$ and $g:R\rightarrow R$ defined by $g(x)=x^2$ is

(a) $(\sin x)^2$ (b) $\sin x^2$

(c) $x^2\sin x$ (d) $\frac{x^2}{\sin x}$

39. If $f(x)=\left(3-x^7\right)^{\frac{1}{7}}$ for all $x\in R$ then $(fof)\,(x)$ is

(a) x (b) $2x$

(c) $3x$ (d) $4x$

40. Damain of the function $f(x)=\sqrt{2x-1}+\sqrt{3-2x}$ on R is

(a) $\left(\frac{1}{2},\frac{3}{2}\right)$ (b) $\left[\frac{1}{2},\frac{3}{2}\right]$

(c) $\left\{x\in R: x\geq\frac{1}{2}\right\}$ (d) None of these

41. If $f:R\rightarrow R$ be defined by $f(x)=4x+3$ then $f^{-1}(x)$ is

(a) $\frac{x-3}{4}$ (b) $3x-4$

(c) $\frac{x-4}{3}$ (d) None of these

42. Total number of one-one function from a set with m elements to a set with n elements, $m\leq n$ is

(a) m^n (b) n^m

(c) $\frac{n!}{(n-m)!}$ (d) None of these

43. If $f(x)=(a-x^n)^{\frac{1}{n}}$ where $a>0$ and $n\in N$ then $fof(x)$ is equal to

(a) x (b) a

(c) x^n (d) a^n

44. The total number of one-one function from a finite set with m elements to a set with n elements for $m > n$ is

(a) $\frac{m!}{(m-n)!}$ (b) $\frac{n!}{(n-m)!}$

(c) n^m (d) none of these

45. The number of bijective functions from a set A to it self when A contains n elements is

(a) n^2 (b) n

(c) $n!$ (d) 2^n

46. Let $f : R \to R$ be a function defined by $f(x) = \cos(5x+2)$ then f is

(a) injective (b) surjective

(c) bijective (d) none of these

47. If $f : R \to R$ be a mapping defined by $f(x) = x^3 + 5$ then $f^{-1}(x)$ is

(a) $(x+5)^{\frac{1}{3}}$ (b) $(x-5)^{\frac{1}{3}}$

(c) $(5-x)^{\frac{1}{3}}$ (d) $5-x$

Answers

1.	(b)	14.	(d)	26.	(c)	38.	(b)
2.	(d)	15.	(b)	27.	(b)	39.	(a)
3.	c)	16.	(c)	28.	(c)	40.	(b)
4.	(d)	17.	(d)	29.	(b)	41.	(a)
6.	(d)	18.	(b)	30.	(a)	42.	(c)
7.	(b)	19.	(b)	31.	(a)	43.	(a)
8.	(a)	20.	(a)	32.	(b)	44.	(d)
9.	(b)	21.	(b)	33.	(b)	45.	(c)
10.	(c)	22.	(c)	34.	(b)	46.	(d)
11.	(b)	23.	(c)	35.	(c)	47.	(b)
12.	(b)	24.	(c)	36.	(c)		
13.	(a)	25.	(a)	37.	(c)		

B. Fill in the blanks

1. The smallest relation on the set $A=\{a,b,c\}$ is ______

2. It $A=\{1,2,3,4,5\}$ and $R:A\to A$ is $\{(1,2),(2,3),(4,5),(3,3)\}$ then $R^{-1}:A\to A$ is _____

3. The sum of two odd functions is _____

4. The sum of two even function is _____

5. If $f:R\to R$ and $g:R\to R$ are given by $f(x)=8x^3$ and $g(x)=x^{\frac{1}{3}}$, then fog is ____

6. If R be a relation from a finite set A having m elements to a finite set B having n elements, then the number of relations from A to B is _________

7. If $n(A)=4$ and $n(B)=6$, then the numebr of one to one functions from A to B is _______

8. If $f:R\to R$ is defined by $f(x)=\left(3-x^3\right)^{\frac{1}{3}}$, then $(fof)(x)=$ _____

9. If $f:R\to R$ and $g:R\to R$ are defined by $f(x)=\sin x$ and $g(x)=5x^2$, then $(gof)(x)=$ _________

10. If $f:R\to R$ is defined by $f(x)=3x+2$ then $f[f(x)]=$ ______

11. If the function $f:R\to R$ defined by $f(x)=3x-4$ is invertible, then $f^{-1}(x)=$ _______

12. If the binary operation $*$ defined on Q is $a*b=2a+b-ab$ for all $a,b\in Q$ then $3*4=$ ______

13. If the binary operation $*$ on set of integers Z is defined as $a*b=2a+b-3b^2$ then $3*4=$ _____

14. If $*$ be a binary operation on the set of integers I defined by $a*b=2a+b-3$ then $3*4=$ _____

15. Let $*:R\times R\to R$ is defined as $a*b=2a+b$ then $(2*3)*4=$ ______

16. Let $*$ be a binary operation on the set of intepers Z defined by $a*b=3a+4b-2$ then $4*5=$ _____

17. The roster form of the relation $R=\{(x,x^3):x$ is a prime number less than $10\}$ is _____

18. If $A=\{1,2,3,4,5,6\}$ and a relation R on a set A is defined by $R=\{(a,b):a,b\in A \text{ and } a \text{ divides } b\}$ then R in roster form is _______

19. Let $A=\{1,2,3,5\}$ and $B=\{4,6,9\}$. A relation R from A to B is defined by $R=\{(x,y):x\in A, y\in B$ and $x-y\, is\, odd\}$ then R in roster form is ______

20. A relation R on the set N is defined by $R=\{(x,y):y=x+5 \text{ and } x<4, x,y\in N\}$ then the relation R in Roster form is ____.

21. If $A=\{x,y,z\}$ and $B=\{a,b\}$ then the total number of relations from A to B is ____.

22. If $f(x)=\sqrt{x}, g(x)=\dfrac{x}{y}$ and $h(x)=4x-8$ then what is the value of $hogof(x)$ ____.

23. If $f:R\to R$ such that $f(x)=\sin x$ and $g:R\to R$ such that $g(x)=x^2$ then $(fog)(x)=$ ________

24. If $f(x)=\dfrac{x+1}{x-1}$ then $f\left[f(x)\right]=$ _____

25. If $f(x) = x^3 - \frac{1}{x^3}$ then $f(x) + f\left(\frac{1}{x}\right) =$ _______

26. A relation which is reflexive symmetric and transitive relation is called as________ relation.

27. A relation R where $aRb \wedge bRc \Rightarrow aRc$ is called a ________ relation.

28. A relation R where $aRb \wedge bRa \Rightarrow a = b$ is called _______ relation.

29. A function defined on a set of real numbers is invertible if it is_______.

30. If $f : R \to R$ be a function define by $f(x) = 3x - 5$, then $f^{-1}(1) =$_______.

31. Let $f : R \to R$ be a function defined by $f(x) = x + 1$ and $g : R \to R$ be another function defined by $g(x) = \sqrt{x}$ then $fog(x) =$_________.

32. Let $f : A \to B$ be a function. Then f is on to if________.

33. one- one and on to function is called a _______ function.

34. Let $f : R \to R$ be a function defined by $f(x) = \frac{1}{\sqrt{x-1}}$, then the domain of $f(x) =$________.

35. If $f : R \to R$ and is defined by $f(x) = |x|$ then $(fof)(x) =$_______.

Answers

1. $\varnothing$
2. $\{(2,1), (3,2), (5,4), (3,3)\}$
3. odd.
4. even
5. $8x$
6. 2^{mn}
7. 360
8. x
9. $5.\sin^2 x$
10. $9x + 8$
11. $\frac{1}{3}(x + 4)$
12. -2
13. 50
14. 7
15. 18
16. 30
17. $R = \{(2,8), (3,27), (5,125), (7,343)\}$
18. $R = \{(1,1), (1,2), (1,3), (1,4), (1,5), (1,6)$ $(2,2), (2,4), (2,6), (3,3), (3,6), (4,4),$ $(5,5), (6,6)\}$
19. $R = \{(1,4), (1,6), (2,9), (3,4), (3,6),$ $(5,4)(5,6)\}$
20. $\{(1,6), (2,7), (3,8)\}$
21. 64
22. $\sqrt{x} - 8$
23. $\sin^2 x$
24. x
25. 0
26. Equivalence
27. transitive
28. Antisymmetric
29. one-one and on to (bijective)
30. 2
31. $\sqrt{x} + 1$
32. $f(A) = B$
33. bijective
34. $\{x \in R : x > 1\}$
35. f

C. Answer the followings in one word.

1. If $A=\{x,y,z\}$ and $A=\{a,b\}$ then what is the number of relation from A to B
2. Write the range set of the function $f(x)=|x|$.
3. If $|X|=5$ then the number of bijective fun ctions from the set X to it self is?
4. The tabular form of the relation $R=\{(x,y):2x-y=0\}$ on $\{1,2,3\}$ is?
5. If $a*b=ab-2$ is a binary operation on Z then $2*(1*5)=?$
6. If R be a relation on A such that $R=R^{-1}$, then write the type of the relation R.
7. Sets A and B have respectively m and n elements. The total number of relations from A to B is 64. If $m<n$ and $m\neq 1$, then write the values of m and n respectively.
8. If $R=\{(x,x^3):x$ is a prime number less than $5\}$ then what is the range of R?
9. What is the least positive integer r such that $185\in[r]_7$?
10. If $R=\{(x,y):x+2y=8\}$ is a relation on N, then write the range of R.
11. Let R be a relation on a finite set A having 'n' elements, then what are the number of relation on A ?
12. Let $A=\{1,2\}, B=\{1,2,3,4\}$. How many relations will be there from A to B ?
13. Let $A=\{1,2,3,5\}, B=\{4,6,9\}$. A relation R from A to B is defined by $R=\{(x,y):x\in A, y\in B \, and \, x-y \, is \, odd\}$ write R in roster form.
14. If $A=\{1,2,3\}, B=\{4,5,6,7\}$ and $f=\{(1,4),(2,5),(3,6)\}$ is a function from **A** to B. State whether f is one one or not.
15. If $f:R\to R$ are given by $f(x)=8x^3$ and $g(x)=x^{\frac{1}{3}}$, then write $(fog)(x)$.
16. If $f:R\to R$ is defined by $f(x)=3x+2$, then find $f[f(x)]$.
17. If the function $f:R\to R$ defined by $f(x)=3x-4$ is invertible, then what is $f^{-1}(x)$?
18. If $f(x)=27x^3$ and $g(x)=x^{\frac{1}{3}}$, then what is $(gof)(x)$?
19. What is the domain of $f(x)=\log\left(\frac{12}{x^2-x}\right)$?
20. If the mapping is $f:R\to R$ given by $f(x)=4x^3-12x$, then what is the image of the intervel $[-1,3]$?

Answers

1. 2^6.
2. $R^+\cup\{0\}$.
3. 120.
4. $\{(1,2)\}$.
5. 4.
6. Symmetric.
7. $m=2$ and $n=3$
8. $\{8,27\}$.
9. 3.
10. $\{1,2,3\}$
11. 2^{n^2}
12. 256

13. $\{(1,4),(1,6),(2,9),(3,4),(3,6),(5,4),(5,6)\}$

14. One-one

15. $8x$

16. $9x+8$

17. $\frac{1}{3}(2x-5)$

18. $3x$

19. Domain $\{x : -3 \le x \le 4\}$

20. $[8, 72]$

D. Answer the following in one sentence

1. Define the equivalence relation on a set A.

2. The graph of an even function is symmetrical about which axis?

3. What is the meaning of $a \equiv b \bmod (5)$ on the set of integers.

4. If f be any real function, then what type of function $\frac{1}{2}[f(x)+f(-x)]$ is ?

5. Express $e^x + \sin x$ as the sum of an even function and odd function.

7. The mapping f and g are given by $f = \{(1,2),(3,4),(5,6),(7,8)\}$ and $g = \{(2,5),(4,7),(6,3),(8,1)\}$ then what is gof .

8. If two functions are odd, then what type of function will be their sum.

9. Express the function $1+x+x^2$ as the sum of an even function and odd function.

10. If $f = \{(1,a),(12,b),(3,c),(4,d)\}$ and $g = \{(a,x),(b,x),(c,y),(d,x)\}$ then what is gof ?

11. What is the natural domain of $f(x)=\sqrt{x}$?

12. Write the identity relation on $\{a,b,c\}$.

13. Write the smallest reflexive relation on $\{a,b,c,d\}$.

14. If R be a relation from $\{11,12,13\}$ to $\{8,10,12\}$ defined by $y = x-3$ then what is R^{-1} ?

15. Defince a symmetric relation.

16. If $A = \{1,2,3\}$, write an example of a relation on A which is reflexive symmetric but not transitive?

17. Let $f = \{(-3,-2),(-1,0),(2,1),(5,3)\}$ and $g = \{(-2,-1),(3,7),(0,2),(1,5)\}$ then what is gof ?

18. If $A = \{1,2,3,4\}, B = \{2,4,6,8\}$ and $f : A \to B$ is defined by $f(x) = 2x$, then find f and f^{-1} as a set of order pairs.

19. Let $f(x) = \frac{x}{\sqrt{1+x^2}}$, then find $(fof)(x)$.

20. Let a function $f : A \to B$ defined by $f(x) = \log(1+x)$ and a function $g : B \to C$ defined by $g(x) = e^x$. Find $(gof)(x)$.

21. If $f(x) = \sqrt{1-x}$ and $g(x) = \log_e x$ are two real functions then find $(fog)(x)$

22. If $A=\{1,2,3\}$ and $B=\{a,b\}$ then write the total number of functions from A to B.

23. Let $A=\{3,5,7\}$ and $B=\{2,4,9\}$ and R is a relation from A to B is given by "is greater than". Write R as a set of order pairs.

24. Let R be a relation on a finete set A having 'n' elements then what is the number of relation on A.

25. If the function $f:R\to R$ is defined by $f(x)=3x-4$ is invertible then find f^{-1}.

Answers

1. A relation R on a set A os said to be an equivalence relation if it reflexive symmetric and transitive.

2. The graph of an even function is symmetrical about $y-axis$.

3. $a-b$ is an integral multiple of 5.

4. $\frac{1}{2}[f(x)+f(-x)]$ is an even function.

5. $e^x+\sin x=\left[\frac{1}{2}\left(e^x+e^{-x}\right)\right]+\left[\frac{1}{2}\left(e^x-e^{-x}\right)+\sin x\right]$

6. $e^x=\frac{1}{2}\left(e^x+e^{-x}\right)+\frac{1}{2}\left(e^x-e^{-x}\right)$

7. $\{(1,5),(3,7),(5,3),(7,1)\}$

8. The sum of two odd functions is odd.

9. $1+x+x^2=\left(1+x^2\right)+x$ where $1+x^2$ is even and x is odd.

10. $gof=\{(1,x),(2,x),(3,y),(4,x)\}$

11. The domain of $f(x)=\sqrt{x}$ is $\{x\in R: x\geq 0\}$.

12. The identity relation on $\{a,b,c\}$ is $\{(a,a),(b,b),(c,c)\}$

13. The smallest reflexive relation on the given set is $\{(a,a),(b,b),(c,c),(d,d)\}$

14. $\{(8,11),(10,13)\}$

15. A relation R on a set A is said to be symmetric if for $a,b\in A,(a,b)\in R\Rightarrow (b,a)\in R$

16. If $A=\{1,2,3\}$ then a relation on A which is reflexive and symmetric but not transitive is

$$R=\{(1,1),(2,2)(3,3),(1,2)(2,1),(2,3),(3,2)\}$$

17. $gof=\{(-3,-2),(-1,2),(2,5),(5,7)\}$

18. $f=\{(1,2),(2,4),(3,6),(4,8)\}$ and $f^{-1}=\{(2,1),(4,2),(6,3),(8,4)\}$

19. $(fof)(x)=\frac{x}{\sqrt{1+2x^2}}$

20. $(fog)(x)=1+x$

21. $(fog)(x)=\sqrt{1-\log x}$

22. Total number of functions from A to B is 8.

23. $R=\{(3,2),(5,2),(5,4),(7,2),(7,4)\}$

24. The number of relation on $A=2^{n^2}$

25. $f^{-1}(x)=\frac{1}{3}(x+4)$

Group - B

B. Short type Question & Answers:

1. Let $X=\{x,y\}$ and $Y=\{u,v\}$. Write down all the functions that can be defined from X to Y. How many of these are (i) one-one (i) on to and (iii) one-one and on to ?

2. Let X and Y be the sets containing m and n elements. What is the total number of function from X to Y.

   How many functions from X to Y are one-one according as $m<n, m>n$ and $m=n$.

3. Examine the function $f:R\to R$ such that $f(x)=x^2$ if it is (i) injective (ii) bijective

4. Show that the function $f(x)=\sin x$ on $\left[0,\frac{\pi}{2}\right]$ is injective

5. Find the composition fog and gof when f and g are function on R given by $f(x)=\cos x, g(x)=\sin x^2$

6. Find the composition fog and gof when f and g are function on R given by $f(x)=g(x)=\left(1-x^3\right)^{\frac{1}{3}}$

7. If f be a real function, then show that (i) $f(x)+f(-x)$ is alongs an even functions and (i) $f(x)-f(-x)$ is always an odd fuctions.

8. Thesf whether the relation $R=\{(m,n):m-n\geq 7\}$ on Z is reflexive, symmetric or transitive.

9. The relation R on the set Z is defined by for $m,n\in Z$, $mRn \Rightarrow -10\leq \quad m+n\leq 10$. Examine whether it is an equivalence relation.

10. Shwo that the relation R defined on the set $A=\{1,2,3,4,5,6\}$ as $R=\{(x,y):$ $x,y\in A$ and y is divisible by $x\}$ is reflexive, and transitive but not symmetric.

11. Show that the operation $*$ given by $a*b=a+b-ab$ is a binary operation on Z,Q and R but not on N.

12. Determine whether the following binary operation on the set R is associaative and commutative. $a*b=\frac{a+b}{2}$ for all $a,b\in R$.

13. Show that $*:R\times R\to R$ given by $a*b=a+4b^2$ a binary operation.

14. Show that $*:R\times R\to R$ defined by $a*b=a+2b$ is not commutative.

15. Show the operation defined by $a*b=a+b-ab$ on $R-\{1\}$ is a binary operation. Show whether it is commutative and associative.

16. Show that the operation $*$ given by $a*b=2a+3b$ on Z is a binary operation. Examine whether it is commutative and associative.

17. Show that the operation $*$ given by $a*b=L.C.M\{a,b\}$ is a binary operation on N. Examine whether it is commutative and associative.

Answers

1. The given sets are

$X=\{x,y\}$ and $Y=\{y,v\}$

The functions that are defined from X to Y

are $f_1=\{(x,u),(y,v)\}$

$f_2=\{(x,v),(y,u\}$

$f_3=\{(x,u),(y,u)\}$

$f_4=\{(x,v),(y,v)\}$

out of these four functions, f_1 and f_2 are one-one function.

Also f_1 and f_2 are on to functions

Also f_1 and f_2 are one-one on to functions.

2. There X and Y are sets containg on and n elements.

$\therefore |X|=m,|Y|=n$

The number of functions from X to $Y=n^m$.

If $m<n,$ then the number of one-one functions $={}^nP_m=\dfrac{n!}{(n-m)!}$

If $m>n,$ then the number of one-one functions $=0$

If $m=n$, then the number of one-one functions $=m!$

3. The given function is

$f:R\to R$ such that $f(x)=x^2$

for $x_1,x_2\in R,$

$f(x_1)=f(x_2)$

$\Rightarrow x_1^2=x_2^2$

$\Rightarrow x_1=\pm x_2$

$x_2\neq x_2$

So f is not one-one

$\Rightarrow f$ is nither injetive nor bijective on R

4. The given function is $f(x)=\sin x$ which defined on $\left[0,\dfrac{\pi}{2}\right]$

Let $x_1,x_2\in\left(0,\dfrac{\pi}{2}\right)$

$\therefore f(x_1)=f(x_2)\Rightarrow \sin x_1=\sin x_2$

$\Rightarrow x_1=x_2$ as $x_1,x_2\in\left[0,\dfrac{\pi}{2}\right]$

So f is one-one.

5. The given function defined on R are $f(x)=\cos x$ and $g(x)=\sin x^2$

$fog(x)=f\left[g(x)\right]=f\left[\sin x^2\right]$

$=\cos\left[\sin x^2\right]$

And $(gof)(x)=g\left[f(x)\right]=g\left[\cos x\right]$

$=\sin\left(\cos^2 x\right)$

Here $\left(fog\right)(x)\neq(gof)(x)$

6. Given that $f(x)=g(x)=\left(1-x^3\right)^{\frac{1}{3}}$

$\left(fog\right)(x)=f\left[g(x)\right]$

$=\left[1-[g(x)]^3\right]^{\frac{1}{3}}$

$=\left[1-\left\{(1-x^3)^{\frac{1}{3}}\right\}^3\right]^{\frac{1}{3}}$

$=\left[1-\left(1-x^3\right)\right]^{\frac{1}{3}}$

$=\left[1-1+x^3\right]^{\frac{1}{3}}=x$

$(gof)(x)=g\left[f(x)\right]=\left[1-\left[f(x)\right]^3\right]^{\frac{1}{3}}$

$$=\left[1-\left[(1-x^3)^{\frac{1}{3}}\right]^3\right]^{\frac{1}{3}}$$

$$=\left[1-(1-x^3)\right]^{\frac{1}{3}}$$

$$(1-1+x^3)^{\frac{1}{3}}=x$$

$\therefore fog = gof$

7. (i) Let $h(x)=f(x)+f(-x)$

$\therefore h(-x)=f(-x)+f(x)$

$=f(x)+f(-x)=h(x)$

So $h(x)$ is an even function.

(ii) Let $g(x)=f(x)-f(-x)$

$\therefore g(-x)=f(-x)-f(x)$

$=-\left[f(x)-f(-x)\right]$

$=-g(x)$

$\Rightarrow g(x)$ is an odd function.

8. The given relation is

$R=\{(m,n):m-n\geq 7\}$ on Z

(i) Reflexive For all $m\in Z$

$m-m=0<7$

$\Rightarrow (m,m)\in R$

$\Rightarrow R$ is reflexive

(ii) Symmetric Let $(m,n)\in R$

$\Rightarrow m-n\geq 7$

$\Rightarrow n-m<7$

$\Rightarrow (n-m)\ngeq 7$

$\Rightarrow (n,m)\notin R$

So the relation R is not symmetric

(iii) Transitive:

Let : $(m,n),(n,p),(n,p)\in R$

$\Rightarrow m,n\geq 7 \ and \ n-p\geq 7$

$\Rightarrow (m,n)+(n+p)\geq 7+7=14$

$\Rightarrow m-p\geq 7$

$\Rightarrow (m,p)\in R$

So R is transitive

9. The given set is Z, the set of in tegers

For $m,n\in z$, $mRn \Rightarrow -10 \ \leq m+n$ ≤ 10

$|m+n|\leq 10$

For all $m\in Z$,

$|m+m|=|2m|\leq 10$ is not true.

$\Rightarrow mRm$ is not true.

so R is not reflexive.

$\Rightarrow R$ is not on equivelence relation

10. The given set is $A=\{1,2,3,4,5,6\}$. The relation R on the set A is defined as $R=\{(x,y: y$ is divisible by $x\}$

$=\{(1,1),(2,2),(3,3),(4,4),(5,5),$
$(6,6),(1,2)\ ,(1,3)(1,4),(1,5),(1,6),\ \ (2,4),$
$(2,6),(3,6)\}$

Reflexive: The relation R on the set A is reflexive if $(x,x)\in R$ for all $x\in A$.

The relation R is reflexive.

Symmetric: The relation R on the set A is

symmetric if for $x,y\in A$,

$(x,y)\in R\Rightarrow (y,x)\in R$

Here $(1,2)\in R$ but $(2,1)\notin R$

$(1,3)\in R$ but $(3,1)\notin R$

So the relation R is not symmetric

Transitive: The relation a on the set A is transifive if for $x, y, z \in A$

$(x, y) \in R, (y, z) \in R \Rightarrow (y, z) \in R$

Here $(1,2) \in R$, $(2,4) \in R$ and also $(1,4) \in R$ $(1,3) \in R, (3,6) \in R$ and also $(1,6) \in R$

So the relation R on the set A is transitive.

11. The operation $*$ given by $a*b = a+b-ab$

For all $a, b \in z$

$a+b-ab \in z$

$\Rightarrow a*b \in z$

$\Rightarrow *$ is a binary operation on z

For all $a, b \in Q$,

$a+b-ab \in Q$

$\Rightarrow a*b \in Q$.

$\Rightarrow *$ is a binary operation on Q

For all $a, b \in R$

$a+b-ab \in R$

$\Rightarrow a*b \in R$

$\Rightarrow *$ is a binary operation on R

Again $3, 4 \in N$

$3+4-3.4 = 7-12 = -5 \notin N$

$\Rightarrow 3*4 \notin N$

So $*$ is not a binary operation on N

12. For all $a, b \in R$,

$\frac{a+b}{2} \in R$

$\Rightarrow a*b \in R$

$\Rightarrow$ The operation $*$ is a binary operation

$a*b = \frac{a+b}{2} = \frac{b+a}{2} = b*a$

$\Rightarrow$ The operation $*$ is commutative

Again for $a, b, c \in R$

$(a*b)*c = \left(\frac{a+b}{2}\right)*c = \frac{\frac{a+b}{2}}{2} + c$

$= \frac{a+b+2c}{4}$ ....(1)

$a*(b*c) = a*\left(\frac{b+c}{2}\right)$

$= \frac{a+\frac{b+c}{2}}{2}$

$= \frac{2a+b+c}{4}$ ....(2)

From (1) and (2) we see that

$(a*b)*c \neq a*(b*c)$.

The operation $*$ is not associateve.

13. The operation $*$ is defined by

For $a, b \in R$

$a*b = a+4b^2 \in R$

$\Rightarrow a*b$ is a binary operation.

14. For $a, b, \in R$, $a*b = a+2b$

$b*a = b+2a$

$\therefore a+2b \neq b+2a$

$\Rightarrow a*b \neq b*a$

$\Rightarrow$ The operation $*$ is not commutative.

15. For $a, b \in R-\{1\}$

$a*b = a+b-ab \in R-\{1\}$

$\Rightarrow$ The operation $*$ is binary operation on $R-\{1\}$

Commutative

$a*b=a+b-ab$ ....(1)

$b*a=b+a-ba$

$=a+b-ab$

$=a*b$ ....(2)

From $a*b=b*a$

$\Rightarrow$ The operation $*$ is commutative.

Associative

$a*(b*c)=a*(b+c-bc)$

$=a+b+c-bc-a(b+c-bc)$

$=a+b+c-bc-ab-ac+abc$

........(1)

Again $(a*b)*c=(a+b-ab)*c$

$=(a+b-ab)+c-(a+b-ab)c$

$=a+b+c-ab-ac-bc+abc$

........(2)

From (1) and (2) , we have

$a*(b*c)=(a*b)*c$

$\Rightarrow$ The operation $*$ is associative.

16. For $a,b\in z$

$2a+3b\in z$

$\Rightarrow a*b\in z$

$\Rightarrow$ The operation $*$ is a binary operation

Commutative

For $a,b\in z$, $a*b=2a+3b$

$b*a=2b+3a$

$\therefore 2a+2b\neq 2b+3a$

$\Rightarrow a*b\neq b*a$

The operation $*$ is not commutative

Associative

$(a*b)*c=(2a+3b)*c$

$=2(2a+3b)+3c$

$=4a+6b+3c$

$a*(b*c)=a*(2b+3c)$

$=2a+6b+9c$

$\therefore (a*b)*c\neq a*(b*c)$

$\Rightarrow$ The operation is not associative.

17. The operation on N is defined by

$a*b=L.C.M,\{a,b\}$

For $a,b\in N$ $L.C.M$ of $\{a,b\}\in N$

$\Rightarrow a*b\in N$

The operation $*$ is a binary operation.

Commutative

For $a,b\in N$

$a*b=L.C.M\{a,b\}$

$=L.C.M\{b,a\}$

$=b*a$

$\Rightarrow$ The operation $*$ is commutative.

Associative

For $a,b,c\in N$

$(a*b)*c=L.C.M\{a,b\}*c$

$=L.C.M$ of $\{a,b,c\}$

$a*(b*c)=a*L.C.M$ of $\{b,c\}$

$=L.C.M\{a,b,c\}$

$\therefore (a*b)*c=a*(b*c)$

The operation $*$ is associative.

Group - C

Long Type Questions

1. Prove that for any $f: X \to Y$, $f \ o \ id_x = f = id_y$ of

2. Test whether the relation

   $R = \{(m,n) : 2 | (m+n)\}$ on z is reflexive, symmetric or transitive.

3. Let R be a relation on the set R of real numbers such that aRb if $a-b$ is an integer. Test whether R is an equivalence relation.

4. Let $\sim$ be defined by $(m,n) \sim (p,q)$ of $mq = np$ where $m, n, p, q \in z - \{0\}$. Show that $\sim$ is an equivalence relation.

5. Test where the relation R defined by $R = \left\{(m,n) : \frac{m}{n} \, is \ a \ power \ of \ 5\right\}$ on the set $Z - \{0\}$ is an equivalence relation?

6. If R and S are two equivalence relation on a set then prove that $R \cap S$ is also an equivalence relation on the set.

7. If m and n are integers and integers and $f(m,n)$ is defined by

   $$f(m,n) = \begin{cases} 5 \, if \ m < n \\ f(m-n, n+2) + m \, if \ m \geq n \end{cases}$$

   Then find $f(5,3)$

8. Prove that the inverse of $f(x) = x^2 - 1$ does not exist in general,

   But $f : [0,\infty) \to [-1,\infty)$ has an inverse given by $f^{-1}(x) = \sqrt{x+1}$ and $f^{-1}[-1,\infty) \to [0,\infty)$

9. Let $f(x) = \sqrt{x}, g(x) = 1 - x^2$. Compute fog and gof and find their natural domain.

10. Let $f : R \to R$ be defined by $f(x) = 3x + 7$ show that f is invertible and find f^{-1}.

11. Prove that $f : X \to Y$ is injective if for all subsets A, B of X,

    $f(A \cap B) = f(A) \cap f(B)$

12. If $f : X \to Y$ and $g : Y \to Z$ are two functions, show that gof is invertible if each of f and g are so and then

    $(gof)^{-1} = f^{-1} o \, g^{-1}$.

13. If $\mathrm{f : A \to B}$ and $\mathrm{g : B \to A}$ such that gof is an identity function of A and fog is an identity function mB, then show that $\mathrm{g = f^{-1}}$.

14. If p is a prime and ab = (mod p) then show that either a = 0 (mod p) or b = 0 (mod p).

Solutions

1. Given that $f : X \to Y$

   For each $x \in X$, there exist $y \in Y$ such that $y = f(x)$

   $(f \, o \, id_x)(x) = f \, o \, (f^{-1} of)(x)$

   $[\because id_x = fof^{-1}]$

   $= (f \, o \, f^{-1}) f(x)$

   $= f(x) = y$ ....(1)

$(id_y o f)(x) = (f o f^{-1}) o f(x)$

$= (f o f^{-1})(y)$

$= f[f^{-1}(y)]$

$= f(x) = y$ ....(2)

From (1) and (2), we get

$f o id_x = f = id_x o f$

2. The given relation is

$R = \{(m,n) : 2 | (m+n)\}$ on Z

$\therefore mRn \Rightarrow 2 | (m,n)\ for\ m,n \in Z$

$\Rightarrow m+n$ is divisible by 2

$\Rightarrow m+n = 2k$ where $k \in Z$

Reflexive

For all $m \in Z$

$m+m = 2m$ which is divisible by 2

$\Rightarrow mRm$ is true for all $m \in Z$

for all $m \in Z$,

$\Rightarrow$ The relation R is reflexive

Symmetric

For $m,n \in Z$,

$mRn \Rightarrow 2 | (m+n)$ for $m,n \in Z$

$\Rightarrow 2$ divides $m+n$

$\Rightarrow m+n = 2k$ where $k \in Z$

$\Rightarrow n+m = 2k$

$\Rightarrow 2$ divides $n+m$

$\Rightarrow nRm$

So the relation R is symmetric.

Transitive:

For $m,n,p \in Z$

$mRn \Rightarrow 2 | (m+n)$ for $m,n \in Z$

$\Rightarrow 2$ divides $m+n$

$\Rightarrow m+n = 2k$ where $k \in Z$ .....(1)

$nRp \Rightarrow 2 | (n+p)$ for $n,p \in Z$

$\Rightarrow 2$ divides $n+p$

$\Rightarrow n+p = 2k_1$ where $k_1 \in Z$ .....(2)

Adding (1) and (2), we get $m+n+n+p = 2k+2k_1-2n$

$\Rightarrow m+p = 2k+2k_1-2n$

$= 2(k+k_1-n)$

$\Rightarrow 2$ divides $m+p$

$\Rightarrow mRp$

$\therefore mRn, nRp \Rightarrow mRp$

So the relation R on the set is transitive

3. The set of real numbers is R.

For $a,b \in R$, the relation R on the set R is defined by

`

$aRb \Rightarrow a-b$ is an integer

$\Rightarrow a-b = k$ where k is an integer.

We shall test whether R is an equvalence relation.

Reflexive :

For all $a \in R$,

$a-a = 0$

$\Rightarrow aRa$ is true

$\Rightarrow R$ is reflexive

Symmetric

For $a,b, \in R$

$aRb \Rightarrow a-b$ is an integer

$\Rightarrow a-b = k$ where k is integer

$\Rightarrow b-a = -k$ whrere $-k$ integer

$\Rightarrow bRa$

So the relation R is symmetric.

Transitive

$aRb \Rightarrow a-b$ is an integer

$\Rightarrow a-b = k$ where k is an integer

.......(1)

$bRc \Rightarrow b-c$ is an integer

$\Rightarrow b-c=k_1$ where k_1 is an integer ....(2)

Adding (1) and (2), we get

$(a-b)+(b-c)=k+k_1$

$\Rightarrow a-c=k+k_1$ which is an integer

$\Rightarrow aRc$.

$\therefore aRb, bRc \Rightarrow aRc$

So R is transitive.

Since R is reflexive, symmetric and transitive it is an equivelence relation.

4. The given set is $A=Z-\{0\}$.

= The set of all non-zero integers.

$Let\ X = A\times A=\{(x,y):x,y\in A\}$

Reflexive

For all $(m,n)\in X$,

$(m,n)\sim(m,n)$ as $mn=nm$

So the relation '$\sim$' is reflexive.

Symmetric

For $(m,n),(p,q)\in X$

$(m,n)\sim(p,q)\Rightarrow mq=np$

$\Rightarrow pn=qm$

$\Rightarrow (p,q)\sim(m,n)$

So the relation R is symmetric

Transitive:

For $(m,n),(p,q),(r,s)\in X$

For $(m,n)\sim(p,q)\Rightarrow mq=np$ ......(1)

$(p,q)\sim(r,s)\Rightarrow ps=qr$ ....(2)

Multiplying (1) and (2), we get

$mq.ps=mp.qr$

$\Rightarrow (m,n)\sim(r,s)$

$\therefore (m,n)\sim(p,q),\ (p,q)\sim(r,s)$

$\Rightarrow (m,n)\sim(r,s)$

So the relation '$\sim$' is transitive.

5. The given set is $A=Z-\{0\}$

The given relation is

$$R=\left\{(m,n):\frac{m}{n} is\ a\ power\ 5\right\}$$

where $m,n\in A$.

Reflexive

For all $m\in A$, $\frac{m}{m}=1$

$\Rightarrow \frac{m}{m}=5^0$

$\Rightarrow (m,n)\in R$

$\Rightarrow R$ is reflexive.

Symmetric

For $(m,n)\in R\Rightarrow \frac{m}{n}=5^k$ where $k\in Z$

$\Rightarrow \frac{n}{m}=5^{-k}$

$\Rightarrow (n,m)\in R$

$\Rightarrow R$ is symmetric

Transitive

For $(m,n),(n,p)\in R$

$\frac{m}{n}\in 5^k$ and $\frac{n}{p}\in 5^{k_1}$

$\Rightarrow \frac{m}{n}.\frac{n}{p}=5^k.5^{k_{1a}}$

$\frac{m}{p}=5^{k+k_1}$

$\Rightarrow (m,p)\in R$

So R is an equivalence relation.

6. Worked out example from the text book

7. Given that

$$f(m,n)=\begin{cases} 5 \, if \; m<n \\ f(m-n,n+2)+m \, if \; m\ge n \end{cases}$$

$\therefore f(5,3)=f(5-3,3+2)+5$

$=f(2,5)+5$

$=5+5=10$

8. Given function is $f:R\to R$ and is defined by $f(x)=x^2-1$

Let $x_1,x_2\in R,\ f(x_1)=f(x_2)$

$\Rightarrow x_1^2-1=x_2^2-1$

$\Rightarrow x_1=\pm x_2$

$\Rightarrow x_1\ne x_2$

So f is not one-one

$\Rightarrow f^{-1}$ does not exist.

Let the function $f:(0,\infty)\to[1,\infty]$ and is defined by $f(x)=x^2-1$

For $x_1,x_2\in[0,\infty),\quad f(x_1)=f(x_2)$

$\Rightarrow x_1^2-1=x_2^2-1$

$\Rightarrow x_1^2=x_2^2$

$\Rightarrow x_1=x_2$

(Such x_1 and x_2 are both positive so -ve sign rejected)

$\Rightarrow f$ is one-one.

Let $y=f(x)=x^2-1$ where $y\in[-1,\infty)$ and is defined by $f(x)=x^2-1$

For $x_1,x_2\in[0,\infty),\quad f(x_1)=f(x_2)$

$\Rightarrow x_1^2-1=x_2^2-1$

$\Rightarrow x_1^2=x_2^2$

$\Rightarrow x_1=x_2$

(Since x_1 and x_2 are both positive so -ve sign is rejected)

$\Rightarrow f$ is one-one

Let $y=f(x)=x^2-1$ where $y\in[-1,\infty)$

$\Rightarrow x^2=y+1$

$\Rightarrow x=\sqrt{y+1}$

Here we see that each $y\in[-1,\infty)$ is the image of $x\in[0,\infty)$. Hence f is on to Since f is one-one and on to, its inverse exists.

Its inverse is $f^{-1}:[-1,\infty)\to(0,\infty)$ and is defined by $f^{-1}(x)=\sqrt{x+1}$.

10. To prove the invertibility of the function f, we have to prove that f is a bijection.

f **__is one-one:__** Let $x,y\in R$

$\therefore f(x)=f(y)\Rightarrow 3x+7=3y+7$

$\Rightarrow 3x=3y$

$\Rightarrow x=y$

Then $f(x)=f(y)\Rightarrow x=y$ for all $x,y\in R$

so f is one-one

f __is into:__

Let y be any arbitrary element of R.

Then $f(x)=y$

$\Rightarrow 3x+7=y$

$\Rightarrow x=\dfrac{y-7}{3}$

Clearly $\dfrac{y-7}{3}\in R$ for all $y\in R$

They for all $y\in R$,

there exists $x=\dfrac{y-7}{3}\in R$

such that $f(x)=f\left(\dfrac{y-7}{3}\right)$

$=3\left(\dfrac{y-7}{3}\right)+7=y$

So f is on to.

$\Rightarrow f$ is a bijection

$\Rightarrow f$ is invertible

$\therefore f^{-1}\ R\to R$ given by $f^{-1}(x)=\dfrac{x-7}{3}$

11. Given that $f : X \to Y$ is injective.

Let A and B are subsets of X.

$Let\ f(x) \in f(A \cap B)$

$\Leftrightarrow x \in A \cap B$

$\Leftrightarrow x \in A$ and $x \in B$

$\Leftrightarrow f(x) \in f(A)$ and $f(x) \in f(B)$

$\Leftrightarrow f(x) \in f(A) \cap f(B)$

So $f(A \cap B) = f(A) \cap f(B)$

12. Two given functions are $f : X \to Y$ and $g : Y \to Z$.

First we shall show that if f and g are invertible then gof is also invertible.

i.e if f and g are one-one and m to then gof is also one-one and on to.

(i) Let f and g are one-one functions

Let $x_1, x_2 \in X$

$\therefore (gof)(x_1) = (gof)(x_2)$

$\Rightarrow g[f(x_1)] = g[f(x_2)]$

$\Rightarrow f(x_1) = f(x_2)$

$\Rightarrow x_1 = x_2 \qquad (\because f \in g\ are\ one)$

Then for $x_1, x_2 \in X$

$(gof)(x_1) = (gof)(x_2)$

$\Rightarrow x_1 = x_2$

So gof is one-one

(ii) Let f and g be two on to functions

Let $z \in Z$,

Given that $g : Y \to Z$ be an on to function.

So there exists an element $y \in Y$ such that $g(y) = z$.

Since $f : X \to Y$ be on to, there exists an element $x \in X$ such that $f(x) = y$.

Now $g(y) = z$

$\Rightarrow g[f(x)] = z$

$\Rightarrow (gof)(x) = z$

Then for any element $z \in Z$ there exists $x \in X$ such that $(gof)(x) = z$

$\Rightarrow gof : X \to Z$ is an on to function.

Then we see that if f and g are one-one and on to, then gof is also an one-one and on to function.

So gof is invertible.

$\Rightarrow (gof)^{-1}$ exists

Here $f : X \to Y$ is bijective.

So there exists $y \in Y$ such that $f(x) = y$

$\Rightarrow x = f^{-1}(y) \qquad$ ....(1)

As an $g : Y \to Z$ is bijective.

So there exists an element $z \in Z$ such that $g(y) = z$

$\Rightarrow y = g^{-1}(z) \qquad$ ....(2)

$(gof)(x) = g[f(x)) = g(y) = 2$

$\Rightarrow x = (gof)^{-1} z. \qquad$ ....(3)

Also $x = f^{-1}(4) = f^{-1}[g^{-1}(2)]$

$= (f^{-1} o\, g^{-1})(z) \qquad$ ......(4)

From (3) and (4), we have

$(gof)^{-1} = f^{-1} o\, g^{-1}$

13. Given that g of I_A and fog = I_B.

We hae to show that g = f_{-1}.

First we shall have to show that f is invertible i.e.

We shall show that f is one-one and onto.

For $x_1, x_2 \in A,\ f(x) = f(x_2)$

$\Rightarrow g[f(x_1) = g[f(x_2)]$

$\Rightarrow (gof)(x_1) = (gof)(x_2)$

$\Rightarrow I_A(x_1) = I_B(x_2)$

$\Rightarrow x_1 = x_2$

This shows that f is one-one.

Again let $y \in B$ and let $g(y) = x$

Here $g(y) = x$

$\Rightarrow f[g(y)] = f(x)$

$\Rightarrow (fog)(y) = f(x)$

$\Rightarrow I_B(y) = f(x)$

$\Rightarrow y = f(x)$

$[\because I_B(y) = y]$

For each $y \in B$ there exists an $x \in A$ such that $f(x) = y$.

So f is on to.

$\Rightarrow$ f is invertible

Now

$(fog) = I_B \Rightarrow f^{-1}0(fog] = f^{-1}$ of $B = f^{-1}$

$\Rightarrow (f^{-1}of)og = f^{-1}$

$\Rightarrow I_A og = f^{-1}$

$\Rightarrow g = f^{-1}$.

14. Given that P is prime.

Also given that ab=0 (mod p)

$\Rightarrow$ p divides ab

$\Rightarrow$ There exists $c \in z$ such that ab = pc ...(1)

We shall show that either

$a \equiv 0 \pmod p$ or $b \equiv 0 \pmod p$

If possible let $a \neq 0 \pmod p$

$\Rightarrow (p, q) = 1$

$\because$ p is prime and a is relatively prime, g.c. f of p p and a is 1.

$\Rightarrow$ There exists m, n $\in$ Z such that pm tan=1

Multiplying both sides by b, we get

bm b tan b = b

$\Rightarrow$ bmp tan b - b

$\Rightarrow$ bmp + pc n = b

$\Rightarrow$ p (mp + cn) = b

$\Rightarrow$ p divides b

$\Rightarrow$ b $\equiv$ 0 ((mod p)

Similalry if b = 0 (mod p), then we can show that a $\equiv$ 0 (mod p)

So either a = o (mod p) or b = 0 (mod p)

CHAPTER - 2

INVERSE TRIGONMETRIC FUNCTIONS

Group - A

A. Choose the correct answer from the given alternatives:(MCQ Questions)

1. If $x+y+z=xyz$ then $\tan^{-1}x+\tan^{-1}y+\tan^{-1}z$?

(a) $\frac{\pi}{2}$ (b) 0

(c) $\frac{\pi}{4}$ (d) $\frac{\pi}{6}$

2. The value of $\sin^{-1}\frac{3}{5}+\tan^{-1}\frac{1}{7}=?$

(a) π (b) $\frac{\pi}{2}$

(c) $\frac{\pi}{3}$ (d) $\frac{\pi}{4}$

3. If $xy+yz+zx=1$ then $\tan^{-1}x+\tan^{-1}y+\tan^{-1}z$ is.

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{3}$

(c) $\frac{\pi}{2}$ (d) π

4. A solution of the equation $\tan^{-1}(1+x)+\tan^{-1}(1-x)=\frac{\pi}{2}$ is

(a) $x=0$ (b) $x=1$

(c) $x=-1$ (d) $x=\pi$

5. The value of $\sin(\cos^{-1}x)$ is

(a) $\frac{x}{\sqrt{1+x^2}}$ (b) $\frac{1}{\sqrt{1+x^2}}$

(c) $\frac{x}{1+x^2}$ (d) $\frac{1}{1+x^2}$

6. The solution of $\sin^{-1}\left(\frac{2a}{1+a^2}\right)-\cos^{-1}\left(\frac{1-b^2}{1+b^2}\right)=\tan^{-1}\left(\frac{2x}{1-x^2}\right)$ is?

(a) $\frac{a-b}{1-ab}$ (b) $\frac{a-b}{1+ab}$

(c) $\frac{1+ab}{a-b}$ (d) $\frac{1-ab}{a-b}$

7. If $\sin^{-1}\frac{x}{5}+\operatorname{cosec}^{-1}\left(\frac{5}{4}\right)=\frac{\pi}{2}$ then $x=?$

(a) 1 (b) 2

(c) 3 (d) 4

8. The value of $2\tan^{-1}\frac{1}{3}+\tan^{-1}\frac{1}{7}$ is

(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{3}$

(c) $\frac{\pi}{4}$ (d) $\frac{\pi}{6}$

9. The value of $\tan^{-1}\frac{x}{y}-\tan^{-1}\left(\frac{x-y}{x+y}\right)$ is

(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{4}$

(c) $\frac{\pi}{2}$ (d) π

10. If $\sin^{-1} x + \sin^{-1} y = \frac{2\pi}{3}$ then $\cos^{-1} x + \cos^{-1} y = ?$

(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{4}$

(c) $\frac{\pi}{3}$ (d) $\frac{\pi}{2}$

11. If $A = \tan^{-1} x$ then the value of $\sin 2x$ is

(a) $\frac{2x}{1-x^2}$ (b) $\frac{2x}{\sqrt{1-x^2}}$

(c) $\frac{2x}{1+x^2}$ (d) $\frac{2x}{\sqrt{1+x^2}}$

12. If the value of $\sin^{-1} x = \frac{\pi}{5}$ for some $x \in (-1,1)$ then the value of $\cos^{-1} x$ is?

(a) $\frac{3\pi}{10}$ (b) $\frac{5\pi}{10}$

(c) $\frac{7\pi}{10}$ (d) $\frac{9\pi}{10}$

13. The value of $\tan^{-1}\left(2\cos\frac{\pi}{3}\right)$ is?

(a) 1 (b) $\frac{\pi}{4}$

(c) $\frac{\pi}{3}$ (d) $\frac{\pi}{2}$

14. If $x + y = 4, xy = 1$ then $\tan^{-1} x + \tan^{-1} y = ?$

(a) $\frac{3\pi}{4}$ (b) $\frac{\pi}{4}$

(c) $\frac{\pi}{2}$ (d) $\frac{\pi}{6}$

15. The value of $\cot^{-1} 2 + \tan^{-1}\frac{1}{3} = ?$

(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{4}$

(c) $\frac{\pi}{2}$ (d) 1

16. The principal value of $\sin^{-1}\left(\sin\frac{2\pi}{3}\right)$ is?

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{3}$

(c) $\frac{2\pi}{3}$ (d) $\frac{4\pi}{3}$

17. If $\sin^{-1}\frac{x}{5} + \operatorname{cosec}^{-1}\frac{5}{4} = \frac{\pi}{2}$, then value of x is

(a) 2 (b) 3

(c) 4 (d) 5

18. The value of $\sin\left(\tan^{-1} x + \tan^{-1}\frac{1}{x}\right)$, $x > 0$ is ?

(a) 0 (b) 1

(c) $\frac{1}{2}$ (d) $\frac{1}{3}$

19. $\cot^{-1}\left[\frac{\sqrt{1-\sin x}+\sqrt{1+\sin x}}{\sqrt{1-\sin x}-\sqrt{1+\sin x}}\right] = ?$

(a) $\pi - \frac{x}{2}$ (b) $\frac{x}{2}$

(c) $2\pi - \frac{x}{2}$ (d) $\pi + \frac{x}{2}$

20. $2\sin^{-1}\frac{4}{5} + \sin\frac{24}{25} = ?$

(a) $-\pi$ (b) π

(c) $\frac{\pi}{2}$ (d) $-\frac{\pi}{2}$

21. $\sec^2(\tan^{-1}2)+\operatorname{cosec}^2(\cot^{-1}3)=?$

(a) 13 (b) 14

(c) 15 (d) 16

22. $\sin(\tan^{-1}x)$ is ?

(a) $\frac{x}{1+x^2}$ (b) $\frac{x}{\sqrt{1+x^2}}$

(c) $\frac{x}{\sqrt{1-x^2}}$ (d) $\frac{1}{\sqrt{1-x^2}}$

23. $\cos(\tan^{-1}x)$ is ?

(a) $\frac{1}{\sqrt{1+x^2}}$ (b) $\frac{1}{\sqrt{1-x^2}}$

(c) $\frac{x}{\sqrt{1+x^2}}$ (d) $\frac{x}{\sqrt{1-x^2}}$

24. $\cos(2\cos^{-1}x)$ is ?

(a) $2x^2+1$ (b) $2x^2-1$

(c) $\frac{1}{2x^2+1}$ (d) $\frac{1}{2x^2-1}$

25. $\tan^{-1}\frac{3}{2}+\tan^{-1}\frac{2}{3}=?$

(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{4}$

(c) $\frac{\pi}{3}$ (d) $\frac{\pi}{2}$

26. $2\tan^{-1}\frac{1}{3}+\cot^{-1}7=?$

(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{4}$

(c) $\frac{\pi}{3}$ (d) $\frac{\pi}{2}$

27. If $\cos^{-1}x+\cos^{-1}y+\cos^{-1}z=\pi$ then $x^2+y^2+z^2+2xyz$ is

(a) xyz (b) $2xyz$

(c) $3xyz$ (d) 1

28. The value of $\cos(2\tan^{-1}x), 0\le x<\infty$ is?

(a) $\frac{1-x^2}{1+x^2}$ (b) $\frac{1+x^2}{1-x^2}$

(c) $\frac{2x}{1-x^2}$ (d) $\frac{2x}{1+x^2}$

29. The value of $\tan^{-1}\frac{3}{5}+\cot^{-1}\frac{5}{4}$ is

(a) $\tan^{-1}1$ (b) $\tan^{-1}\frac{35}{13}$

(c) $\tan^{-1}\frac{13}{25}$ (d) $\tan^{-1}\frac{1}{2}$

30. The value of $sin^{-1}\frac{1}{2}+\sin^{-1}\frac{\sqrt{3}}{2}$ is

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{3}$

(c) $\frac{\pi}{2}$ (d) π

31. The value of $\tan\left(2\tan^{-1}\frac{1}{5}-\frac{\pi}{4}\right)$ is

(a) $\frac{7}{17}$ (b) $-\frac{7}{17}$

(c) $\frac{17}{7}$ (d) $-\frac{17}{7}$

32. If $x+\frac{1}{x}=2$ then the principal of $\sin^{-1}x$ is?

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{2}$

(c) π (d) $\frac{2\pi}{3}$

33. Write the value of $\cos\left(\sin^{-1}\frac{1}{2}\right)$

(a) $\frac{1}{2}$ (b) $\frac{\sqrt{3}}{2}$

(c) $\frac{1}{\sqrt{2}}$ (d) $\sqrt{3}$

34. The range of $\cos^{-1}$ is

(a) $\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$ (b) $[0,\pi]-\left\{\frac{\pi}{2}\right\}$

(c) $[0,\pi]$ (d) $[-\pi,\pi]$

35. The value of $\cos\tan^{-1}\cot\cos^{-1}\frac{\sqrt{3}}{2}$ is ?

(a) $\frac{1}{3}$ (b) $\frac{1}{2}$

(c) 1 (d) $\frac{3}{2}$

36. The value of $\sin^{-1}\left[\cos\left(\frac{33\pi}{5}\right)\right]$ is ?

(a) $-\frac{\pi}{10}$ (b) $\frac{\pi}{10}$

(c) $\frac{3\pi}{10}$ (d) $\frac{-3\pi}{10}$

37. The value of $\sin\left(2\sin^{-1}0.8\right)$ is?

(a) 0.66 (b) 0.76

(c) 0.86 (d) 0.96

38. The principal value of $\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)$ is?

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{3}$

(c) $\frac{-\pi}{3}$ (d) $\frac{\pi}{4}$

39. The value of $\cot\left(\cos^{-1}\frac{7}{25}\right)$ is?

(a) $-\frac{7}{24}$ (b) $\frac{7}{24}$

(c) $\frac{24}{7}$ (d) $-\frac{24}{7}$

40. The value of $\cot\left[\cos^{-1}\left(-\frac{1}{3}\right)\right]$ is?

(a) $\frac{1}{2\sqrt{3}}$ (b) $-\frac{1}{2\sqrt{2}}$

(c) $-\frac{1}{\sqrt{2}}$ (d) $\frac{1}{\sqrt{2}}$

41. The value of $\cot\left(\frac{\pi}{4}-2\cot^{-1}3\right)$ is?

(a) 5 (b) 6

(c) 7 (d) 8

42. If $\tan(x+y)=33$ and $x=\tan^{-1}3$ then the value of y is

(a) $\tan^{-1}(0.5)$ (b) $\tan^{-1}(0.4)$

(c) $\tan^{-1}(0.2)$ (d) $\tan^{-1}(0.3)$

43. Find the value of

$\sin\left[\cot^{-1}\left\{\tan\left(\cos^{-1}x\right)\right\}\right]$

(a) $\sin x$ (b) x

(c) $\cos x$ (d) $\tan x$

44. $\cos\left[2\cos^{-1}(0.8)\right]$ is ?

(a) 2.8 (b) 0.28

(c) 0.028 (d) 0.82

45. The value of $2\sin^{-1}\frac{1}{2}+\cos^{-1}\frac{1}{2}$ is

(a) $\frac{2\pi}{3}$ (b) $\frac{\pi}{2}$

(c) $\frac{\pi}{3}$ (d) $\frac{\pi}{4}$

46. If $\cot^{-1}x+\cot^{-1}y+\cot^{-1}z=\frac{\pi}{2}$ then the value of $x+y+z$ is

(a) $3xyz$ (b) $2xyz$

(d) xyz (d) $4xyz$

47. The value of $\sin\left[\cos^{-1}\left\{\tan\left(\cos^{-1}x\right)\right\}\right]$ is?

(a) x (b) $2x$

(c) $3x$ (d) $4x$

48. If $r^2 = x^2 + y^2 + z^2$ then the value of $\tan^{-1}\left(\frac{yz}{xr}\right) + \tan^{-1}\left(\frac{xz}{yr}\right) + \tan^{-1}\left(\frac{xy}{zr}\right)$ is

(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{4}$

(c) $\frac{\pi}{3}$ (d) $\frac{\pi}{2}$

49. The value of $\sin\left[\frac{1}{4}\sin^{-1}\frac{\sqrt{63}}{8}\right]$ is

(a) $\frac{1}{\sqrt{2}}$ (b) $\frac{1}{2\sqrt{2}}$

(c) $\frac{1}{3\sqrt{2}}$ (d) $\frac{1}{4\sqrt{2}}$

50. The value of

$\tan\left[\frac{1}{2}\sin^{-1}\frac{2x}{1+x^2} + \frac{1}{2}\cos^{-1}\frac{1-y^2}{1+y^2}\right]$ is

(a) $\frac{x-y}{1-xy}$ (b) $\frac{x-y}{1+xy}$

(c) $\frac{x+y}{1-xy}$ (d) $\frac{x+y}{1+xy}$

Answers

1. (b)	2. (d)	3. (c)	4. (a)
5. (b)	6. (b)	7. (c)	8. (c)
9. (b)	10. (c)	11. (a)	12. (a)
13. (b)	14. (c)	15. (b)	16. (b)
17. (b)	18. (b)	19. (a)	20. (b)
21. (c)	22. (b)	23. (a)	24. (b)
25. (d)	26. (b)	27. (d)	28. (a)
29. (b)	30. (c)	31. (b)	32. (b)
33. (b)	34. (c)	35. (b)	36. (a)
37. (d)	38. (c)	39. (b)	40. (b)
41. (c)	42. (d)	43. (b)	44. (b)
45. (a)	46. (c)	47. (a)	48. (d)
49. (b)	50. (c)		

B. Fill in the blanks

1. The value of $\tan\left(2\tan^{-1}\frac{1}{5}-\frac{\pi}{4}\right)=$ ______

2. The value of $\tan^{-1}1+\tan^{-1}2+\tan^{-1}3$ $=$ ______

3. $\tan^{-1}\frac{3}{2}+\tan^{-1}\frac{2}{3}=$ ______

4. If $\sin^{-1}x=\frac{\pi}{5}$ for some $x\in(-1,1)$, then the value of $\cos^{-1}x$ is ______

5. The principal value of $\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)$ is ______.

6. The value of $4\tan^{-1}\frac{1}{5}-\tan^{-1}\frac{1}{239}$ is ______.

7. If $A=\tan^{-1}x$ then the value of $\sin 2A=$ ______

8. If $\sin(\sin^{-1}\frac{1}{5}+\cos^{-1}x)=1$ then the value of x is ______

9. The value of $\sin\left(2\sin^{-1}0.8\right)$ is ______

10. The value of $\cot\left(\cos^{-1}\frac{7}{25}\right)$ is ______

11. The value of $\cos^{-1}\frac{1}{2}+2\sin^{-1}\frac{1}{2}$ is ______

12. The value of $\sin^{-1}\left[\cos\left(\frac{33\pi}{5}\right)\right]$ is ______

13. The value of $\cos\tan^{-1}\cot\cos^{-1}\frac{\sqrt{3}}{2}$ is ______

14. If the value of $\sin^{-1}x=\frac{\pi}{5}$ for same $x\in(-1,1)$ then the value of $\cos^{-1}x$ is______.

15. The value of $\tan^{-1}\left(2\cos\frac{\pi}{3}\right)=$ ______

16. If $x+y=4, xy=1$ then $\tan^{-1}x+\tan^{-1}y=$ ______

17. The value of $\cot^{-1}2+\tan^{-1}\frac{1}{3}$ is______

18. If $\sin^{-1}\frac{x}{5}+\operatorname{cos}ec^{-1}\frac{5}{4}=\frac{\pi}{2}$ then the value of x is ______

19. The value of $\sin\left(\tan^{-1}x+\tan^{-1}\frac{1}{x}\right), x>0$ is ______

20. $2\sin^{-1}\frac{4}{5}+\sin^{-1}\frac{24}{25}=$ ______

21. Using the principal values, the value of $\cos^{-1}\left(\cos\frac{13\pi}{16}\right)=$ ______

22. $sin\left(2sin^{-1}0.6\right)=$ ______

23. $\tan\left(\frac{\pi}{4}+2\cot^{-1}3\right)=$ ______

24. $\cos\left(2sin^{-1}x\right)=$ ______

25. $\tan^{-1}\left(\frac{x}{y}\right)-\tan\left(\frac{x-y}{x+y}\right)=$ ______

26. $\operatorname{cosec}\left(\cos^{-1}\frac{3}{5}+\cos^{-1}\frac{4}{5}\right)=$ ________

27. $sen^{-1}\frac{1}{\sqrt{5}}+\cos^{-1}\frac{3}{\sqrt{10}}=$ ________

28. $\sin\cos^{-1}\tan\sec^{-1}\sqrt{2}=$ ________

29. $\sin\left(2\tan^{-1}\sqrt{\frac{1-x}{1+x}}\right)=$ ________

30. $\sin\cot^{-1}\cos\tan^{-1}x=$ ________

Answers

1. $\frac{-7}{17}$	2. π	3. $\frac{\pi}{2}$	4. $\frac{3\pi}{10}$
5. $\frac{-\pi}{4}$	6. $\frac{\pi}{4}$	7. $\frac{2x}{1+x^2}$	8. $\frac{1}{5}$
9. 0.96	10. $\frac{7}{24}$	11. $\frac{2\pi}{3}$	12. $\frac{-\pi}{10}$
13. $\frac{1}{2}$	14. $\frac{3\pi}{10}$	15. $\frac{\pi}{4}$	16. $\frac{\pi}{2}$
17. $\frac{\pi}{4}$	18. 3	19. 1	20. π
21. $\frac{\pi}{6}$	22. 0.96	23. 7	24. $1-2x^2$
25. $\frac{\pi}{4}$	26. 1	27. $\frac{\pi}{4}$	28. 0
29. $\sqrt{1-x^2}$	30. $\sqrt{\frac{1+x^3}{2+x^2}}$		

C. Answer in one word

1. Write the value of $\cos\left(\sin^{-1}\frac{1}{2}\right)$
2. Write the range of $\cos^{-1}$.
3. Write the value of

$$\tan\left(\frac{\pi}{4}+\frac{1}{2}\cos^{-1}\frac{a}{b}\right)+\tan\left(\frac{\pi}{4}-\frac{1}{2}\cos^{-1}\frac{a}{b}\right)$$

4. Writ the value of $\cos^{-1}\left(\cos 3\frac{\pi}{2}\right)$
5. Write the principal value of

$$\sin^{-1}\left(-\frac{1}{2}\right)+\cos^{-1}\cos\left(-\frac{\pi}{2}\right)$$

6. What is the principal value of $\sin^{-1}\left(\sin\frac{2\pi}{3}\right)$?
7. What is the value of $\tan^{-1}\left(2\cos\frac{\pi}{3}\right)$?
8. If the value of $\sin^{-1}x=\frac{\pi}{5}$ for some $x\in[-1,1]$, then what is the value of $\cos^{-1}x$?
9. What is the value of $\cot^{-1}2+\tan^{-1}\frac{1}{3}$?
10. If $x+y=4, xy=1$, then what is the value of $\tan^{-1}x+\tan^{-1}y$?
11. If $\sin^{-1}\frac{x}{5}+\operatorname{cosec}^{-1}\frac{5}{4}=\frac{\pi}{2}$, then what is the value of x?
12. What is the value of

$$\sin\left(\tan^{-1}x+\tan^{-1}\frac{1}{x}\right), x>0 \ ?$$

13. What is the value of $2\sin^{-1}\frac{1}{2}+\cos^{-1}\frac{1}{2}$?
14. What is the value of

$$\sin\left[\cos^{-1}\left\{\tan\left(\cos^{-1}x\right)\right\}\right] \ ?$$

15. Find the value of $\cot\left(\frac{\pi}{4}-2\cos^{-1}3\right)$?

16. If $\tan^{-1}x+\tan^{-1}y=\frac{\pi}{4}$, $xy<1$, then write the value of $x+y+xy$.

17. If $\sin\left(\sin^{-1}\frac{1}{5}+\cos^{-1}x\right)=1$, then find the value of x

18. Write the value of $\cos^{-1}\left(-\frac{1}{2}\right)+2\sin^{-1}\frac{1}{2}$.

19. Write the principal value of $\tan^{-1}\left[\sin\left(\frac{-\pi}{2}\right)\right]$.

20. Write the principal value of $\cos^{-1}\cos(680)$.

Solutions

1. $\frac{\sqrt{3}}{2}$

2. $[0,\pi]$

3. $\frac{2b}{a}$

4. $\frac{\pi}{2}$

5. $-\frac{\pi}{6}+1$

Hints $\sin^{-1}\left(-\frac{1}{2}\right)+\cos^{-1}\cos\left(-\frac{\pi}{2}\right)$

$=-\frac{\pi}{6}+\cos^{-1}\cos\frac{\pi}{2}$

$=-\frac{\pi}{6}+\cos^{-1}0=-\frac{\pi}{6}+1$

6. $\frac{\pi}{3}$

Hints $\sin^{-1}\sin\frac{2\pi}{3}=\sin^{-1}\left(\sin\left(\pi-\frac{\pi}{3}\right)\right)$

$=\sin^{-1}\sin\frac{\pi}{3}$

$=\sin^{-1}\sqrt{3}=\frac{\pi}{3}$.

7. $\frac{\pi}{4}$

8. $\frac{3\pi}{10}$

Hints $\sin^{-1}x+\cos^{-1}x=\frac{\pi}{2}$

$\Rightarrow\frac{\pi}{5}+\cos^{-1}x=\frac{\pi}{2}$

$\Rightarrow\cos^{-1}x=\frac{\pi}{2}-\frac{\pi}{5}=\frac{3\pi}{10}$

9. $\frac{\pi}{4}$

10. $\frac{\pi}{2}$

Hints $\tan^{-1}x+\tan^{-1}y=\tan^{-1}\frac{x+y}{1-xy}$

$=\tan^{-1}\left(\frac{4}{1-1}\right)$

$=\tan^{-1}\infty=\frac{\pi}{2}$.

11. 3

Hints $\sin^{-1}\frac{x}{5}+\cos ec^{-1}\frac{5}{4}=\frac{\pi}{2}$

$\Rightarrow\sin^{-1}\frac{x}{5}+\sin^{-1}\frac{4}{5}=\frac{\pi}{2}$

$\Rightarrow\sin^{-1}\frac{x}{5}=\frac{\pi}{2}-\sin^{-1}\frac{4}{5}=\cos^{-1}\frac{4}{5}$

$\Rightarrow x=3$

12. 1

13. $\frac{2\pi}{3}$

Hints $2\sin^{-1}\frac{1}{2}+\cos^{-1}\frac{1}{2}=2.\frac{\pi}{6}+\frac{\pi}{3}$

$=\frac{\pi}{3}+\frac{\pi}{3}=\frac{2\pi}{3}$

14. x

Hints $\sin\left[\cot^{-1}\left\{\tan\left(\cos^{-1}x\right)\right\}\right]$

$$=\sin\cot^{-1}\left[\tan\tan^{-1}\frac{\sqrt{1-x^2}}{x}\right]$$

$$=\sin\cot^{-1}\frac{\sqrt{1-x^2}}{x}$$

$$=\sin\sin^{-1}x=x$$

15. 7.

Hints: we know $2\cot^{-1}3=2\tan^{-1}\frac{1}{3}$

$$=\tan^{-1}\frac{2.\frac{1}{3}}{1-\frac{1}{9}}$$

$$=\tan^{-1}\frac{3}{4}=\cot^{-1}\frac{4}{3}$$

$$\therefore \cot\left(\frac{\pi}{4}-2\cot^{-1}3\right)=\cot\left(\frac{\pi}{4}-\cot^{-1}\frac{4}{3}\right)$$

$$=\frac{\cot\frac{\pi}{4}-\frac{4}{3}+1}{\frac{4}{3}-\cot\frac{\pi}{4}}$$

$$=\frac{\frac{4}{3}+1}{\frac{4}{3}-1}=7$$

16. 1

Hints: $\tan^{-1}x+\tan^{-1}y=\frac{\pi}{4}$

$$\Rightarrow \tan^{-1}\left(\frac{x+y}{1-xy}\right)=\frac{\pi}{4}$$

$$\Rightarrow \frac{x+y}{1-xy}=\tan\frac{\pi}{4}=1$$

$$\Rightarrow x+y=1-xy$$

$$\Rightarrow x+y+xy=1$$

17. $x=\frac{1}{5}$

Hints : $\sin\left(\sin^{-1}\frac{1}{5}+\cos^{-1}x\right)=1$

$$\Rightarrow \sin^{-1}\frac{1}{5}+\cos^{-1}x=\sin^{-1}1$$

$$\Rightarrow \sin^{-1}\frac{1}{5}+\cos^{-1}x=\frac{\pi}{2}$$

$$\Rightarrow \cos^{-1}x-=\frac{\pi}{2}-\sin^{-1}\frac{1}{5}=\cos^{-1}\frac{1}{5}$$

$$\Rightarrow x=\frac{1}{5}$$

18. π .

Hints $\cos^{-1}\left(\frac{-1}{2}\right)+2\sin^{-1}\frac{1}{2}$

$$=\pi-\cos^{-1}\frac{1}{2}+2\sin^{-1}\frac{1}{2}$$

$$=\pi-\frac{\pi}{3}+2.\frac{\pi}{6}=\pi\ .$$

19. $-\frac{\pi}{4}$

Hints: We know the principal value branch of

$\tan^{-1}$ is $\left(\frac{-\pi}{2},\frac{\pi}{2}\right)$

$$\therefore \tan^{-1}\sin\left(-\frac{\pi}{2}\right)=\tan^{-1}(-1)$$

$$=\tan^{-1}\left(-\tan\frac{\pi}{4}\right)$$

$$=\tan^{-1}\left[\tan\left(-\frac{\pi}{4}\right)\right]$$

$$=-\frac{\pi}{4}$$

20. 40.

Hints:

$$\cos^{-1}\left(\cos 680^0\right)=\cos^{-1}\left[\cos 2.360-40\right]$$

$$=\cos^{-1}\cos 40^0$$

$$=40^0$$

D. Answer in one sentence:

1. Write the principal value of $\cos^{-1}\frac{\sqrt{3}}{2}+\cos^{-1}\left(-\frac{1}{2}\right)$

2. Write the principal value of $\tan^{-1}\left(\frac{a}{b}\right)-\tan^{-1}\left(\frac{a-b}{a+b}\right)$

3. Write the principal value of $\tan^{-1}(1)+\cos^{-1}\left(\frac{-1}{2}\right)$

4. Write the value of $\tan^{-1}\left[2\sin\left(2\cos^{-1}\frac{\sqrt{3}}{2}\right)\right]$

5. Write the principal value of $\tan^{-1}\left(\sqrt{3}\right)-\cot^{-1}\left(-\sqrt{3}\right)$

6. What is the principal value of $\tan^{-1}\sqrt{3}-\sin^{-1}(-2)$?

7. If $x+y+z=xyz$ then what is the value of $\tan^{-1}x+\tan^{-1}y+\tan^{-1}z$?

8. Find the value of $\sin^{-1}\frac{3}{5}+\tan^{-1}\frac{1}{7}$

9. If $xy+yz+zx=1$, then what is the value of $\tan^{-1}x+\tan^{-1}y+\tan^{-1}z$?

10. What is the value of $\sin\left(2\sin^{-1}\frac{3}{5}\right)$?

11. Find the value of $\tan\left(\frac{\pi}{4}+2\cot^{-1}3\right)$.

12. Whata is the value of $\cos ec\left(\cos^{-1}\frac{3}{5}+\cos^{-1}\frac{4}{5}\right)$?

13. Write value of $\sin\cos^{-1}\tan\left(\sec^{-1}\sqrt{2}\right)$ in simplesst form.

14. What is the value of $\sin\cos^{-1}\cos\tan^{-1}x$?

15. What is the value of $2\tan^{-1}\frac{1}{3}+\tan^{-1}\frac{1}{7}$?

16. Write the principal value of $\cos^{-1}\left(\cos\frac{2\pi}{3}\right)+\sin^{-1}\left(\sin\frac{2\pi}{3}\right)$.

17. What is the principal value of $\tan^{-1}(-1)$?

18. Find the principal value of $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$.

19. Using the principal value evaluate $\tan^{-1}(1)+\sin^{-1}\left(-\frac{1}{2}\right)$.

20. Write the principal value of $\sin^{-1}\left(\sin\frac{3\pi}{5}\right)$.

<u>Solution</u>

1. The principal value of $\cos^{-1}\frac{\sqrt{3}}{2}+\cos^{-1}\left(-\frac{1}{2}\right)$ is $\frac{5\pi}{6}$

Hints: $\cos^{-1}\frac{\sqrt{3}}{2}+\cos^{-1}\left(-\frac{1}{2}\right)$

$$=\cos^{-1}\frac{\sqrt{3}}{2}+\left[\pi-\cos^{-1}\left(\frac{1}{2}\right)\right]$$

$$=\frac{\pi}{6}+\pi-\frac{\pi}{3}=\frac{5\pi}{6}$$

2. The principal value of $\tan^{-1}\left(\frac{a}{b}\right)-\tan^{-1}\left(\frac{a-b}{a+b}\right)$ is $\frac{\pi}{4}$

3. The principal value of $\tan^{-1}(1)+\cos^{-1}\left(-\frac{1}{2}\right)$ is $\frac{11\pi}{12}$.

Hints: $\tan^{-1}1+\cos^{-1}\left(-\frac{1}{2}\right)$

$$=\tan^{-1}\left(\tan\frac{\pi}{4}\right)+\pi-\cos^{-1}\frac{1}{2}$$

$$=\frac{\pi}{4}+\pi-\frac{\pi}{3}=\frac{11\pi}{12}$$

4. The value of

$$\tan^{-1}\left[2\sin\left(2\cos^{-1}\frac{\sqrt{3}}{2}\right)\right] \text{ is } \frac{\pi}{3}$$

Hints: $\tan^{-1}\left[2\sin\left(2\cos^{-1}\frac{\sqrt{3}}{2}\right)\right]$

$$= \tan^{-1}\left[2\sin\left\{\cos^{-1}\left(2.\frac{3}{4}-1\right)\right\}\right]$$

$$\left[\because 2\cos^{-1}x = \cos^{-1}\left(2x^2-1\right)\right]$$

$$= \tan^{-1}\left[2\sin\left(\cos^{-1}\frac{1}{2}\right)\right]$$

$$= \tan^{-1}\left(2\sin\frac{\pi}{3}\right)$$

$$= \tan^{-1}\left(2.\frac{\sqrt{3}}{2}\right) = \tan^{-1}\sqrt{3} = \frac{\pi}{3}$$

5. The principal value of $\tan^{-1}\left(\sqrt{3}\right) - \cos^{-1}\left(-\sqrt{3}\right)$ is $-\frac{\pi}{2}$

Hints: $\tan^{-1}\sqrt{3} - \cot^{-1}\left(-\sqrt{3}\right)$

$$= \tan^{-1}\sqrt{3} - \left[\pi - \cot^{-1}\sqrt{3}\right]$$

$$= \left(\tan^{-1}\sqrt{3} + \cot^{-1}\sqrt{3}\right) - \pi$$

$$= \frac{\pi}{2} - \pi = -\frac{\pi}{2}$$

6. The principal value of

$\tan^{-1}\sqrt{3} - \sec^{-1}(-2)$ is $-\frac{\pi}{3}$

7. If $x+y+z = xyz$ then the value of $\tan^{-1}x + \tan^{-1}y + \tan^{-1}z$ is 0.

8. The value of $\sin^{-1}\frac{3}{5} + \tan^{-1}\frac{1}{7}$ is $\frac{\pi}{4}$.

9. If $xy+yz+zx = 1$, then the value of $\tan^{-1}x + \tan^{-1}y + \tan^{-1}z = \frac{\pi}{2}$.

10. The value of $\sin\left(2\sin^{-1}\frac{3}{5}\right)$ is $\frac{24}{25}$

Hints:

$$\sin\left(2\sin^{-1}\frac{3}{5}\right) = \sin^{-1}\left[2.\frac{3}{5}\sqrt{1-\frac{9}{25}}\right]$$

$$[\because 2\sin^{-1}x = \sin^{-1}\left[2x\sqrt{1-x^2}\right]$$

$$= \frac{24}{25}$$

11. The value of

$\tan\left(\frac{\pi}{4} + 2\cot^{-1}3\right)$ is 7.

Hints:

$$\tan\left(\frac{\pi}{4} + 2\cot^{-1}3\right) = \tan\left(\frac{\pi}{4} + 2\tan^{-1}\frac{1}{3}\right)$$

$$= \tan[\frac{\pi}{4} + \tan^{-1}\left[\frac{2.\frac{1}{3}}{1-\frac{1}{9}}\right]$$

$$= \tan\left[\tan^{-1}1 + \tan^{-1}\frac{3}{4}\right]$$

$$= \tan\tan^{-1}\left[\frac{1+\frac{3}{4}}{1-1.\frac{3}{4}}\right]$$

$$= 7$$

12. The value of

$$\operatorname{cosec}\left(\cos^{-1}\frac{3}{5}+\cos^{-1}\frac{4}{5}\right) \text{ is } 1.$$

HInts: $\operatorname{cosec}\left[\cos^{-1}\frac{3}{5}+\cos^{-1}\frac{4}{5}\right]$

$$=\operatorname{cosec}\left[\cos^{-1}\frac{3}{5}+\sin^{-1}\sqrt{1-\frac{16}{25}}\right]$$

$$=\operatorname{cosec}\left[\cos^{-1}\frac{3}{5}+\sin^{-1}\frac{3}{5}\right]$$

$$=\operatorname{cosec}\frac{\pi}{2}=1$$

13. The value of $\sin^{-1}\cos^{-1}\tan\sec^{-1}\sqrt{2}$ in simplest form is 0.

14. The value of $\sin\cos^{-1}\cos\tan^{-1}x$ is $\sqrt{\frac{1+x^2}{2+x^2}}$.

15. The value of $2\tan^{-1}\frac{1}{3}+\tan^{-1}\frac{1}{7}$ is $\frac{\pi}{4}$.

16. The principal value of

$$\cos^{-1}\left(\cos\frac{2\pi}{3}\right)+\sin^{-1}\left(\sin\frac{2\pi}{3}\right) \text{ is } \pi$$

Hints: We know the principal value branch

$\sin^{-1}$ is $\left[\frac{-\pi}{2},\frac{\pi}{2}\right]$ and for $\cos^{-1}$ is $[0,\pi]$.

$$\because \cos^{-1}\cos\left(\frac{2\pi}{3}\right)+\sin^{-1}\left(\sin\frac{2\pi}{3}\right)$$

$$=\frac{2\pi}{3}+\sin^{-1}\sin\left(\pi-\frac{\pi}{3}\right)$$

$$=\frac{2\pi}{3}+\sin^{-1}\sin\frac{\pi}{3}$$

$$=\frac{2\pi}{3}+\frac{\pi}{3}=\pi$$

17. The principal value of $\tan^{-1}(-1)$ is $-\frac{\pi}{4}$

18. The principal value of $\cos^{-1}\frac{\sqrt{3}}{2}$ is $\frac{\pi}{6}$.

19. $\tan^{-1}(1)+\sin^{-1}\left(-\frac{1}{2}\right)$

$$=\tan^{-1}\left(\tan\frac{\pi}{4}\right)+\sin^{-1}\left[\sin\left(-\frac{\pi}{6}\right)\right]$$

$$=\frac{\pi}{4}-\frac{\pi}{6}=\frac{\pi}{12}$$

20. The principal value of $\sin^{-1}\left[\sin\left(\pi-\frac{2\pi}{5}\right)\right]$ is $\frac{2\pi}{5}$.

Hints:

$$\sin^{-1}\left(\sin\frac{3\pi}{5}\right)=\sin^{-1}\left[\sin\left(\pi-\frac{2\pi}{5}\right)\right]$$

$$=\sin^{-1}\sin\frac{2\pi}{5}$$

$$=\frac{2\pi}{5}.$$

Group - B

B. Short type Question & Answers:

1. Show that

$$\sin^{-1}\left(\frac{1}{\sqrt{10}}\right)+\cos^{-1}\left(\frac{2}{\sqrt{5}}\right)=\frac{\pi}{4}$$

2. Find the value of $\tan\left[\cos^{-1}\frac{4}{5}\tan^{-1}\left(\frac{2}{3}\right)\right]$

3. Prove that $\cot^{-1}9+\operatorname{cosec}^{-1}\left(\frac{\sqrt{41}}{4}\right)=\frac{\pi}{4}$

4. Show that $\sin^{-1}\frac{4}{5}+2\tan^{-1}\frac{1}{3}=\frac{\pi}{2}$

5. Show that

$$\tan^2\cos^{-1}\frac{1}{\sqrt{3}}+\cot^2\sin^{-1}\frac{1}{\sqrt{5}}=6$$

6. Solve

$$\sin^{-1}\frac{2a}{1+a^2}+\sin^{-1}\frac{2b}{1+b^2}=2\tan^{-1}x$$

7. Prove that

$$\sec^2\left(\tan^{-1}3\right)-\operatorname{cosec}^2\left(\cot^{-1}3\right)=0$$

8. Evaluate $\cos(2\cos^{-1}x+\sin^{-1}x)\ at\ x=\frac{1}{5}$

9. Prove that

$$\tan^{-1}\sqrt{x}=\frac{1}{2}\cos^{-1}\left(\frac{1-x}{1+x}\right)\text{ when } x\in(0,1)$$

10. Prove that

$$\cos^{-1}\left(\frac{12}{13}\right)+\sin^{-1}\left(\frac{3}{5}\right)=\sin^{-1}\left(\frac{56}{65}\right)$$

11. Prove that

$$\tan^{-1}x+\tan^{-1}\left(\frac{2x}{1-x^2}\right)=\tan^{-1}\left(\frac{3x-x^2}{1-3x^2}\right)$$

12. Solve $\cos\left(\tan^{-1}x\right)=\sin\left(\cot^{-1}\frac{3}{4}\right)$

13. Solve $\tan^{-1}\left(\frac{1-x}{1+x}\right)=\frac{1}{2}\tan^{-1}x$

14. Solve $2\tan^{-1}\left(\cos x\right)=\tan^{-1}(2\operatorname{cosec} x)$

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15. Solve $\tan^{-1}x+2\cot^{-1}x=\frac{2\pi}{3}$

16. Prove that

$$\cos^{-1}x+\cos^{-1}\left[\frac{x}{2}+\frac{\sqrt{3-3x^2}}{2}\right]=\frac{\pi}{3}$$

17. Prove that

$$\cot^{-1}7+\cot^{-1}8+\cot^{-1}18=\cot^{-1}3$$

18. Prove that

$$\sin^{-1}\left(\frac{5}{13}\right)+\cos^{-1}\left(\frac{3}{5}\right)=\sin^{-1}\left(\frac{63}{65}\right)$$

19. Prove that

$$\tan^{-1}\frac{1}{4}+\tan^{-1}\frac{2}{9}=\frac{1}{2}.\tan^{-1}\frac{4}{3}$$

20. Solve

$$3\tan^{-1}\left(\frac{1}{2+\sqrt{3}}\right)-\tan^{-1}\frac{1}{x}=\tan^{-1}\frac{1}{3}$$

21. Prove that

$$\tan\left(\frac{\pi}{4}+\frac{1}{2}\cos^{-1}\frac{a}{b}\right)+\tan\left(\frac{\pi}{4}-\frac{1}{2}\cos^{-1}\frac{a}{b}\right)=\frac{2b}{a}$$

22. If $\cos^{-1}\left(\frac{x}{a}\right)+\cos^{-1}\left(\frac{y}{b}\right)=\theta$, then prove that $\frac{x^2}{a^2}-\frac{2xy}{ab}\cos\theta+\frac{y^2}{b^2}=\sin^2\theta$

23. If $\cos^{-1}x+\cos^{-1}y+\cos^{-1}z=\pi$ then prove that $x^2+y^2+z^2+2xyz=1$.

24. If $\sin^{-1}x+\sin^{-1}y+\sin^{-1}z=\pi$, then show that $x\sqrt{1-x^2}+y\sqrt{1-y^2}+z\sqrt{1-z^2}$ $=2xyz$

25. If $r=x+y+z$, then prove that $\tan^{-1}\sqrt{\frac{xr}{yz}}+\tan^{-1}\sqrt{\frac{yr}{zx}}+\tan^{-1}\sqrt{\frac{zr}{xy}}=\pi$

26. If $u=\cos^{-1}\sqrt{\cos\propto}-\tan^{-1}\sqrt{\cos\propto}$ then prove that $\sin u=\tan^2\frac{\propto}{2}$.

27. Prove that

$$\sin^{-1}\sqrt{\frac{x-q}{p-q}}=\cos^{-1}\sqrt{\frac{p-x}{p-q}}=$$

$$\tan^{-1}\sqrt{\frac{x-q}{p-x}}$$

28. In a $\Delta ABC, < A=90^0$, then prove that $\tan^{-1}\left(\frac{b}{a+c}\right)+\tan^{-1}\left(\frac{c}{a+b}\right)=\frac{\pi}{4}$ when a,b,c are sides of a triangle.

29. Prove that

$$\tan^{-1}x+\cot^{-1}(x-1)=\tan^{-1}(x^2+x+1)$$

30. Show that

$$4\left(\cot^{-1}\frac{3}{2}+\operatorname{cosec}^{-1}\sqrt{26}\right)=\pi$$

Solutions (Hints)

1. $\sin^{-1}\left(\frac{1}{\sqrt{10}}\right)+\cos^{-1}\left(\frac{2}{\sqrt{5}}\right)=\frac{\pi}{4}$

let $\sin^{-1}\left(\frac{1}{\sqrt{10}}\right)=\alpha, \cos^{-1}\left(\frac{2}{\sqrt{5}}\right)=\beta$

$\therefore \sin\alpha=\frac{1}{\sqrt{10}}, \cos\beta=\frac{2}{\sqrt{5}}$

$\cos\alpha=\sqrt{1-\sin^2\alpha}=\frac{3}{\sqrt{10}}$

$\therefore \tan\alpha=\frac{\sin\alpha}{\cos\alpha}=\frac{1}{3}$

$\alpha=\tan^{-1}\frac{1}{3}$

$\Rightarrow \sin^{-1}\left(\frac{1}{\sqrt{10}}\right)=\tan^{-1}\frac{1}{3}$

$\sin\beta=\sqrt{1-\cos^2\beta}=\frac{1}{\sqrt{5}}$

$\therefore \tan\beta=\frac{\sin\beta}{\cos\beta}=\frac{1/\sqrt{5}}{2/\sqrt{5}}=\frac{1}{2}$

$\therefore \beta=\tan^{-1}\frac{1}{2}$

$\Rightarrow \cos^{-1}\left(\frac{2}{\sqrt{5}}\right)=\tan^{-1}\frac{1}{2}$

L.HS $=\tan^{-1}\frac{1}{3}+\tan^{-1}\frac{1}{2}$

$=\tan^{-1}\frac{\frac{1}{3}+\frac{1}{2}}{1-\frac{1}{3}.\frac{1}{2}}=\tan^{-1}\left(\frac{5}{5}\right)$

$=\tan^{-1}1=\frac{\pi}{4}$

3. $\cot^{-1}9+\operatorname{cosec}^{-1}\frac{\sqrt{41}}{4}=\frac{\pi}{4}$

We know $\cot^{-1}9=\tan^{-1}\frac{1}{9}$

$Let \operatorname{cosec}^{-1}\left(\frac{\sqrt{41}}{4}\right)=\theta$

$\Rightarrow \operatorname{cosec}\theta=\frac{\sqrt{41}}{4}$

$\Rightarrow \sin\theta=\frac{4}{\sqrt{41}}$

$\therefore \theta=\sin^{-1}\frac{4}{\sqrt{41}}=\tan^{-1}\frac{\frac{4}{\sqrt{41}}}{\sqrt{1-\frac{16}{41}}}=\tan^{-1}\frac{4}{5}$

$\Rightarrow \operatorname{cosec}^{-1}\frac{\sqrt{41}}{4}\ \ \tan^{-1}\frac{4}{5}$

$L.H.S=\cot^{-1}9+\operatorname{cosec}^{-1}\frac{\sqrt{41}}{4}$

$=\tan^{-1}\frac{1}{9}+\tan^{-1}\frac{4}{5}$

$=\tan^{-1}\left(\frac{\frac{1}{9}+\frac{4}{5}}{1-\frac{1}{9}.\frac{4}{5}}\right)$

5. $\tan^2\left(\cos^{-1}\frac{1}{\sqrt{3}}\right)+\cot^2\left(\sin^{-1}\frac{1}{\sqrt{5}}\right)=6$

Let $\cos^{-1}\frac{1}{\sqrt{3}}=\alpha \Rightarrow \cos\alpha=\frac{1}{\sqrt{3}}$

$\sin\alpha=\sqrt{1-\cos^2\alpha}=\sqrt{1-\frac{1}{3}}=\sqrt{\frac{2}{3}}$

$\tan\alpha=\frac{\sin\alpha}{\cos\alpha}=\frac{\sqrt{\frac{2}{3}}}{\frac{1}{\sqrt{3}}}=\sqrt{2}$

$\Rightarrow \alpha=\tan^{-1}\sqrt{2}$

$\Rightarrow \cos^{-1}\frac{1}{\sqrt{3}}=\tan^{-1}\sqrt{2}$

Let $\sin^{-1}\frac{1}{\sqrt{5}}=\beta \Rightarrow \sin\beta=\frac{1}{\sqrt{5}}$

$\cos\beta=\sqrt{1-\sin^2\beta}=\sqrt{1-\frac{1}{5}}=\frac{2}{\sqrt{5}}$

$\therefore \cot\beta=\frac{\cos\beta}{\sin\beta}=2$

$\Rightarrow \beta=\cot^{-1}2$

$\Rightarrow \sin^{-1}\frac{1}{\sqrt{5}}=\cot^{-1}2$

L.H.S $=\tan^2\cos^{-1}\frac{1}{\sqrt{3}}+\cot^2\sin^{-1}\frac{1}{\sqrt{5}}$

$=\tan^2\tan^{-1}\sqrt{2}+\cot^2\cot^{-1}2$

$\left[\tan\tan^{-1}\sqrt{2}\right]^2+\left[\cot\cot^{-1}2\right]^2$

$=\left(\sqrt{2}\right)^2+2^2$

$=2+4=6$

6. $\sin^{-1}\frac{2a}{1+a^2}+\sin^{-1}\frac{2b}{1+b^2}=2\tan^{-1}x$

$\Rightarrow 2\tan^{-1}a+2\tan^{-1}b=2\tan^{-1}x$

$\Rightarrow \tan^{-1}x=\tan^{-1}a+\tan^{-1}b=\tan^{-1}\frac{a+b}{1-ab}$

$\Rightarrow x=\frac{a+b}{1-ab}$

8. When $x=\frac{1}{5}$

$\cos\left(2\cos^{-1}x+\sin^{-1}x\right)\ =\cos$

$\left(2\cos^{-1}\frac{1}{5}+\sin^{-1}\frac{1}{5}\right)$

$=\cos\left(\cos^{-1}\frac{1}{5}+\cos^{-1}\frac{1}{5}+\sin^{-1}\frac{1}{5}\right)$

$$= \cos\left(\frac{\pi}{2} + \cos^{-1}\frac{1}{5}\right)$$

$$= -\sin\cos^{-1}\frac{1}{5} = -\sin\sin^{-1}\sqrt{1-\frac{1}{25}}$$

$$= -\sqrt{\frac{24}{25}} = \frac{-2\sqrt{6}}{5}$$

10. Let $\cos^{-1}\left(\frac{12}{13}\right) = x,\ \sin^{-1}\left(\frac{3}{5}\right) = y$

$$\therefore \cos x = \frac{12}{13}, \sin y = \frac{3}{5}$$

$$\sin x = \sqrt{1-\cos^2 x} = \sqrt{1-\frac{144}{169}} = \frac{5}{13}$$

$$\cos y = \sqrt{1-\sin^2 y} = \sqrt{1-\frac{9}{25}} = \frac{4}{5}$$

$$\therefore \sin(x+y) = \sin x\cos y + \cos x\sin y$$

$$= \frac{5}{13}.\frac{4}{5} + \frac{12}{13}.\frac{3}{5} = \frac{56}{65}$$

$$\Rightarrow x + y = \sin^{-1}\frac{56}{65}$$

$$\Rightarrow \cos^{-1}\left(\frac{12}{13}\right) + \sin^{-1}\left(\frac{3}{5}\right) = \sin^{-1}\left(\frac{56}{65}\right)$$

12. $\cos\left(\tan^{-1} x\right) = \sin\left(\cot^{-1}\frac{3}{4}\right)$

$$\Rightarrow \sin\left(\frac{\pi}{2} - \tan^{-1} x\right) = \sin\left(\cot^{-1}\frac{3}{4}\right)$$

$$\Rightarrow \frac{\pi}{2} - \tan^{-1} x = \cot^{-1}\frac{3}{4}$$

$$\Rightarrow \tan^{-1} x + \cot^{-1}\frac{3}{4} = \frac{\pi}{2}$$

$$\Rightarrow x = \frac{3}{4}.$$

13. $\tan^{-1}\left(\frac{1-x}{1+x}\right) = \frac{1}{2}\tan^{-1} x$

$$\Rightarrow 2\tan^{-1}\left(\frac{1-x}{1+x}\right) = \tan^{-1} x$$

$$\Rightarrow \tan^{-1}\left[\frac{2\left(\frac{1-x}{1+x}\right)}{1-\left(\frac{1-x}{1+x}\right)^2}\right] = \tan^{-1} x$$

$$\Rightarrow \tan^{-1}\left[\frac{2(1-x)(1+x)}{(1+x)^2-(1-x)^2}\right] = \tan^{-1} x$$

$$\Rightarrow \tan^{-1}\frac{1-x^2}{2x} = \tan^{-1} x$$

$$\Rightarrow \frac{1-x^2}{2x} = x$$

$$\Rightarrow 1 - x^2 = 2x^2$$

$$\Rightarrow 3x^2 = 1$$

$$\Rightarrow x^2 = \frac{1}{3} \qquad \Rightarrow x = \pm\frac{1}{\sqrt{3}}$$

14. $2\tan^{-1}(\cos x) = \tan^{-1}(2\operatorname{cosec} x)$

$$\Rightarrow \tan^{-1}\left(\frac{2\cos x}{1-\cos^2 x}\right) = \tan^{-1}\left(\frac{2}{\sin x}\right)$$

$$\Rightarrow \frac{2\cos x}{1-\cos^2 x} = \frac{2}{\sin x}$$

$$\Rightarrow \frac{2\cos x}{\sin^2 x} = \frac{2}{\sin x}$$

$$\Rightarrow \sin x\cos x = \sin^2 x$$

$$\Rightarrow \sin x(\cos x - \sin x) = 0$$

$$\Rightarrow \sin = 0 \ or \cos x = \sin x$$

$$\Rightarrow \sin x = 0 \ or \cot x = 1$$

$$\Rightarrow x = 0, \frac{\pi}{2}$$

17. $\cot^{-1}7+\cot^{-1}8+\cot^{-1}18$

$$=\tan^{-1}\frac{\frac{1}{7}+\frac{1}{8}}{1-\frac{1}{7}.\frac{1}{8}}+\tan^{-1}\frac{1}{18}$$

$$=\tan^{-1}\frac{15}{55}+\tan^{-1}\frac{1}{18}$$

$$=\tan^{-1}\frac{65}{195}=\tan^{-1}\frac{1}{3}=\cot^{-1}3.$$

18. Let $\sin^{-1}\left(\frac{5}{13}\right)=\alpha,\cos^{-1}\left(\frac{3}{5}\right)=\beta$

$$\Rightarrow \sin\alpha=\frac{5}{13},\cos\beta=\frac{3}{5}$$

$$\cos\alpha=\sqrt{1-\sin^2\alpha}=\frac{12}{13}$$

$$\sin\beta=\sqrt{1-\cos^2\beta}=\frac{4}{5}$$

$$\therefore \sin(\alpha+\beta)=\sin\alpha\cos\beta+\cos\alpha\sin\beta$$

$$=\frac{5}{3}.\frac{3}{5}+\frac{12}{13}.\frac{4}{5}$$

$$=\frac{63}{65}$$

$$\Rightarrow \alpha+\beta=\sin^{-1}\left(\frac{63}{65}\right)$$

$$\Rightarrow \sin\left(\frac{5}{13}\right)+\cos^{-1}\left(\frac{3}{5}\right)=\sin^{-1}\left(\frac{63}{65}\right)$$

20. The given equation is

$$3\tan^{-1}\left(\frac{1}{2+\sqrt{3}}\right)-\tan^{-1}\frac{1}{x}=\tan^{-1}\frac{1}{3}$$

Let $\tan^{-1}\frac{1}{2+\sqrt{3}}=\theta$

$$\Rightarrow \tan\theta=\frac{1}{2+\sqrt{3}} \quad ....(1)$$

Also $\tan 15=\tan(45^0-30^0)$

$$=\frac{\tan 45^0-\tan 30^0}{1+\tan 45^0\tan 30^0}$$

$$=\frac{1-\frac{1}{\sqrt{3}}}{1+\frac{1}{\sqrt{3}}}=\frac{\sqrt{3}-1}{\sqrt{3}+1}=\frac{\left(\sqrt{3}-1\right)\left(\sqrt{3}+1\right)}{\left(\sqrt{3}+1\right)^2} \quad (2)$$

From (1) and (2), we see that

$$\tan\theta=\tan 15$$

$$\Rightarrow \theta=15$$

$$\Rightarrow \tan^{-1}\left(\frac{1}{2+\sqrt{3}}\right)=15$$

Given equation is

$$3.15-\tan^{-1}\frac{1}{x}=\tan^{-1}\frac{1}{3}$$

$$\Rightarrow \tan^{-1}1-\tan^{-1}\frac{1}{x}=\tan^{-1}\frac{1}{3}$$

$$\Rightarrow \tan^{-1}\frac{1}{x}=\tan^{-1}1-\tan^{-1}\frac{1}{3}$$

$$=\tan^{-1}\frac{1-\frac{1}{3}}{1+1.\frac{1}{3}}=\tan^{-1}\frac{1}{2}$$

$$\Rightarrow x=2$$

23. $\cos^{-1}x+\cos^{-1}y+\cos^{-1}z=\pi$

$$\Rightarrow \cos^{-1}x+\cos^{-1}y=\pi-\cos^{-1}z$$

$$\Rightarrow \cos^{-1}\left[xy-\sqrt{\left(1-x^2\right)\left(1-y^2\right)}\right]$$

$$=\pi-\cos^2 z$$

$$\Rightarrow xy-\sqrt{\left(1-x^2\right)\left(1-y^2\right)}=\cos\left(\pi-\cos^{-1}z\right)$$

$$\Rightarrow xy-\sqrt{\left(1-x^2\right)\left(1-y^2\right)}=-\cos\cos^{-1}$$

$$z=-z$$

$$\Rightarrow xy+z=\sqrt{\left(1-x^2\right)\left(1-y^2\right)}$$

$$\Rightarrow (xy+z)^2 = (1-x^2)(1-y^2)$$

$$\Rightarrow x^2y^2z^2 + 2xyz = 1 - x^2 - y^2 + x^2y^2$$

$$\Rightarrow x^2 + y^2 + z^2 + 2xyz = 1$$

24. Let $\sin^{-1} x = A,\ \sin^{-1} y = B, \sin^{-1} z = C$

$\therefore A + B + C = \pi$

Also $x = \sin A,\ y = \sin B,\ z = \sin C$

$$\therefore \cos A = \sqrt{1-\sin^2 A} = \sqrt{1-x^2}$$

$$\cos B = \sqrt{1-y^2},\ \cos C = \sqrt{1-z^2}$$

L.H.S $= x\sqrt{1-x^2} + y\sqrt{1-y^2} + z\sqrt{1-z^2}$

$= \sin A \cos A + \sin B + \cos B + \sin C \cos C$

$= \frac{1}{2}[2\sin A\cos A + 2\sin B\cos B + 2$

$\sin C \cos C]$

$= \frac{1}{2}[\sin 2A + \sin 2B + \sin 2C]$

$= \frac{1}{2}.4\sin A.\sin B.\sin C$

$[\because A + B + C = \pi]$

$= 2\sin A \sin B \sin C = 2xyz$

25. Given that $r = x + y + z$

L..H.S $= \tan^{-1}\sqrt{\frac{xy}{yz}} + \tan^{-1}\sqrt{\frac{yr}{zx}} + \tan^{-1}\sqrt{\frac{zr}{xy}}$

$$= \tan^{-1}\left[\frac{\sqrt{\frac{xr}{yz}} + \sqrt{\frac{yr}{zx}} + \sqrt{\frac{zr}{xy}} - \sqrt{\frac{xr}{yz}}\sqrt{\frac{yr}{zx}}\sqrt{\frac{zr}{xy}}}{1 - \sqrt{\frac{xr}{yz}.\frac{yr}{zx}} - \sqrt{\frac{yr}{zx}.\frac{zr}{xy}} - \sqrt{\frac{xr}{yz}.\frac{zr}{xy}}}\right]$$

$$= \tan^{-1}\left[\frac{\frac{(x+y+z)\sqrt{r}}{\sqrt{xyz}} - \frac{\sqrt{r^2}}{\sqrt{xy^2}}}{1 - \sqrt{\frac{r^2}{z^2}} - \sqrt{\frac{r^2}{x^2}} - \sqrt{\frac{r^2}{y^2}}}\right]$$

$$= \tan^{-1}\left[\frac{\frac{(x+y+z)\sqrt{r} - r^{3/2}}{\sqrt{xyz}}}{1 - \left(\frac{r}{x} + \frac{r}{y} + \frac{r}{z}\right)}\right]$$

$$= \tan^{-1}\left[\frac{r\sqrt{r} - r\sqrt{2}}{1 - \frac{r}{x} + \frac{r}{y} + \frac{r}{z}}\right]$$

$= \tan^{-1} 0 = \pi$

26. Given that

$u = \cot^{-1}\sqrt{\cos\alpha} - \sqrt{\tan\alpha}\sqrt{\cos\alpha}$

$$\Rightarrow u = \tan^{-1}\frac{1}{\sqrt{\cos\alpha}} - \tan^{-1}\sqrt{\cos\alpha}$$

$$= \tan^{-1}\left[\frac{\frac{1}{\sqrt{\cos\alpha}} - \sqrt{\cos\alpha}}{1 + \frac{1}{\sqrt{\cos\alpha}}.\sqrt{\cos\alpha}}\right]$$

$$= \tan^{-1}\frac{1-\cos\alpha}{2\sqrt{\cos\alpha}}$$

$$\Rightarrow \tan u = \frac{1-\cos\alpha}{2\sqrt{\cos\alpha}}$$

$$\Rightarrow 1 + \tan^2 u = 1 + \frac{(1-\cos\alpha)^2}{4\cos\alpha}$$

$$\Rightarrow \sin^2 u = \frac{(1+\cos\alpha)^2}{4\cos\alpha}$$

$$\sin u = \frac{\tan u}{\sec u} = \frac{1-\cos\alpha}{1+\cos\alpha} = \frac{2\sin^2\frac{\alpha}{2}}{2\cos^2\frac{\alpha}{2}}$$

$$= \tan^2\frac{\alpha}{2}$$

$$\Rightarrow \sin u = \tan^2\frac{\alpha}{2}$$

28. Given that $< A = 90^0$

$$\Rightarrow b^2 + c^2 = a^2$$

$$\tan^{-1}\frac{b}{a+c} + \tan^{-1}\frac{c}{a+b}$$

$$= \tan^{-1}\left(\frac{\frac{b}{a+c}+\frac{c}{a+b}}{1-\frac{b}{a+C}.\frac{c}{a+b}}\right)$$

$$= \tan^{-1}\left[\frac{b(a+b)+c(a+c)}{(a+c)(a+b)-bc}\right]$$

$$= \tan^{-1}\left(\frac{ab+ac+b^2+c^2}{a^2+ac+ab}\right)$$

$$= \tan^{-1}\left(\frac{ab+ac+a^2}{a^2+ab+ac}\right)$$

$$= \tan^{-1}1 = \tan^{-1}\frac{\pi}{4}$$

29. $\tan^{-1}x + \cot^{-1}(x+1)$

$$= \tan^{-1}x + \tan^{-1}\frac{1}{x+1}$$

$$= \tan^{-1}\left(\frac{x+\frac{1}{x+1}}{1-x.\frac{1}{x+1}}\right)$$

$$= \tan^{-1}\left[\frac{x(x+1)+1}{(x+1)-x}\right]$$

$$= \tan^{-1}\left(x^2+x+1\right)$$

30. L.HS $= 4\left(\cot^{-1}\frac{3}{2} + \cos ec^{-1}\sqrt{26}\right)$

We know $\cot^{-1}\frac{3}{2} = \tan^{-1}\frac{2}{3}$

Let $\cos ec^{-1}\sqrt{26} = \theta \Rightarrow \cos ec\theta = \sqrt{26}$

$$\therefore \cos ec^2\theta = 26$$

$$\Rightarrow 1+\cot^2\theta = 26$$

$$\Rightarrow \cot^2\theta = 25$$

$$\Rightarrow \cot\theta = 5$$

$$\therefore \tan\theta = \frac{1}{\cot\theta} = \frac{1}{5}$$

$$\Rightarrow \theta = \tan^{-1}.\frac{1}{5}$$

$$\Rightarrow \cos ec^{-1}\sqrt{26} = \tan^{-1}\frac{1}{5}$$

L.HS $= 4 = \left(\cot^{-1}\frac{3}{2} + \cos ec^{-1}\sqrt{26}\right)$

$$= 4\left(\tan^{-1}\frac{2}{3} + \tan^{-1}\frac{1}{5}\right)$$

$$= 4\tan^{-1}\left(\frac{\frac{2}{3}+\frac{1}{5}}{1-\frac{2}{3}.\frac{1}{5}}\right)$$

$$= r\tan^{-1}\frac{13}{13} = 4\tan^{-1}1 = 4.\frac{\pi}{4}$$

$$= \pi$$

Group - C

Long Type Questions

1. If $\cos^{-1}\frac{x}{2}+\cos^{-1}\frac{y}{3}=\theta$ then prove that $9x^2-12xy\cos\theta+4y^2=36\sin^2\theta$.

2. If $\sin^{-1}x+\sin^{-1}y+\sin^{-1}z=\pi$, then prove that $x^4+y^4+z^4+4x^2y^2z^2 = 2\left(x^2y^2z^2+z^2x^2\right)$.

3. If $\sin^{-1}\frac{x}{a}+\sin^{-1}\frac{y}{b}=\sin^{-1}\frac{c^2}{ab}$, then prove that $y^2x^2+2xy\sqrt{a^2b^2-c^2}+a^2y^2=c^4$.

4. If $\sin^{-1}\left(\frac{x}{a}\right)+\sin^{-1}\left(\frac{y}{b}\right)=\alpha$, then prove that $\frac{x^2}{a^2}+\frac{2xy}{ab}\cos\alpha+\frac{y^2}{b^2}=\sin^2\alpha$.

5. Prove that $\cos^{-1}\left(\frac{b+a\cos x}{a+b\cos x}\right)= 2\tan^{-1}\left(\sqrt{\frac{a-b}{a+b}}\tan\frac{x}{2}\right)$

6. Prove that

$$\tan^{-1}x=2\tan^{-1}(\cos ec\tan^{-1}x+\tan\cot^{-1}x).$$

7. Prove that

$$\cot^{-1}\left[\frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right]=\frac{x}{2},$$

$$x\in\left(0,\frac{\pi}{2}\right)$$

8. Show that

$$\tan^{-1}\left(\frac{2a-b}{b\sqrt{3}}\right)+\tan^{-1}\left(\frac{2b-a}{a\sqrt{3}}\right)=\frac{\pi}{3}$$

9. Show that

$$\tan^{-1}\frac{1}{x+y}+\tan^{-1}\frac{y}{x^2+xy+1}=\tan^{-1}\frac{1}{x}$$

10. Solve:

$$\tan^{-1}\frac{1}{3}+\tan^{-1}\frac{1}{5}+\tan^{-1}\frac{1}{7}+\tan^{-1}x=\frac{\pi}{4}$$

11. Solve:

$$3\sin^{-1}\frac{2x}{1+x^2}-4\cos^{-1}\frac{1-x^2}{1+x^2}+2\tan^{-1}\frac{2x}{1-x^2}=\frac{\pi}{3}$$

12. Solve:

$$\cot^{-1}\frac{1}{x-1}+\cot^{-1}\frac{1}{x}+\cot^{-1}\left(\frac{1}{x+1}\right)=\cot^{-1}\left(\frac{1}{3x}\right)$$

13. Prove that $\tan^{-1}\left[\frac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}\right]=\frac{\pi}{4}-\frac{1}{2}\cos^{-1}x$

14. Show that $\tan^{-1}\left[\frac{\sqrt{1+x^2}+\sqrt{1-x^2}}{\sqrt{1+x^2}+\sqrt{1-x^2}}\right]=\alpha$ if $x^2=\sin^2\alpha$

15. Prove $\tan^{-1}\left(\frac{1}{3}\right)+\tan^{-1}\left(\frac{1}{5}\right)+\tan^{-1}\left(\frac{1}{7}\right)+\tan^{-1}\left(\frac{1}{8}\right)=\frac{\pi}{4}$

Solutions & Hints

1. Given that $\cos^{-1}\left(\frac{x}{2}\right)+\cos^{-1}\left(\frac{y}{3}\right)=\theta$

$$\Rightarrow cos^{-1}\left[\frac{x}{2}.\frac{y}{3}-\sqrt{\left(1-\frac{x^2}{4}\right)\left(1-\frac{y^2}{9}\right)}\right]=\theta$$

$$\Rightarrow \frac{xy}{6}-\frac{\sqrt{\left(4-x^2\right)\left(9-y^2\right)}}{6}=\cos\theta$$

$$\Rightarrow xy-\sqrt{\left(4-x^2\right)\left(9-y^2\right)}=6\cos\theta$$

$$\Rightarrow \sqrt{\left(4-x^2\right)\left(9-y^2\right)}=xy-6\cos\theta$$

Squaring both sides, we get

$$\left(4-x^2\right)\left(a-b^2\right)=\left(xy-6\cos\theta\right)^2$$

$$\Rightarrow 36-9x^2-4y^2+x^2y^2=x^2y^2$$

$$-12xy\cos\theta+36\cos^2\theta$$

$$\Rightarrow 9x^2+4y^2-12xy\cos\theta=36-36\cos^2\theta$$

$$\Rightarrow 9x^2+4y^2-12xy\cos\theta=36\sin^2\theta$$

2. Let $\sin^{-1}x=A,\sin^{-1}y=B,\sin^{-1}z=C$

$$\Rightarrow \sin A=x,\sin B=y,\sin C=z$$

$$\therefore \sin^{-1}+\sin^{-1}y+\sin^{-1}z=\pi$$

$$\Rightarrow A+B+C=\pi$$

$$\Rightarrow A+C=\pi-B$$

$$\Rightarrow \cos\left(A+C\right)=\cos\left(\pi-B\right)$$

$$\Rightarrow \cos(A+C)=-\cos B$$

$$x^2-y^2+z^2=\sin^2A-\sin^2B+\sin^2C$$

$$=\frac{1}{2}\left(1-\cos 2A\right)-\frac{1}{2}\left(1-\cos 2B\right)+$$

$$\frac{1}{2}\left(1-\cos 2C\right)$$

$$=\frac{1}{2}-\frac{1}{2}\left[\cos 2A+\cos 2C-\cos 2B\right]$$

$$=\frac{1}{2}-\frac{1}{2}[2.\cos(A+C).\cos(A-C)$$

$$-(2\cos^2B-1)]$$

$$=\frac{1}{2}-\cos(A+C).\cos(A-C)+\cos^2B-\frac{1}{2}$$

$$=\cos B.\cos(A-C)+\cos^2B$$

$$=\cos B\left[\cos(A-C)+\cos B\right]$$

$$=\cos B\left[\cos(A-C)-\cos(A+C)\right]$$

$$=\cos B.2\sin A\sin C$$

$$=2\sin A\sin C\sqrt{1-\sin^2B}$$

$$=2xz\sqrt{1-y^2}$$

$$\therefore x^2-y^2+z^2=2xz\sqrt{1-y^2}$$

Squaring both sides, we get

$$\left(x^2-y^2+z^2\right)^2=4x^2z^2\left(1-y^2\right)$$

$$\left(x^2-y^2+z^2\right)^2=4x^2z^2(1-y^2)$$

$$\Rightarrow x^4+y^4+z^4-2x^2y^2-2y^2z^2 2x^2z^2$$

$$=4x^2z^2-4x^2y^2z^2$$

$$\Rightarrow x^4+y^4+z^4+4x^2y^2z^2$$

$$=2(x^2y^2+y^2z^2+z^2x^2)$$

3. Let $\sin^{-1}\frac{x}{a}=\alpha,\sin^{-1}\frac{y}{b}=\beta$

$$\therefore \frac{x}{a}=\sin\alpha,\frac{y}{b}=\sin\beta$$

$$\cos\alpha=\sqrt{1-\frac{x^2}{a^2}},$$

$$\cos\beta=\sqrt{1-\frac{y^2}{b^2}}$$

Given that $\sin^{-1}\frac{x}{a}+\sin^{-1}\frac{y}{b}=\sin^{-1}\frac{c^2}{ab}$

$$\Rightarrow \alpha+\beta=\sin^{-1}\frac{c^2}{ab}$$

$$\Rightarrow \cos(\alpha+\beta)=\cos\sin^{-1}\frac{c^2}{ab}$$

$\Rightarrow \cos\alpha\cos\beta-\sin\alpha.\sin\beta=$

$\cos\cos^{-1}\sqrt{1-\frac{c^4}{a^2b^2}}$

$$\Rightarrow \sqrt{1-\frac{x^2}{a^2}}.\sqrt{1-\frac{y^2}{b^2}}-\frac{x}{a}.\frac{y}{b}=\sqrt{1-\frac{c^4}{a^2b^2}}$$

$$\Rightarrow \sqrt{(a^2-x^2)(b^2-y^2)}-xy=\sqrt{a^2b^2-c^4}$$

$$\Rightarrow (a^2-x^2)(b^2-y^2)=$$

$$\left[xy+\sqrt{a^2b^2-c^4}\right]^2$$

$\Rightarrow a^2b^2-a^2y^2-b^2x^2+x^2y^2 \quad =x^2y^2+$

$2xy\sqrt{a^2b^2-c^4}\ +a^2b^2-c^4$

$$\Rightarrow b^2x^2+2xy\sqrt{a^2c^2-c^4}\ +a^2y^2=c^4$$

5. Let $\tan^{-1}\sqrt{\frac{a-b}{a+b}}\tan\frac{x}{2}=\theta$

$$\Rightarrow \tan\theta=\sqrt{\frac{a-b}{a+b}}\tan\frac{x}{2}$$

$$\Rightarrow \tan^2\theta=\frac{a-b}{a+b}.\tan^2\frac{x}{2}.$$

We shall show that

$$\cos^{-1}\left(\frac{a\cos x+b}{a+b\cos x}\right)=2\theta$$

$$\Rightarrow \frac{a\cos x+b}{a+b\cos x}=\cos 2\theta$$

$$\text{RHS}=\cos 2\theta=\frac{1-\tan^2\theta}{1+\tan^2\theta}$$

$$=\frac{1-\frac{a-b}{a+b}\tan^2\frac{x}{2}}{1+\frac{a-b}{a+b}\tan^2\frac{x}{2}}$$

$$=\frac{(a+b)-(a-b)\tan^2\frac{x}{2}}{a+b+(a-b)\tan^2\frac{x}{2}}$$

$$=\frac{a+b-a\tan^2\frac{x}{2}+b\tan^2\frac{x}{2}}{a+b+a\tan^2\frac{x}{2}-b\tan^2\frac{x}{2}}$$

$$=\frac{a\left(1-\tan^2\frac{x}{2}\right)+b\left(1+\tan^2\frac{x}{2}\right)}{a\left(1+\tan^2\frac{x}{2}\right)+b\left(1-\tan^2\frac{x}{2}\right)}$$

$$=x\frac{a^2\left(\frac{1-\tan^2\frac{x}{2}}{1+\tan^2\frac{x}{2}}\right)+b}{a+b\frac{1-\tan^2\frac{x}{2}}{1+\tan^2\frac{x}{2}}}$$

$$=\frac{a\cos x+b}{a+b\cos x}.$$

6. R.H.S $\quad =2\tan^{-1}[\cos ec\tan^{-1}x-\tan$

$\cos^{-1}x]$.

$$=2\tan^{-1}\left[\cos ec\tan^{-1}x-\tan.\tan^{-1}\frac{1}{x}\right]$$

$$=2\tan^{-1}\left[\cos ec\tan^{-1}x-\frac{1}{x}\right] \quad ....(1)$$

Let $\tan^{-1}x=\theta$

$$\Rightarrow x=\tan\theta$$

$$\cos ec\theta=\sqrt{1=\cot^2\theta}$$

$$=\frac{\sqrt{\tan^2\theta+1}}{\tan\theta}=\frac{\sqrt{x^2+1}}{x}$$

From (1)

$$\text{R.H.S} = 2\tan^{-1}\left[\frac{\sqrt{x^2+1}}{x}-\frac{1}{x}\right]$$

$$= 2\tan^{-1}\left(\frac{\sqrt{x^2+1} \quad -1}{x}\right)$$

$$= 2\tan^{-1}\left(\frac{\sqrt{\tan^2\theta+1} \quad -1}{\tan\theta}\right)$$

$$= 2\tan^{-1}\left(\frac{\sec\theta-1}{\tan\theta}\right)$$

$$= 2\tan^{-1}\left(\frac{1-\cos\theta}{\sin\theta}\right)$$

$$= 2\tan^{-1}\left(\frac{2\sin^2\frac{\theta}{2}}{2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}\right)$$

$$= 2\tan^{-1}\tan\frac{\theta}{2}$$

$$= 2.\frac{\theta}{2} = \theta = \tan^{-1}x = LHS.$$

7. $\text{L.HS} = \cot^{-1}\left[\frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right]$ ..(1)

We know $1+\sin x = \left(\cos\frac{x}{2}+\sin\frac{x}{2}\right)^2$

$$\Rightarrow \sqrt{1+\sin x} = \cos\frac{x}{2}+\sin\frac{x}{2}$$

Similarly $\sqrt{1-\sin x} = \cos\frac{x}{2}-\sin\frac{x}{2}$

From (1) we have

L.H.S

$$= \cot^{-1}\left(\frac{\cos\frac{x}{2}+\sin\frac{x}{2}+\cos\frac{x}{2}-\sin\frac{x}{2}}{\cos\frac{x}{2}+\sin\frac{x}{2}-\left(\cos\frac{x}{2}-\sin\frac{x}{2}\right)}\right)$$

$$= \cot^{-1}\left(\frac{2\cos\frac{x}{2}}{2\sin\frac{x}{2}}\right) = \cos^{-1}\cot\frac{x}{2} = \frac{x}{2}$$

8. $\text{L.HS} = \tan^{-1}\left(\frac{2a-b}{b\sqrt{3}}\right)+\tan^{-1}\left(\frac{2b-a}{a\sqrt{3}}\right)$

$$= \tan^{-1}\left(\frac{\frac{2a-b}{b\sqrt{3}}+\frac{2b-a}{a\sqrt{3}}}{1-\frac{2a-b}{b\sqrt{3}}.\frac{2b-a}{a\sqrt{3}}}\right)$$

$$= \tan^{-1}\left[\frac{a(2a-b)+b(2b-a)}{\frac{3ab-(2a-b)(2b-a)}{\sqrt{3}}}\right]$$

$$= \tan^{-1}\frac{(2a^2-ab+2b^2-ab)\sqrt{3}}{3ab-4ab+2a^2+2b^2-ab}$$

$$= \tan^{-1}\left(\frac{2a^2+2b^2-2ab}{2a^2+2b^2-2ab}\right)\sqrt{3}$$

$$= \tan^{-1}\sqrt{3} = \frac{\pi}{3}.$$

10. $\tan^{-1}\frac{1}{3}+\tan^{-1}\frac{1}{5}+\tan\frac{1}{7}+\tan^{-1}x = \frac{\pi}{4}$

$$\Rightarrow \left(\tan^{-1}\frac{1}{3}+\tan^{-1}\frac{1}{5}\right)+\tan^{-1}x = \frac{\pi}{4}$$

$$-\tan^{-1}\frac{1}{7}$$

$$\Rightarrow \left(\tan^{-1}\frac{1}{3}+\tan^{-1}\frac{1}{5}\right)+\tan^{-1}x = \tan^{-1}1$$

$$-\tan^{-1}\frac{1}{7}$$

$$\Rightarrow \tan^{-1}\left(\frac{\frac{1}{3}+\frac{1}{5}}{1-\frac{1}{3}.\frac{1}{5}}\right)+\tan^{-1}x = \tan^{-1}\frac{1-\frac{1}{7}}{1+1.\frac{1}{7}}$$

$$\Rightarrow \tan^{-1}\frac{8}{14}+\tan^{-1}x = \tan^{-1}\frac{6}{8}$$

$$\Rightarrow \tan^{-1}\frac{4}{7}+\tan^{-1}x=\tan^{-1}\frac{3}{4}$$

$$\Rightarrow \tan^{-1}x=\tan^{-1}\frac{3}{4}-\tan^{-1}\frac{4}{7}$$

$$=\tan^{-1}\frac{\frac{3}{4}-\frac{4}{7}}{1+\frac{3}{4}.\frac{4}{7}}$$

$$=\tan^{-1}\left(\frac{21-16}{28+12}\right)=\tan^{-1}\frac{5}{40}=\tan^{-1}\frac{1}{8}$$

$$\Rightarrow \tan^{-1}x=\tan^{-1}\frac{1}{8}$$

$$\Rightarrow x=\frac{1}{8}.$$

11. Given equation is

$$3\sin^{-1}\frac{2x}{1+x^2}-4\cos^{-1}\frac{1-x^2}{1+x^2}+2\tan^{-1}\frac{2x}{1-x^2}=\frac{\pi}{3}$$

$$\Rightarrow 3.2\tan^{-1}x-4.2\tan^{-1}x+2.2\tan^{-1}x=\frac{\pi}{3}$$

$$\Rightarrow 6\tan^{-1}x-8\tan^{-1}x+4\tan^{-1}x=\frac{\pi}{3}$$

$$\Rightarrow 2\tan^{-1}x=\frac{\pi}{3}$$

$$\Rightarrow \tan^{-1}x=\frac{\pi}{6}$$

$$\Rightarrow x=\tan\frac{\pi}{6}=\frac{1}{\sqrt{3}}$$

12. Given equation is

$$\cot^{-1}\frac{1}{x-1}+\cot^{-1}\frac{1}{x}+\cot^{-1}\frac{1}{x+1}=\cot^{-1}\frac{1}{3x}$$

$$\Rightarrow \tan^{-1}(x-1)+\tan^{-1}x+\tan^{-1}(x+1)=\tan^{-1}3x$$

$$\Rightarrow \tan^{-1}(x-1)+\tan^{-1}(x+1)=\tan^{-1}3x-\tan^{-1}x$$

$$\Rightarrow \tan^{-1}\frac{(x-1)+(x+1)}{1-(x-1)(x+1)}=\tan^{-1}\frac{3x-x}{1+3x.x}$$

$$\Rightarrow \tan^{-1}\frac{2x}{1-(x^2-1)}=\tan^{-1}\frac{2x}{1+3x^2}$$

$$\Rightarrow \frac{2x}{2-x^2}=\frac{2x}{1+3x^2}$$

$$\Rightarrow 1+3x^2=2-x^2$$

$$\Rightarrow 4x^2=1$$

$$\Rightarrow x^2=\frac{1}{4} \quad \Rightarrow x=\pm\frac{1}{2}$$

13. Let $x=\cos\theta \qquad \Rightarrow \theta=\cos^{-1}x$

$$\text{L.H.S}=\tan^{-1}\left[\frac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}\right]$$

$$=\tan^{-1}\left[\frac{\sqrt{1+\cos\theta}-\sqrt{1-\cos\theta}}{\sqrt{1+\cos\theta}+\sqrt{1-\cos\theta}}\right]$$

$$=\tan^{-1}\left[\frac{\sqrt{2}\cos\frac{\theta}{2}-\sqrt{2}\sin\frac{\theta}{2}}{\sqrt{2}\cos\frac{\theta}{2}+\sqrt{2}\sin\frac{\theta}{2}}\right]$$

$$=\tan^{-1}\left[\frac{\cos\frac{\theta}{2}-\sin\frac{\theta}{2}}{\cos\frac{\theta}{2}+\sin\frac{\theta}{2}}\right]$$

$$=\tan^{-1}\left[\frac{1-\tan\frac{\theta}{2}}{1+\tan\frac{\theta}{2}}\right]$$

$$=\tan^{-1}\left[\frac{\tan\frac{\pi}{4}-\tan\frac{\theta}{2}}{1+\tan\frac{\pi}{4}.\tan\frac{\theta}{2}}\right]$$

$$=\tan^{-1}\tan\left(\frac{\pi}{4}-\frac{\theta}{2}\right)$$

$$=\frac{\pi}{4}-\frac{\theta}{2}=\frac{\pi}{4}\frac{-1}{2}\cos^{-1}x.$$

CHAPTER - 3

LINEAR PROGRAMMING

GROUP-A

A. Choose the correct answer from the given choices (MCQ)

1. The maximum value of $x+y$ subject to $3x+4y\leq 12, x\geq 0, y\geq 0$ is ?
   (a) 3 (b) 4
   (c) 5 (d) 6

2. The maximum value of $z=2x+3y$ subject to $x+y\leq 2, x, y\geq 0$ is
   (a) 6 (b) 3
   (c) 4 (d) 5

3. The maximum value of $z=3x+2y$ subject to $2x+3y\leq 6, x\geq 0, y\geq 0$ is
   (a) 9 (b) 4
   (c) 3 (d) 2

4. The maximum vlaue of $z=5x+6y$ subject to $2x+3y\leq 12, x\geq 0, y\geq 0$ is
   (a) 25 (b) 30
   (c) 24 (d) 34

5. The manimum vlaue of $z=2x+3y$ subject to $3x+2y\geq 6, x\geq 0, y\geq 0$ is
   (a) 4 (b) 5
   (c) 6 (d) 7

6. For the L.P.P, Maximum value of $z=20x+30y$ subject to $3x+5y\leq 15, x, y\geq 0$ is
   (a) 80 (b) 90
   (c) 100 (d) 110

7. In a L.P.P manimize $z=6x_1+7x_2$ subject to $x_1+2x_2\geq 2, x_1, x_2\geq 0$, the manimum value of z is
   (a) 5 (b) 6
   (c) 7 (d) 8

8. In a L.P.P, maximize $z=5x+3y$ subject to $3x+5y\leq 15$, $5x+2y\leq 10$, $x\geq 0, y\geq 0$, the maximum value of z is
   (a) $\frac{141}{19}$ (b) $\frac{142}{19}$
   (c) $\frac{143}{19}$ (d) $\frac{144}{19}$

9. In a L.P.P, maximize $z=x+y$
   subject to $2x+y\leq 50$
   $x+2y\leq 40$
   $x\geq 0, y\geq 0$
   The maximum value of z is
   (a) 20 (b) 30
   (c) 40 (d) 50

10. For a L.P.P, maximize $z=2.5x+y$ subject to $x+3y\leq 12$
    $3x+y\leq 12$
    $x\geq 0, y\geq 0$,
    The maximum value of z is
    (a) 10 (b) 10.5
    (c) 9 (d) 9.5

ANSWERS

1.	(b)	3.	(a)	5.	(a)	7.	(c)	9.	(b)
2.	(a)	4.	(b)	6.	(c)	8.	(b)	10.	(b)

B. Fill in the blanks

1. In a L.P.P, maximize

   $z = 8000x + 12000y$

   subject ot $3x + 4y \leq 60$

   $x + 3y \leq 30$

   $x \geq 0, y \geq 0$

   the maximum value of z is _________.

2. For a L.P.P, Maximize $z = 6x + 8y$

   subject ot $x + y \leq 500$

   $x \leq 400$

   $y \geq 200$

   $x \geq 0, y \geq 0,$

   the maximum value of z is ________.

3. For a L.P.P, maximize $z = 5x + 3y$

   subject to $2x + y \leq 12$

   $3x + 2y \leq 20$

   $x \geq 0, y \geq 0,$

   the maximum value of z is ________.

4. For a L.P.P, maximize $z = 20x + 10y$

   subject to $1.5x + 3y \leq 42$

   $3x + y \leq 24$

   $x \geq 0, y \geq 0$

   the maximum value of z is ________.

5. For a L.P.P, maximize $z = 5x + 3y$

   subject to $2x + y \leq 12$

   $3x + 2y \leq 20$

   $x \geq 0, y \geq 0$

   the maximum value of z is _________.

6. Solution of manimize $z = 5x + 7y$

   subject ot $2x + y \geq 8$

   $x + 2y \geq 10$

   $x \geq 0, y \geq 0$ is _________.

7. Solution of L.P.P

   maximize $z = 5x + 3y$

   subject to $3x + 5y \leq 15$

   $5x + 2y \leq 10$

   $x \geq 0, y \geq 0$ is ___________.

8. Solution of L.P.P maximize $z = 3x + 2y$

   subject to $x + y \leq 400$

   $2x + y \leq 500$

   $x \geq 0, y \geq 0$ is ___________

9. The solution of L.P.P

   maximize $z = x + 2y$

   subject to $2x + y \leq 4$

   $x \geq 0, y \geq 0$ is ___________

10. The solution of L.P.P

    minimize $z = 3x + 5y$

    subject to $x + 3y \geq 3$

    $x + y \geq 2$

    $x \geq 0, y \geq 0$ is ________.

<u>Answers</u>

1.	168000	2.	4000
3.	160	4.	200
5.	32	6.	15
7.	$\frac{235}{19}$	8.	900
9.	4	10.	7

C. Write the answers in one word;

1. Write the maximum value of $x+y$ subject to $2x+3y \le 6$, $x \ge 0$, $y \ge 0$.

2. Write the solution of the following L.P.P

   Maximize $z = 2x+3y$

   subject to $x+y \le 1, xy \ge 0$

3. Write the maximum value of

   $z = 20x+30y$ subject to

   $3x+5y \le 15, x, y \ge 0$

4. What is the minimum value of

   $z = 6x_1 + 7x_2$ subject to

   $x_1 + 2x_2 \ge 2, \quad x_1, x_2 \ge 0$?

5. Write the maximum value of $3x+y$

   subject to $2x+3y \ge 6$, $x \ge 0, y \ge 0$

6. What is the maximum value of $3x+2y$,

   subject ot $5x+y \le 10$

   $x+y \ge 6$

   $x \ge 0, y \ge 0$

7. What is the maximum value of $22x+18y$, subject to $x+y \le 20$

   $3x+2y \le 48$

   $x \ge 0, y \ge 0$

8. Write the maximum value to $5x_1 + 7x_2$ subject to $x_1 + x_2 \le 4$

   $3x_1 + 8x_2 \le 24$

   $10x_1 + 7x_2 \le 35,$

   $x_1, x_2 \ge 0$

Answers

1. The maximum value of z is 3.
2. The maximum value of z is 3.
3. Maximum value =100
4. Minimum value = 7
5. Maximum value = 9
6. Minimum value= 13
7. Maximum value = 392
8. Maximum value= $\frac{124}{5}$

D. Write the answer in One Sentence

1. State the feasible solution.
2. Mention the quadrant in which the solution of an L.P.P with two decision variable lies when the graphical method is adopted.
3. When a linear programming problem has infinetely many solutions?
4. When a L.P.P has no solution?
5. State the extreme point theorem.
6. Define objective function.
7. Define constraints
8. What is the optimal vlaue of a L.P.P
9. Define optmization problem
10. Define feasible region.

Answers

1. Any solution of a L.P.P which satisfies the non negative restructions is called a feasoble solution.

2. In the graphical method, the solution of a L.P.P lies in the first quadrant.

3. If the two vertices of a convex polygon give the optimal value of the objective function then all ponts on the line segment joining two vertices give the optinal vlaeu and the L.P.P is said to have infinitely many solutions.

4. If the convex polygon is an empty set then the linear progarmming problem has no solution.

5. An optimal solution of a L.P.P if exits, occurs at one of the extreme points of a convex polygon.

6. If $c_1, c_2, ...c_n$ are constants and $x_1, x_2, ...x_n$ are variable then the linear function $z = c_1x_1 + c_2x_2 + ...+ c_nx_n$ which is to be maximized or minimized is called an objective function.

7. The linear in equations on the variables of a linear programming problem are called constraints. The conditions $x \geq 0, y \geq 0$ are called non negative restriction.

8. The maximum or minimum value of an objective function is called its optimal value.

9. A problem which seeks to maximize and minimize a linear function subject to certain constraints as determined by a set of linear function subject to certain constraints as determined by a set of linear inegualites is called an optimization problem.

10. A common region determined by all the constraints including the non-negative restriction $x \geq 0, y \geq 0$ of a linear programming problem is called the feasible region. The feasible region is always a convex polygon.

Group-B

Short type questins & Answers:

1. Find the feasible region on the following system.

   $2x + y \geq 6, x - y \leq 3, \; x \geq 0, y \geq 0$

2. Solve the following L.P.P

   Maximize $z = 20x + 30y$

   Subject to $2x + 5y \leq 5,$

   $x, y \geq 0$

3. Solve the following L.P.P

   Minimize $z = 6x_1 + 7x_2$

   Subject to $x_1 + 2x_2 \geq 2$

   $x_1, x_2 \geq 0$

4. Let a L.P.P be

   Maximize $z = 3x_1 + 5x_2$

   Subject to $x_1 + 3x_2 \leq 30$

   $x_1 + 2x_2 \leq 12$

   $2x_1 + 5x_2 \leq 20$

   $x_1, x_2 \geq 0$

   Test whether the point (2,3) and (-3,4) are feasible solutions or not.

5. Solve the following L.P.P

   Minimize $z = 5x + 7y$

   subject to $2x + y \geq 8$

   $x + 2y \geq 10$

   $x \geq 0, y \geq 0$

6. Solve the L.P.P

Miximize $z = 5x + 3y$

Subject to $3x + 5y \le 5$

$5x + 2y \le 10$

$x \ge 0, y \ge 0$

7. Solve the L.P.P

Maximize $z = 3x + zy$

Subject to $x + y \le 400$

$2x + y \le 500$

$x \ge 0, y \ge 0$

8. Fond the feasible region for the following constraints in a graph

$2x + y \le 4$, $x \ge -0, y \ge 0$

9. Shade the feasible region satisfying

$2x + 3y \le 6$, $x \ge 0, y \ge 0$

10. Two types of food x and y are mixed to prepare a mixture in such a way that the mixture containts at least 10 units of vitamin A. 12 units of vitamine B and 8 units of vitamine C. These vitamines are avialable in 1kg of food as per the table before

Food	**Vitamins**		
	A	B	C
x	1	2	3
y	2	2	1

1kg of food x costs Rs. 16 and 1kg of food y costs Rs. 20. Formulate the L.P.P. so as to determine the least cost of the mixture containing the required amount of vitamins.

Answers

1. The given in equations are

$2x + y \ge 6$ ....(1)

$x - y \le 3,$ ....(2)

$x \ge 0, y \ge 0$ ....(3)

Changing the in equations to equations, are get

$2x + y = 6$ ....(4)

$x - y = 3$ ....(5)

$x = 0, y = 0$ ....(6)

The line (4) intersect the axes at two points (3,0) and (0,6)

Putting (0,0) in (1), we get

$2.0 + 0 \ge 0 \Rightarrow 0 \ge 6$ which is false so the half plane is away from the origin.

The line (5) intersect the axis is (3,0) and (0,-3).

Putting (0,0) in the equatity (2) we get

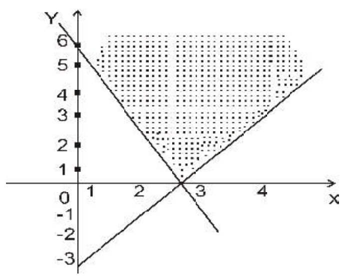
$0 - 0 \le 3$

$\Rightarrow 0 \le 3$

Which is true.

So the half plane is towards the origin.

The shaded portion is the given teasible region.



2. The given L.P.P is

Maximize $z = 20x + 30y$ ....(1)

Subject to $3x + 5y \le 15$ ....(2)

$x, y \ge 0$ ....(3)

Writing the inequations as equations, we get

$3x+5y=15$ ....(4)

$x=0, y=0$ ....(5)

The line (4) intersect the axis at (5,0) and (0,3).

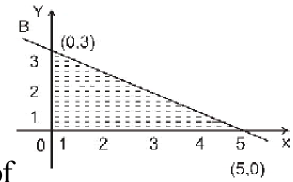
Putting (0,0) in the in equation (2), we get

$3.0+5.0 \le 15$

$\Rightarrow 0 \le 15$ which true.

So the half plane is towards the origin.

oAB is the feasible rigion where 0(0,0) A(5,0) and B(0,3). The value of different points are as follows.



Point	x	y	z = 20x + 30y
0	0	0	z = 0
A	5	0	z = 100
B	0	3	z = 90

The maximum value of $z=100$

It is obtained at A where $x=5, y=0$

3. The given L.P.P is

Minimize $z=6x_1+7x_1$ ....(1)

Subject to $x_1+2x_2 \ge 2$ ....(2)

$x_1, x_2 \ge 0$ ....(3)

Changing the inequations to equations we get

$x_{-1}+2x_2=2$ ....(4)

$x_{-1}=0, x_2=0$ ....(5)

The line (4) intersect the axis is A (2,0) and B (0,1)

Putting (0,0) is (2) we get

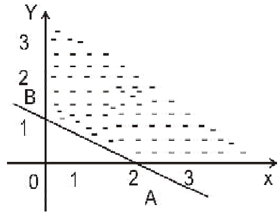
$0+2.0 \ge 2$

$\Rightarrow 0 \ge 2$ which is false.

The half plane is away from the origin.

The feasible region is X AB Y

The value of the objective funtion at the corner points are given below



Point	x_1	x_2	$z=6x_1+7x_2$
A	2	0	$z=6\times2+7\times0=12$
B	0	1	$z=6\times0+7\times1=7$

The minimum value of z is 7

It is obtained at B where $x_1=0, x_2=1$

4. The given L.P.P is

Minimize $z=3x_1+5x_2$ ....(1)

Subject to $5x_1+3x_2 \le 30$ ....(2)

$x_1+2x_2 \le 12$ ....(3)

$2x_1+5x_2 \le 20$ ....(4)

and $x_1, x_2 \ge 0$ ....(5)

The point (2,3) satisfies the constraints (2), (3) and (4) and non negative restriction (5)

So the point (2,3) is a feasible solution.

But the point (-3,4) does not satisfres the constraints and non negative restriction (5)

So the point (-3,4) is not a feasible solution.

5. Given L.P.P is

Minimize $z=5x+7y$ ....(1)

Subject to $2x+y \ge 8$ ....(2)

$x+2y \ge 10$ ....(3)

$x \ge 0, y \ge 0$ ....(4)

Changing the in equations to equations, we get

$2x+y=8$ .... (5)

$x+2y+2=10$ .... (6)

$x=0, y=0$ .... (7)

The line (5) intersects the coordinate axes at (4,0) and (0,8)

Putting (0,0) in (2) , we get

$$2.0+0\geq 8$$

$$\Rightarrow 0\geq 8$$ which is false.

So the half plane is away from the origin.

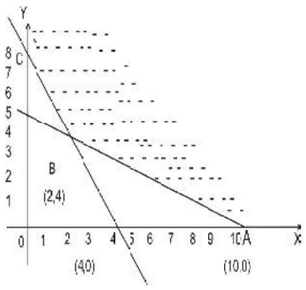
The line (6) intersects the coordinate axes at (10,0) and (0,5)

$$0+2.0\geq 10$$

$$\Rightarrow 0\geq 10$$ which is false.

So the helf plane is away from the origin.

The graph of the problem is as shown is the figure. XA B CY is the feasible region its vertices are (10,0), (2,4)and (0,8)



The value the objective function is given is the following table

Point	x	y	$z=5x+7y$
A	10	0	$z=5\times10+7\times0=50$
B	2	4	$z=5\times2+7\times4=10+28=38$
C	0	8	$z=5\times0+7\times8=56$

The minimum value of z is 38

it is obtained when $x=2, y=4$

6. The L.P.P is

Miximize $z=5x+3y$ ....(1)

subject to $3x+5y\leq 15$ ....(2)

$5x+2y\leq 10$ ....(3)

$x\geq 0, y\geq 0$ ....(4)

Changing the inequations to equations, we get

$3x+5y=15$ ....(5)

$5x+2y=10$ ....(6)

$x=0, y=0$ ....(7)

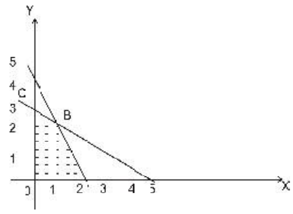
The line (5) intersect the coordinate axes at (5,0) and (0,3)

The line (6) intersect the coordinate axes at (2,0) and (0,5).

Putting (0,0) in equations (2) & (3) we have $0\leq 5$ and $0\leq 10$ which is true.

So the half plane is towards the origin.

The graph of the problem is an shown is the figure.0ABC is the feasible region whose vertices are



0(0,0), A(2,0), B$\left(\frac{20}{19},\frac{45}{19}\right)$,

C(0,3). The value z at the verfices are gien in the following table.

The the value of z at the vertices are given in the following table.

Point	x	y	$z=5x+3y$
0	0	0	$2=5\times0+3\times0=0$
A	2	0	$2=5\times2+3\times0=10$
B	$\frac{20}{19}$	$\frac{45}{19}$	$z=5\times\frac{20}{19}+3\times\frac{45}{19}=\frac{235}{19}$
C	0	3	$z=5\times0+3\times3=9$

Maximum valeu of $z=\frac{235}{19}$

It is obtained when $x=\frac{20}{19}, y=\frac{45}{19}$

7. It is same as No-6

8. The given constraints are

$2x+y\leq 4$ ....(1)

$x\geq 0, y\geq 0$ ....(2)

changing the equation to equation, we get

$2x+y=4$ ....(3)

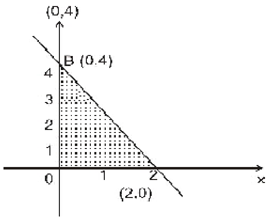
$x=0, y=0$

when $x=0, y=4$

when $y=0, x=2$

The line (3) passes through (0,4) and (2,0)

The graph is as shown in the figure.



0AB is the feasible region where 0(0,0), A(2,0) and B is (0,4)

9. Same as no.8

10. Let x kg of food X and y kg of food Y are required to prepare a mixture at least cost.

Given that 1kg of food x costs Rs. 16.00

x kg of food X costs Rs. $16x$.

Also 1kg of food Y costs Rs 20.00

y kg of food Y costs Rs. 20 y

Total cost is $z = 16x + 20y$

Total units of vitamine $A = x + 2y$

Total units of vitamine $B = 2x + 2y$

Total units of vitamine $C = 3x + y$

The formulation of the linear programming problem is as follows.

Minimize $z = 16x + 20y$

Subject to $x + 2y \geq 20$

$2x + 2y \geq 12$

$3x + y \geq 8$

and $x, y \geq 0$

GROUP-C

Long Questions

1. A furniture dealer deals in only two items, tables and chairs. He has Rs 50,000/- to invest and has a storage space of atmost 60 pieces. A table costs Rs. 2500/- and a chair is Rs. 500/-. He estimates that from the sale of one table, he can make a profit of Rs 250 and that form the sale of chairs, a profit of Rs. 75/-. He wants to know how many tables and chairs he should buy form the available mony so as to maximize his total profit, assuming that he can sell all the items which he buy?

2. One kind of cake required 200 gms of flour and 25 gms of fat, anothr kind of cake requires 100 gms of flour and 50 gms of fat. Find the maximum number of cakes which can be made from 5kg of flour and 1kg of fat, assuming that there no shortage of other ingredints used in making the cakes. Make it a L.P.P and solve it graphically.

3. Solve the following L.P.P graphically

Maximize $z = 22x + 18y$

Subject to $x + y \leq 20$

$3x + 2y \leq 48$

$x \geq 0$, $y \geq 0$

4. Solve the following L.P.P

Minimize $z = 20x_1 + 40x_2$

Subject to $36x_1 + 6x_2 \geq 108$

$3x_1 + 12x_2 \geq 36$

$2x_1 + x_2 \geq 10$

$x_1, x_2 \geq 0$

5. Solve the following L.P.P graphically

Maximize $z = 4x_1 + 3x_2$

Subject to $x_1 + x_2 \leq 50$

$x_1 + 2x_2 \leq 80$

$2x_1 + x_2 \geq 20$

6. Solve the following L.P.P graphically

Maximize $z = 2x + 10y$

Subject to $x + 2y \le 40$

$3x + y \ge 30$

$4x + 3y \ge 60$

$x, y \ge 0$

Hints & Solutions

1. Let the dealer buys x number of tables and y number of chairs. As the cost of one table is Rs. 2500, the cost of x number of tables is 2500 x. As the cost of one chair is Rs 500, the cost of y number of chairs in 500 y.

Total cost x tables and y chairs is 2500 x +500 y.

As the maximum amount he can inveis Rs 50, 000/- so

$2500x + 500y \le 50,000$

$\Rightarrow 5x + y \le 100$

The storage capacity is 60.

Here x and y must be non negative.

$x \ge 0, y \ge 0$

Let z be the profit function

$z = 250x + 75y$

Linear programming problem is as followis.

Maximize $z = 250x + 75y$ ....(1)

Subject to $5x + y \le 100$ ....(2)

$x + y \le 60$ ....(3)

$x \ge 0, y \ge 0$ ....(4)

First we convert each inequations to equations.

$5x + y = 100$ ....(5)

$x + y = 60$ ....(6)

To draw the line (5), we take the following data.

x	0	20
y	100	0

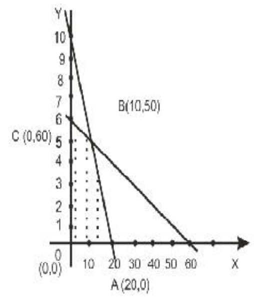
The line (5) passes through (20,0) and (0,100) putting (0,0) in the inquality.

(2) we see

$5.0 + 0 \le 100$

$\Rightarrow 0 \le 100$ which

is trueSo half plane



(2) is towards the origin. To drow the line (6) we take the following data

x	0	60
y	60	0

The line (6) passing through two points (60,0) and (10,60).

Putting (0,0) in inequation (3), we have $0 + 0 \le 60$.

$\Rightarrow 0 \le 60$ which is true.

The half plane (3) is towards the origin.

Here O ABC is the feasible region where O(0,0), A(20,0), B(10,50) and C(0,60).

The vertices of the feasible region are O(0,0), A(20,0), B(10,50) and (0,60).

We shall find the value of the object function at each of the vertices.

Point	x	y	$z = 250x + 75y$
0	0	0	$z = 250 \times 0 + 75 \times 0 = 0$
A	20	0	$z = 250 \times 20 + 75 \times 0 = 5000$
B	10	50	$z = 250 \times 10 + 75 \times 50 = 6250$
C	0	60	$z = 250 \times 0 + 75 \times 60 = 4500$

Maximum value of z is 6250.

It is obtained at B where $x = 10, y = 50$

2. Let the number of 1st kind of cakes by x and the number of 2nd kind of cakes by y . Required numbr of cakes is $z = x + y$.

We write the dat in tabular form as follows.

Case	Flour	Fat
1st kind	200 gm	25 gm
2nd kind	100 gm	50 gm

The quantity of flours used in two types of cakes is $200x + 100y$.

$$\therefore 200x + 100y \leq 5000$$

$$\Rightarrow 2x + y = 50$$

The quantity of used in two types of cakes is $25x + 50y$

$$\therefore 25x + 50y \leq 1000$$

$$\Rightarrow x + 2y \leq 40$$

The given L.P.P is

Maximize $z = x + y$ ....(1)

Subject to $2x + y \leq 50$ ....(2)

$x + 2y \leq 40$ ....(3)

$x \geq 0, y \geq 0$ ....(4)

Taking the inequations an equations

We have $2x + y = 50$ ....(5)

$x + 2y = 40$ ....(6)

$x = 0, y = 0$ ....(7)

From equation (5), we see that

X	0	25
y	50	0

The line (5) passes through the points (25,0) and (0,50).

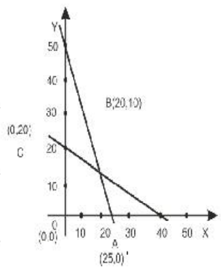
Putting (0,0) in (2), we get

$2.0 + 0 \leq 50$

$\Rightarrow 0 \leq 50$ which towards the origin.

From the equation (6), we get

X	0	40
y	20	0



The line (6) passes througn the points (40,0) and (0,20)

Putting (0,0) in (3), we have

$$0 + 2.0 \leq 40$$

$\Rightarrow 0 \leq 40$ which is true.

So the half plane (3) is twords the origin.

O ABC is the feasible region,

Where O(0,0), A(25,0),B(20,10),C(0,20)

The value of z at the corner points are

Point	x	y	$z = x + y$
O	0	0	$z = 0 + 0 = 0$
A	25	0	$z = 25 + 0 = 25$
B	20	10	$z = 20 - 10 = 30(Maximum)$
C	0	20	$z = 0 + 20 = 20$

The maximum number of cakes =30

20 cakes are of 1st kind

10 cakes are of 2nd kind.

3. The given L.P.P is

Maximize $z = 22x + 18y$ ....(1)

Subject to $x + y \le 20$ ....(2)

$3x + 2y \le 48$ ....(3)

$x \ge 0, y \ge 0$ ....(4)

Changing the inequations to equations, we get

$x + y = 20$ ....(5)

$3x + 2y = 48$ ....(6)

$x = 0, y = 0$ ....(7)

From the equation (5), we have

X	0	20
y	20	0

The line (5) passes through (20,0) and (0,20).

Putting (0,0) in (2), we get

$0 + 0 \le 20$

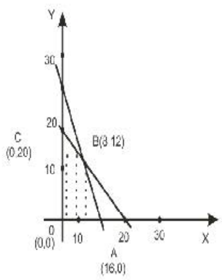
$\Rightarrow 0 \le 20$ which is true.

So the half plane (2) is towards the origin.

From the equation (6),

we get

X	0	16
y	24	0



The line (6) pass through

(16,0) and (0,24)

Putting (0,0) in (3) we get

$3.0 + 2.0 \le 45$

$\Rightarrow 0 \le 45$ which is twords

O ABC is the feasible region where

O(0,0), A(16,0), B(8,12), C(0,20)

The value of z at the corner points are given below.

Point	x	y	$z = 22x + 18y$
O	0	0	$z = 22 \times 0 + 18 \times 0 = 0$
A	16	0	$z = 22 \times 16 + 18.0 = 352$
B	8	12	$z = 22 \times 18 \times 12 = 392$
C	0	20	$z = 22 \times 0 + 18 \times 20 = 362$

The maximum value of z is 392

It is obtained when $x = 8, y = 12$

4. The given L.P.P is

Minimize $z = 20x_1 + 40x_2$ ....(1)

Subject to $36x_1 + 6x_2 \ge 108$ ......(2)

$3x_1 + 12x_2 \ge 36$ ....(3)

$2x_1 + x_2 \ge 10$ ....(4)

$x_1, x_2 \ge 0$ ....(5)

Changing the in equations to equations, we get

$36x_1 + 6x_2 = 108$ .... (6)

$3x_1 + 12x_2 = 36$ ....(7)

$2x_1 + x_2 = 10$ ....(8)

$x_1 = 0, x_2 = 0$ ....(9)

To drow the line (6), we take the following data

x_1	0	12
x_2	3	0

The line (7) passes through the points (0,3) and (12,0)

To draw the line (8), we have the following data

The line (8) passes through the poitns (0,10) and (5,0)

Putting (0,0) in inequation (2) we get

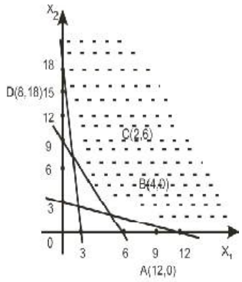
$$36.0 + 6.0 \geq 108$$

$\Rightarrow 0 \geq 108$ which is false.

The half plane (2) is away from the origin. Semilarly the half planes (3) & (4) are away from the origin.

The graph of the L.P.P is as shown in the figure.

Here $X_1\, ABCD\, X_2$ is the feasible region.



When $A(12,0), B(4,2)$ $C(2,6)$ & $D(0,18)$

The value at these objective function at these poins are given below.

Point	x_1	x_2	$z = 20x_1 + 40x_2$
A	12	0	$z = 20 + 12 + 40 \times 0 = 240$
B	4	2	$z = 20 \times 1 - 10 \times 2 = 160$
C	2	6	$z = 20 \times 2 + 40 \times 6 = 280$
D	0	18	$z = 20 \times 0 + 40 \times 18 = 720$

The minimum value of z is 160 and it is obtained when $x_1 = 4, x_2 = 2$

Note, other problems can be done as previous problems.

CHAPTER - 4

MATRICES

Multiple Choice Questions (MCQ)

A. Choose the correct answer from the given choices:

1. If $A_\alpha = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$ then $A_\alpha A_\beta$ is

(a) $A_{\alpha+\beta}$ (b) $A_{\alpha\beta}$

(c) $A_{\alpha-\beta}$ (d) none of these

2. If $A = \begin{bmatrix} -i & 0 \\ 0 & i \end{bmatrix}$ then $A^1 A$ is equal to

(a) 1 (b) $-iA$

(c) -1 (d) iA

3. The inverse of a symmetric matrix (if it exists) is

(a) a symmetric matirx

(b) a skew symmetric matrix

(c) a diagonal matrix

(d) none of these

4. If A is square matrix of order 3 and $|2A| = n|A|$ then the value of n is

(a) 2 (b) 4

(c) 8 (d) 16

5. If A is a square matirx of order 3 and $|A| = 3$ then the matrix represented by A (adj A) is

(a) $\begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}$ (b) $\begin{bmatrix} 0 & 0 & 3 \\ 0 & 3 & 0 \\ 3 & 0 & 0 \end{bmatrix}$

(c) $\begin{bmatrix} 0 & 3 & 0 \\ 3 & 0 & 3 \\ 0 & 3 & 0 \end{bmatrix}$ (d) $\begin{bmatrix} 0 & 0 & 3 \\ 0 & 3 & 0 \\ 3 & 0 & 0 \end{bmatrix}$

6. If $\begin{bmatrix} 2 & 4 & 3 \\ 2 & 4 & 2 \\ 9 & \lambda & 8 \end{bmatrix}$ is a singular matrix then the value λ is

(a) 18 (b) 22

(c) 26 (d) 30

7. If $\begin{bmatrix} 3 & 2 \\ 7 & x \end{bmatrix}\begin{bmatrix} 5 & -2 \\ -7 & y \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ then the value of $x + y$ is

(a) 7 (b) 8

(c) 9 (d) 10

8. If $\begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix} A = \begin{bmatrix} 1 & 1 \\ 0 & -1 \end{bmatrix}$ then the value of A is

(a) $\begin{bmatrix} 1 & -1 \\ 0 & 4 \end{bmatrix}$ (b) $\begin{bmatrix} 0 & -1 \\ 0 & 1 \end{bmatrix}$

(d) $\begin{bmatrix} -1 & 4 \\ 0 & 1 \end{bmatrix}$ (d) $\begin{bmatrix} 1 & 4 \\ 4 & -1 \end{bmatrix}$

9. If $A + \begin{bmatrix} 1 & 2 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$ then A is

(a) $\begin{bmatrix} 1 & -1 \\ -2 & 1 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & -2 \\ -1 & 1 \end{bmatrix}$

(c) $\begin{bmatrix} -1 & 1 \\ 1 & -2 \end{bmatrix}$ (d) $\begin{bmatrix} 1 & -1 \\ 2 & 1 \end{bmatrix}$

10. If $A=\begin{bmatrix}1 & 0\\ \frac{1}{2} & 1\end{bmatrix}$ then A^{100} is

(a) $\begin{bmatrix}1 & 0\\ 50 & 1\end{bmatrix}$ (b) $\begin{bmatrix}1 & 0\\ \left(\frac{1}{20}\right)^{10} & 1\end{bmatrix}$

(c) $\begin{bmatrix}1 & 0\\ 25 & 0\end{bmatrix}$ (d) none of these

11. If $\begin{bmatrix}3 & 5 & 3\\ 2 & 4 & 2\\ \lambda & 7 & 8\end{bmatrix}$ is a singular matrix then the value of λ is

(a) 11 (b) 22

(c) 33 (d) 44

12. If the matrix A satisfies the equation $\begin{bmatrix}1 & 3\\ 0 & 1\end{bmatrix}A=\begin{bmatrix}1 & 1\\ 0 & -1\end{bmatrix}$ then the value of A is

(a) $\begin{bmatrix}4 & 1\\ 1 & 0\end{bmatrix}$ (b) $\begin{bmatrix}1 & 4\\ 0 & -1\end{bmatrix}$

(c) $\begin{bmatrix}1 & 0\\ 4 & 1\end{bmatrix}$ (d) $\begin{bmatrix}-1 & 0\\ 4 & 1\end{bmatrix}$

13. If $2\begin{bmatrix}1 & 3\\ 0 & x\end{bmatrix}+\begin{bmatrix}y & 0\\ 1 & 2\end{bmatrix}=\begin{bmatrix}5 & 6\\ 1 & 8\end{bmatrix}$ then $x+y$ is

(a) 2 (b) 4

(c) 6 (d) 8

14. If $A^T=\begin{bmatrix}3 & 4\\ -1 & 2\\ 0 & 1\end{bmatrix}$ and $B=\begin{bmatrix}-1 & 2 & 1\\ 1 & 2 & 3\end{bmatrix}$ then A^T-B^T is

(a) $\begin{bmatrix}4 & 3\\ -3 & 0\\ -1 & 2\end{bmatrix}$ (b) $\begin{bmatrix}-3 & 0\\ 4 & 3\\ -1 & 2\end{bmatrix}$

(c) $\begin{bmatrix}-1 & 2\\ 4 & 3\\ -3 & 0\end{bmatrix}$ (d) $\begin{bmatrix}-3 & 0\\ -1 & 2\\ -3 & 0\end{bmatrix}$

15. If $A=\begin{bmatrix}1 & 2\\ 3 & 4\end{bmatrix}$ then $A+A^T$ is

(a) $\begin{bmatrix}5 & 8\\ 2 & 5\end{bmatrix}$ (b) $\begin{bmatrix}2 & 5\\ 5 & 8\end{bmatrix}$

(c) $\begin{bmatrix}5 & 2\\ 8 & 5\end{bmatrix}$ (d) $\begin{bmatrix}8 & 5\\ 5 & 2\end{bmatrix}$

16. If $2\begin{bmatrix}3 & 4\\ 5 & x\end{bmatrix}+\begin{bmatrix}1 & y\\ 0 & 1\end{bmatrix}=\begin{bmatrix}7 & 0\\ 10 & 5\end{bmatrix}$ then $x-y$ is

(a) 4 (b) 6

(c) 8 (d) 10

17. If $\begin{bmatrix}x & 1\end{bmatrix}\begin{bmatrix}1 & 0\\ -2 & 0\end{bmatrix}=0$ then x is

(a) 2 (b) 4

(c) 6 (d) 8

18. If $\begin{bmatrix}x-y & z\\ 2x-y & w\end{bmatrix}=\begin{bmatrix}-1 & 4\\ 0 & 5\end{bmatrix}$ then $x+y$ is

(a) 1 (b) 2

(c) 3 (d) 4

19. If $\begin{bmatrix}a+4 & 3b\\ 8 & -6\end{bmatrix}=\begin{bmatrix}2a+2 & b+2\\ 8 & a-8b\end{bmatrix}$ then the value of $a-2b$ is

(a) 0 (b) 1

(c) 2 (d) 3

20. If $\begin{bmatrix} xy & 4 \\ 2+6 & x+y \end{bmatrix} = \begin{bmatrix} 8 & w \\ 0 & 6 \end{bmatrix}$ then the value of $x+y+z$ is

(a) 4 (b) 3

(c) 1 (d) 0

21. If $\begin{bmatrix} 2x & 4 \end{bmatrix} \begin{bmatrix} x \\ -1 \end{bmatrix} = 0$

then the positive value of x is

(a) 2 (b) 3

(c) 4 (d) 6

22. If $\begin{bmatrix} a-b & 2a-c \\ 2a-b & 3c+d \end{bmatrix} = \begin{bmatrix} -1 & 5 \\ 0 & 13 \end{bmatrix}$ then a is

(a) 4 (b) 3

(c) 2 (d) 1

23. If $x\begin{bmatrix} 2 \\ 3 \end{bmatrix} + y\begin{bmatrix} -1 \\ 1 \end{bmatrix} = \begin{bmatrix} 10 \\ 5 \end{bmatrix}$

then the value of x is

(a) 1 (b) 2

(c) 3 (d) 4

24. If $\begin{bmatrix} 9 & -14 \\ -2 & 13 \end{bmatrix} = A + \begin{bmatrix} 1 & 2 & -1 \\ 0 & 4 & 9 \end{bmatrix}$ then A is

(a) $\begin{bmatrix} 2 & 3 & 6 \\ 8 & -3 & 5 \end{bmatrix}$ (b) $\begin{bmatrix} 8 & -3 & 5 \\ -2 & -3 & 6 \end{bmatrix}$

(c) $\begin{bmatrix} 8 & 3 & 5 \\ 2 & 3 & 6 \end{bmatrix}$ (d) $\begin{bmatrix} 8 & -3 & -5 \\ 2 & 3 & -6 \end{bmatrix}$

25. If $A = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$ and $A^2 = kA$, then the value of k is

(a) 1 (b) 2

(c) 3 (d) 4

26. If $A = \begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix}$ and $A^2 = pA$, then the value of p is

(a) 1 (b) 2

(c) 3 (d) 4

27. If $2\begin{bmatrix} x & 5 \\ 7 & y-3 \end{bmatrix} + \begin{bmatrix} 3 & -4 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 7 & 6 \\ 15 & 14 \end{bmatrix}$

$y-x$ is

(a) 5 (b) 6

(c) 7 (d) 8

28. If $3A - B = \begin{bmatrix} 5 & 0 \\ 1 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 4 & 3 \\ 2 & 5 \end{bmatrix}$ then the value of A is

(a) $\begin{bmatrix} 1 & 2 \\ 3 & 1 \end{bmatrix}$ (b) $\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 3 \\ 2 & 1 \end{bmatrix}$ (d) $\begin{bmatrix} 1 & 3 \\ 2 & 1 \end{bmatrix}$

29. If $\begin{bmatrix} x+y+z \\ x+z \\ y+z \end{bmatrix} = \begin{bmatrix} 9 \\ 5 \\ 7 \end{bmatrix}$

then the value of $x-y+z$ is

(a) 1 (b) 2

(c) 3 (d) 4

30. What is the order of the product matrix

$\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \begin{bmatrix} 2 & 3 & 4 \end{bmatrix}$?

(a) 1×1 (b) 2×2

(c) 3×3 (d) 4×4

31. If $\begin{bmatrix} x & x-y \\ 2x+y & 7 \end{bmatrix} = \begin{bmatrix} 3 & 1 \\ 8 & 7 \end{bmatrix}$ then the value of y is

(a) 1 (b) 2

(c) 3 (d) 4

32. Form the matrix equation

$\begin{bmatrix} x+y & 4 \\ -5 & 3y \end{bmatrix} = \begin{bmatrix} 3 & 4 \\ -5 & 6 \end{bmatrix}$ then the value of x is

(a) 1 (b) 2

(c) 3 (d) 4

33. If $\begin{bmatrix} 1 & 3 \\ 4 & 5 \end{bmatrix}\begin{bmatrix} x \\ 2 \end{bmatrix} = \begin{bmatrix} 5 \\ 6 \end{bmatrix}$ then the value of x is

(a) 1 (b) -1

(c) 2 (d) -2

34. If $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} 3 & 1 \\ 2 & 5 \end{bmatrix} = \begin{bmatrix} 7 & 11 \\ k & 23 \end{bmatrix}$ then the value of k is

(a) 11 (b) 13

(c) 15 (d) 17

35. If $\begin{bmatrix} x+y & 1 \\ 2y & 5 \end{bmatrix} = \begin{bmatrix} 7 & 1 \\ 4 & 5 \end{bmatrix}$ then the value of x ?

(a) 3 (b) 4

(c) 5 (d) 6

36. If $\begin{bmatrix} 2x+y & 3y \\ 0 & 4 \end{bmatrix} = \begin{bmatrix} 6 & 0 \\ 0 & 4 \end{bmatrix}$ then the value of x is

(a) 2 (b) 3

(c) 4 (d) 5

37. If $\begin{bmatrix} 3y-x & -2x \\ 3 & 7 \end{bmatrix} = \begin{bmatrix} 5 & -2 \\ 3 & 7 \end{bmatrix}$ then the value of y is

(a) 1 (b) 2

(c) 3 (d) 4

38. If $\begin{bmatrix} 2x & 1 \\ 5 & x+2y \end{bmatrix} = \begin{bmatrix} 4 & 1 \\ 5 & 0 \end{bmatrix}$ then what is the value of y is

(a) -1 (b) 1

(c) 2 (d) -2

Answers

1. (a)	2. (c)	3. (a)
4. (c)	5. (a)	6. (a)
7. (b)	8. (d)	9. (b)
10. (a)	11. (c)	12. (b)
13. (c)	14. (a)	15. (b)
16. (d)	17. (a)	18. (c)
19. (a)	20. (d)	21. (c)
22. (d)	23. (c)	24. (b)
25. (b)	26. (d)	27. (c)
28. (b)	29. (a)	30. (c)
31. (b)	32. (a)	33. (b)
34. (d)	35. (c)	36. (b)
37. (b)	38. (a)	

B. Fill in the blanks:

1. If $\begin{bmatrix} y+2x & 5 \\ -x & 3 \end{bmatrix} = \begin{bmatrix} 7 & 5 \\ -2 & 3 \end{bmatrix}$

then the value of y is__________

2. If $A + \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$

then the value of A is________

3. If $\begin{bmatrix} x & y & x \end{bmatrix} - \begin{bmatrix} -4 & 3 & 1 \end{bmatrix} = \begin{bmatrix} -5 & 1 & 0 \end{bmatrix}$

then the value of $y+z-x$ is_______

4. If $A + \begin{bmatrix} 2 & -3 \\ -4 & 7 \end{bmatrix} = \begin{bmatrix} 4 & 1 \\ 3 & 2 \end{bmatrix}$

then the value of A is________

5. If $\begin{bmatrix} 2x & y \\ 1 & 3 \end{bmatrix} + \begin{bmatrix} 4 & 2 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} 8 & 3 \\ 1 & 2 \end{bmatrix}$

then the value of $x+y$ is_______

6. If $\begin{bmatrix} x+y & x-z \\ 2x-y & 0 \end{bmatrix} = \begin{bmatrix} 2 & 2 \\ 1 & 0 \end{bmatrix}$

then the value of $x+y+z$ is_______

7. If $\begin{bmatrix} 1 & 3 \\ 2 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ 1 \end{bmatrix}$

then the value of $x+y$ is_________

8. If $A = \begin{bmatrix} 3 & 1 \\ 2 & -3 \end{bmatrix}$

then the value of $|adj\ A|$ is_______

9. If $\begin{bmatrix} 2x & 4 \\ x+2 & 3 \end{bmatrix}$

is a singular matrix then the value of x is ________

10. If $\begin{bmatrix} 2x+4 & 4 \\ x+5 & 3 \end{bmatrix}$ is a singular matrix then the value of x is__________

11. If $\begin{bmatrix} 15 & x+y \\ 2 & y \end{bmatrix} = \begin{bmatrix} 15 & 8 \\ x-y & 3 \end{bmatrix}$

then the value of x is________

12. A is a square matrix of order 3. The value of n when $|2A| = n|A|$ is__________

13. If $\begin{bmatrix} 3 & 5 & 3 \\ 2 & 4 & 2 \\ \lambda & 7 & 8 \end{bmatrix}$

is a singular matrix, then the value of λ is______

14. In a matrix equation

$\begin{bmatrix} 1 & 3 \\ 4 & 5 \end{bmatrix} \begin{bmatrix} x \\ 2 \end{bmatrix} = \begin{bmatrix} 5 \\ 6 \end{bmatrix}$

the value of x is_________

15. If $\begin{bmatrix} 3 & 2 \\ 7 & x \end{bmatrix} \begin{bmatrix} 5 & -2 \\ -7 & y \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

then the value of $x+y =$ ________

16. If A is a square matrix of order 3 and $|A| = 3$ then the matrix represented by $A(adj\ A) =$ _________

17. If the matrix A satifying the equation $\begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix} A = \begin{bmatrix} 1 & 1 \\ 0 & -1 \end{bmatrix}$,

then the value of A is________

18. The number of entries in a 3×4 matrix is _________

19. If $A=\begin{bmatrix}2&3&4&5\\6&7&8&9\\1&-3&5&6\end{bmatrix}$

then the order of A^T is ________

20. If $A+\begin{bmatrix}1&2\\1&1\end{bmatrix}=\begin{bmatrix}2&0\\0&2\end{bmatrix}$

then $A=$ ______

21. If $\begin{bmatrix}x\\y\end{bmatrix}+\begin{bmatrix}3\\4\end{bmatrix}=\begin{bmatrix}2\\-1\end{bmatrix}$

then $x+y=$ ______

22. If $\begin{bmatrix}x-y&z\\2x-y&w\end{bmatrix}=\begin{bmatrix}-1&4\\0&5\end{bmatrix}$

then $x+y=$ _______

23. If $A=\begin{bmatrix}1&0\\0&1\end{bmatrix}$ then $A^{-1}=$ _______

24. If $A=\begin{bmatrix}1&-1\\2&3\end{bmatrix}, B=\begin{bmatrix}4&2\\-1&-2\end{bmatrix}$

then $AB=$ _______

25. If $\begin{bmatrix}2x&y\\1&3\end{bmatrix}+\begin{bmatrix}4&2\\0&-1\end{bmatrix}=\begin{bmatrix}8&3\\1&2\end{bmatrix}$

then $x+y=$ _______

Answers

1. 3
2. $\begin{bmatrix}2&-1\\-1&2\end{bmatrix}$
3. 14
4. $\begin{bmatrix}2&4\\7&-4\end{bmatrix}$
5. 3
6. 1
7. 2
8. -11
9. 4
10. 4
11. 5
12. 8
13. 33
14. -1
15. 8
16. $\begin{bmatrix}3&0&0\\0&3&0\\0&0&3\end{bmatrix}$

Hints. A is a square matrix of order 3 and $|A|=3$

We know $A^{-1}=\frac{1}{|A|}(adj\,A)$

$$\therefore A\,A^{-1}=\frac{1}{|A|}A(adj\,A)$$

$$\Rightarrow I=\frac{1}{3}A(adj\,A)$$

$$\Rightarrow A(adj\,A)=3I$$

$$=3\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}=\begin{bmatrix}3&0&0\\0&3&0\\0&0&3\end{bmatrix}$$

17. $\begin{bmatrix}1&4\\0&-1\end{bmatrix}$
18. 12
19. 4×3
20. $\begin{bmatrix}1&-2\\-1&1\end{bmatrix}$
21. -6
22. 3
23. $\begin{bmatrix}1&0\\0&1\end{bmatrix}$
24. $\begin{bmatrix}5&4\\5&-2\end{bmatrix}$
25. 3

C. Answer in one word

1. If I_n is an identity matrix of order n and k a natural number then what is the value of I_n^k

2. If A is a 4×5 matrix and B is a matrix such that $A^T B$ and BA^T both are defined, then what is the order of B

3. If $\begin{bmatrix} 3 & 5 & 3 \\ 2 & 4 & 2 \\ \lambda & 7 & 8 \end{bmatrix}$ is a singular matrix then what is the value of λ?

4. If A be an invertible matrix then what is $\det\left(A^{-1}\right)$?

5. What is the value of

$$\begin{bmatrix} \cos^2 x & \sin^2 x \\ \sin^2 x & \cos^2 x \end{bmatrix} + \begin{bmatrix} \sin^2 x & \cos^2 x \\ \cos^2 x & \sin^2 x \end{bmatrix}?$$

6. What is the value of

$$\cos\theta \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} + \sin\theta$$

$$\begin{bmatrix} \sin\theta & -\cos\theta \\ \cos\theta & \sin\theta \end{bmatrix}$$

7. If $2X + Y = \begin{bmatrix} 1 & 0 \\ -3 & 2 \end{bmatrix}, Y = \begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix}$

then what is value of X ?

8. If $2\begin{bmatrix} 1 & 3 \\ 0 & x \end{bmatrix} + \begin{bmatrix} y & 0 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 6 \\ 1 & 8 \end{bmatrix}$

then the value of $x + y$ is ?

9. If $\begin{bmatrix} 3 & 2 \\ 7 & x \end{bmatrix}\begin{bmatrix} 5 & -2 \\ -7 & y \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

then $x + y = ?$

10. If $\begin{bmatrix} 1 & 2 & 3 \end{bmatrix} A = \begin{bmatrix} 0 \end{bmatrix}$ then the order of the matrix A is?

11. If $\begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix} A = \begin{bmatrix} 1 & 1 \\ 0 & -1 \end{bmatrix}$ then $A = ?$

12. If $\begin{bmatrix} 1 & x & 1 \end{bmatrix}\begin{bmatrix} 1 & 3 & 2 \\ 0 & 5 & 1 \\ 0 & 3 & 2 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \\ x \end{bmatrix} = 0$ then $x = ?$

13. If $A = \begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix}, B = \begin{bmatrix} 2 & 1 \\ 2 & 1 \end{bmatrix}$ then $AB = ?$

14. If A is a square matrix of order 3, and $|2A| = n|A|$ then what is the value of n?

15. If $\begin{bmatrix} 2x & 4 \end{bmatrix}\begin{bmatrix} x \\ -8 \end{bmatrix} = 0$ then the +ve value of x is?

Answers

1. I_n

Hints. I_n is an identity matrix of order n k is a natural number.

$I_n^k = I_n \times I_n x............k$ times $= I_n$

2. 4×5

Hints: A is a 4×5 matrix.

$\therefore A^T$ is a 5×4 matrix.

Let B be as $m \times n$ matrix.

In order that $A^T B$ is definded; the number of column of $A^T =$ the number of rows of B.

$\therefore m = 4$

In order that BA^T is defined, the number of columns of $B =$ the number of rows of $A^T = 5$

$\Rightarrow n = 5$

Then B is of order 4×5.

3. 33

4. $\dfrac{1}{\det A}$

5. $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

6. $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

7. $\begin{bmatrix} -1 & -1 \\ -2 & -1 \end{bmatrix}$

8. 6

9. 8

10. 3×1

11. $\begin{bmatrix} 1 & 4 \\ 0 & -1 \end{bmatrix}$

12. $\dfrac{19 \pm \sqrt{53}}{2}$

13. $\begin{bmatrix} 4 & 2 \\ 8 & 4 \end{bmatrix}$

14. 8

15. 4

D. <u>Write the answer in one sentence</u>

1. Define a square matrix.

2. What is a diagonal matrix.

3. What is an identity matrix?

4. When two matrices are said to be equal?

5. Define lower triangular matrix.

6. Define a null matrix.

7. Define upper triangular matrix.

8. If $\begin{bmatrix} x+y & 2 \\ 5+z & xy \end{bmatrix} = \begin{bmatrix} 6 & 2 \\ 5 & 8 \end{bmatrix}$

   then find the values of x, y and z.

9. Write down the matrix

   $\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix}$ if $aij = 2i + 3j$

10. Define adjoint of a matrix.

11. Define the inverse of a matrix.

12. If $A = \begin{bmatrix} 1 & 3 \\ 9 & 2 \end{bmatrix}$

    then what is the adjoint of A.

13. Evaluate

    $$\begin{bmatrix} a & b \end{bmatrix} \begin{bmatrix} c \\ d \end{bmatrix} + \begin{bmatrix} a & b & c & d \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}$$

14. Given a matrix $A = \begin{bmatrix} 2 & -1 \\ 4 & 2 \end{bmatrix}$,

    find matrix kA whare $k = -\dfrac{1}{2}$

15. If $A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ satisfies $A^4 = \lambda A$

    then write the value of λ

Answers

1. A matrix A is said to be a square matrix if the number of rows and columns of A are equal. If the number of rows and column are same, then the matrix is called a square matrix of order n.

2. A square matrix A is said to be a diagonal matrix if all the elements except the elements in the main digonal are zero.

   Then $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$ is a diagonal matrix.

3. A square matrix is said to be an identity matrix or unit matrix if all the off diagonal elements are zero and the elements along the main diagonal are equal to 1.

   Then $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ is an identity matrix.

4. Two matrices A and B are said to be equal if (i) they are of the same order and (ii) each element of A is equal to the corresponding elements of B

   be $a_{ij} = b_{ij}$ for all i & j

5. A square matrix $A = \left[a_{ij} \right]_{n \times n}$ is said to be a lower triangular matrix if all the lements lying above the prinipal diagonal are zero i.e $a_{ij} = 0 \, \forall i < j$

6. A matrix having all its elements as zero is called a null matrix or zero matrix.

   i.e $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ is a null matrix

7. A square matrix $A = \left[a_{ij} \right]_{n \times n}$ is said to be an upper triangular matrix if all the elements lying below the main diagonal are zero.

   i.e $a_{ij} = 0$ all $i > j$

8. Given that

$$\begin{bmatrix} x+y & 2 \\ 5+z & xy \end{bmatrix} = \begin{bmatrix} 6 & 2 \\ 5 & 8 \end{bmatrix}$$

$\Rightarrow x + y = 6$

$xy = 8$

$5 + z = 5 \quad \Rightarrow z = 0$

$(x-y)^2 = (x+y)^2 - 4xy$

$= 36 - 4.8$

$= 36 - 32 = 4$

$\Rightarrow x - y = 2$

$\therefore x = 4, y = 2, z = 0$

9. $a_{ij} = 2i + 3j$

$\therefore a_{11} = 2.1 + 3.1 = 2 + 3 = 5$

$a_{12} = 2.1 + 3.2 = 2 + 6 = 8$

$a_{13} = 2.1 + 3.3 = 2 + 9 = 11$

$a_{21} = 2.2 + 3.1 = 4 + 3 = 7$

$a_{22} = 2.2 + 3.2 = 4 + 6 = 10$

$a_{23} = 2.2 + 3.3 = 4 + 9 = 13$

$$\therefore \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix} = \begin{bmatrix} 5 & 8 & 11 \\ 7 & 10 & 13 \end{bmatrix}$$

10. If A be a square matrix then the matrix obtained by transposing the matrix formed by the cofactors of the elements of A is called the adjoint of A.

Let $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$

Matrix of the cofactors $= \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix}$

Adjoint of $A = \begin{bmatrix} A_{11} & A_{21} & A_{31} \\ A_{12} & A_{22} & A_{32} \\ A_{13} & A_{23} & A_{33} \end{bmatrix}$

11. If A and B be square matrices of the same order such that $AB = I = BA$ where I is the identity matrix then B is called the multiplicative inverse of A. It is written as A^{-1}.

12. $A = \begin{bmatrix} 1 & 3 \\ 9 & 2 \end{bmatrix}$

$A_{11} =$ cofactor of $1 = (-1)^{1+1}.2 = 2$

$A_{12} =$ cofactor $3 = (-1)^{1+2}9 = -9$

$A_{21} =$ cofactor of $9 = (-1)^{2+1}3 = -3$

$A_{22} =$ cofactor $2 = (-1)^{2+2}.1 = 1$

$\therefore$ Matrix of the cofators $= \begin{bmatrix} 2 & -9 \\ -3 & 1 \end{bmatrix}$

Adjoint of $A = \begin{bmatrix} 2 & -3 \\ -9 & 1 \end{bmatrix}$

13. $\left[ac + bd + a^2 + b^2 + c^2 + d^2\right]$

14. $\begin{bmatrix} -1 & \frac{1}{2} \\ -2 & -1 \end{bmatrix}$

15. The value of $\lambda = 8$

GROUP-B

Short type (Questions & Answers)

1. If the matrix A is such that

$$\begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix} A = \begin{bmatrix} -4 & 1 \\ 7 & 1 \end{bmatrix}$$

then find A

2. Find the inverse of the matrix

$$\begin{bmatrix} 0 & 0 & 2 \\ 0 & 2 & 0 \\ 2 & 0 & 0 \end{bmatrix}$$

3. If $A = \begin{bmatrix} 1 & -2 & 2 \\ 3 & 1 & -1 \end{bmatrix}$ and

$$B = \begin{bmatrix} 2 & 4 \\ 1 & 2 \\ 3 & -1 \end{bmatrix}$$

then verify that $(AB)^T = B^T A^T$

4. If $A = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$

then show that

$$A^k = \begin{bmatrix} 1+2k & -4k \\ k & 1-2k \end{bmatrix}$$

5. If A, B, C are matrices of order 2×2 each and

$$2A + B + C = \begin{bmatrix} 1 & 2 \\ 3 & 0 \end{bmatrix},$$

$$A + B + C = \begin{bmatrix} 0 & 1 \\ 2 & 1 \end{bmatrix} \text{ and}$$

$$A + B - C \begin{bmatrix} 1 & 2 \\ 1 & 0 \end{bmatrix}$$

then find A, B, C.

6. Find the inverse of the matrix

$$\begin{bmatrix} 1 & 1 & 2 \\ 0 & 1 & 2 \\ 1 & 2 & 1 \end{bmatrix}$$

7. If $A = \begin{bmatrix} 0 & -\tan^2 \dfrac{\theta}{2} \\ \tan^2 \dfrac{\theta}{2} & 0 \end{bmatrix}$

then prove that

$$\det\left[(I + A)(I - A)^{-1}\right] = \sec^2 \frac{\theta}{2}$$

8. If $A = \begin{bmatrix} 1 & 2 & 0 \\ 0 & 1 & 3 \\ -2 & 5 & 3 \end{bmatrix}$

then verify that $A + A^1$ is symmetric and $A - A^1$ is skew symmetric.

9. If $A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & -2 & 1 \\ 4 & 2 & 1 \end{bmatrix}$

then show that

$$A^3 - 23A - 40I = 0$$

10. If $A = \begin{bmatrix} \alpha & 0 \\ 1 & 1 \end{bmatrix}$ and

$$B = \begin{bmatrix} 1 & 0 \\ 5 & 1 \end{bmatrix}$$

then show that for no values of $\alpha, A^2 = B$

11. If $A = \begin{bmatrix} -1 & 3 & 5 \\ 1 & -3 & -5 \\ -1 & 3 & 5 \end{bmatrix}$ then find $A^3 - A^2$

12. Prove that $A=\begin{bmatrix}3 & 1\\ -1 & 2\end{bmatrix}$

$\Rightarrow A^2-5A+7I=0$

13. Construct the matrix $\left[a_{ij}\right]_{2\times 3}$

Where $a_{ij}=|i-j|$

14. If $A=\begin{bmatrix}1 & 1\\ 2 & 2\end{bmatrix}$, and $B=\begin{bmatrix}2 & 1\\ 2 & 1\end{bmatrix}$

then find AB and BA

15. If $\begin{bmatrix}4x & x-2u\\ 2u+v & 3v-2w\end{bmatrix}=\begin{bmatrix}8 & 6\\ 3 & 5\end{bmatrix}$

then find x,u,v,w

Hints & Solutions

1. Given matrix equation is

$$\begin{bmatrix}1 & -1\\ 2 & 3\end{bmatrix}A=\begin{bmatrix}-4 & 1\\ 7 & 7\end{bmatrix}$$

$$\Rightarrow A=\begin{bmatrix}1 & -2\\ 2 & 3\end{bmatrix}^{-1}\begin{bmatrix}-4 & 1\\ 7 & 7\end{bmatrix} \quad .....(1)$$

$$\begin{bmatrix}1 & -2\\ 2 & 3\end{bmatrix}^{-1}=\frac{1}{5}\begin{bmatrix}3 & 1\\ -2 & 1\end{bmatrix} \quad .....(2)$$

From (1) & (2) are get

$$A=\frac{1}{5}\begin{bmatrix}3 & 1\\ -2 & 1\end{bmatrix}\begin{bmatrix}-4 & 1\\ 7 & 7\end{bmatrix}$$

$$=\frac{1}{5}\begin{bmatrix}-12+7 & 3+7\\ 8+7 & -2+7\end{bmatrix}$$

$$=\frac{1}{5}\begin{bmatrix}-5 & 10\\ 15 & 5\end{bmatrix}$$

$$=\begin{bmatrix}-1 & 2\\ 3 & 1\end{bmatrix}$$

2. Let $A=\begin{bmatrix}0 & 0 & 2\\ 0 & 2 & 0\\ 2 & 0 & 0\end{bmatrix}$ $|A|=\begin{vmatrix}0 & 0 & 2\\ 0 & 2 & 0\\ 2 & 0 & 0\end{vmatrix}=-8$

$A_{11}=$ cofactor of $0=(1-1)^{1+1}\begin{vmatrix}2 & 0\\ 0 & 0\end{vmatrix}=0$

$A_{12}=0,\ A_{13}=-4,\ A_{21}=0,\ A_{22}=-4,$

$A_{23}=0,\ A_{31}=-4, A_{32}=0, A_{33}=0$

Matrix of the cofactors

$$=\begin{bmatrix}0 & 0 & -4\\ 0 & -4 & 0\\ -4 & 0 & 0\end{bmatrix}$$

Adjoint of $A=$ transpose of the matrix of cofactors

$$=\begin{bmatrix}0 & 0 & -4\\ 0 & -4 & 0\\ -4 & 0 & 0\end{bmatrix} \quad A^{-1}=\frac{1}{|A|}(adjA)$$

$$=\frac{1}{-8}\begin{bmatrix}0 & 0 & -4\\ 0 & -4 & 0\\ -4 & 0 & 0\end{bmatrix}=\begin{bmatrix}0 & 0 & \frac{1}{2}\\ 0 & \frac{1}{2} & 0\\ 0 & 0 & \frac{1}{2}\end{bmatrix}$$

3. Given that

$$A=\begin{bmatrix}1 & -2 & 2\\ 3 & 1 & -1\end{bmatrix}, B=\begin{bmatrix}2 & 4\\ 4 & 2\\ 3 & -1\end{bmatrix}$$

$$AB=\begin{bmatrix}1 & -2 & 2\\ 3 & 1 & -1\end{bmatrix}\begin{bmatrix}2 & 4\\ 1 & 2\\ 3 & -1\end{bmatrix}=\begin{bmatrix}6 & -2\\ -2 & 15\end{bmatrix}$$

$$(AB)^T=\begin{bmatrix}6 & 4\\ -2 & 15\end{bmatrix} \quad ......(1)$$

$n \ B^T = \begin{bmatrix} 2 & 1 & 3 \\ 4 & 2 & -1 \end{bmatrix} \ A^T = \begin{bmatrix} 1 & 3 \\ -2 & 1 \\ 2 & -1 \end{bmatrix}$

$\therefore B^T A^T = \begin{bmatrix} 2 & 1 & 3 \\ 4 & 2 & -1 \end{bmatrix} \begin{bmatrix} 1 & 3 \\ -2 & 1 \\ 2 & -1 \end{bmatrix}$

$= \begin{bmatrix} 6 & 4 \\ -2 & 15 \end{bmatrix}$ ......(2)

From (1) & (2) we have

$(AB)^T = B^T A^T$

4. Let $A = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$

We shall show that

$A^k = \begin{bmatrix} 1+2k & -4k \\ k & 1-2k \end{bmatrix}$ ....(1)

Let $P(k)$ be the above statement (1) forst we shall show that $P(1)$ is true.

Taking $k = 1$ in (1), we get

$A^1 = \begin{bmatrix} 1+2 & -4 \\ 1 & 1-2 \end{bmatrix} = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$

which is true.

so P(1) is true.

let P(m) be true.

i.e $A^m = \begin{bmatrix} 1+2m & -4m \\ m & 1-2m \end{bmatrix}$...........(2)

We shall have to show that $P(m+1)$ is true.

L.H.S of $P(m+1) = A^{m+1} = A^m.A$

$= \begin{bmatrix} 1+2m & -4m \\ m & 1-2m \end{bmatrix} \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$

$= \begin{bmatrix} (1+2m)3-4m & -4(1+2m)-4m(-1) \\ m.3+(1-2m) & -4m-(1-2m) \end{bmatrix}$

$= \begin{bmatrix} 3+2m & -4-4m \\ m+1 & -2m-1 \end{bmatrix}$

$= \begin{bmatrix} 1+2(m+1) & -4(m+1) \\ m+1 & 1-2(m+1) \end{bmatrix}$

$= R.H.S$ of $P(m+1)$

so $P(m+1)$ is true.

Here we see that (1) P(i) is ture

(ii) $P(m)$ is true $\Rightarrow P(m+1)$ is true.

So according to the principle of induction $P(k)$ is true.

i.e $A^k = \begin{bmatrix} 1+2k & -4k \\ k & 1-2k \end{bmatrix}$

5. Given that

$2A+B+C = \begin{bmatrix} 1 & 2 \\ 3 & 0 \end{bmatrix}$ ....(1)

$A+B+C = \begin{bmatrix} 0 & 1 \\ 2 & 1 \end{bmatrix}$ ....(2)

$A+B-C = \begin{bmatrix} 1 & 2 \\ 1 & 0 \end{bmatrix}$ ....(3)

Subtracting (2) from (1), we get

$A = \begin{bmatrix} 1 & 2 \\ 3 & 0 \end{bmatrix} - \begin{bmatrix} 0 & 1 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$

Subtracting (3) from (2), we get

$$2C=\begin{bmatrix}0 & 1\\ 2 & 1\end{bmatrix}-\begin{bmatrix}1 & 2\\ 1 & 0\end{bmatrix}=\begin{bmatrix}-1 & -1\\ 1 & 1\end{bmatrix}$$

$$\Rightarrow C=\frac{1}{2}\begin{bmatrix}-1 & -1\\ 1 & 1\end{bmatrix}=\begin{bmatrix}-\frac{1}{2} & -\frac{1}{2}\\ \frac{1}{2} & \frac{1}{2}\end{bmatrix}$$

From (2), we get

$$\begin{bmatrix}1 & 1\\ 1 & -1\end{bmatrix}+B+\begin{bmatrix}-\frac{1}{2} & -\frac{1}{2}\\ \frac{1}{2} & \frac{1}{2}\end{bmatrix}=\begin{bmatrix}0 & 1\\ 2 & 1\end{bmatrix}$$

$$\Rightarrow B=\begin{bmatrix}-\frac{1}{2} & \frac{1}{2}\\ \frac{1}{2} & \frac{3}{2}\end{bmatrix}$$

6. Let $A=\begin{bmatrix}1 & 1 & 2\\ 0 & 1 & 2\\ 1 & 2 & 1\end{bmatrix}$

$$|A|=\begin{vmatrix}1 & 1 & 2\\ 0 & 1 & 2\\ 1 & 2 & 1\end{vmatrix}$$

$$=1(1-4)-1(0-2)+2(0-1)=-3$$

Again the matrix of the cotactros

$$=\begin{bmatrix}-3 & 2 & -1\\ 3 & -1 & -1\\ 0 & -2 & 1\end{bmatrix}$$

Adjoint of $A=\begin{bmatrix}-3 & 3 & 0\\ 2 & -1 & -2\\ -1 & -1 & 1\end{bmatrix}$

$$A^{-1}=\frac{1}{|A|}.adj\ A=\frac{1}{3}\begin{bmatrix}-3 & 3 & 0\\ 2 & -1 & -2\\ -1 & -1 & 1\end{bmatrix}$$

7. Given that $A=\begin{bmatrix}0 & -\tan\frac{\theta}{2}\\ \tan\frac{\theta}{2} & 0\end{bmatrix}$

$$I+A=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}+\begin{bmatrix}0 & -\tan\frac{\theta}{2}\\ \tan\frac{\theta}{2} & 0\end{bmatrix}$$

$$=\begin{bmatrix}1 & -\tan\frac{\theta}{2}\\ \tan\frac{\theta}{2} & 1\end{bmatrix} \qquad ....(1)$$

$$I-A=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}-\begin{bmatrix}0 & -\tan\frac{\theta}{2}\\ \tan\frac{\theta}{2} & 0\end{bmatrix}$$

$$=\begin{bmatrix}1 & \tan\frac{\theta}{2}\\ -\tan\frac{\theta}{2} & 0\end{bmatrix}$$

We shall find the matrix of the cofactors of the elements of $I-A$

Here

$A_{11}=1 \quad A_{12}=(-1)^{1+2}\left(-\tan\frac{\theta}{2}\right)=\tan\frac{\theta}{2}$

$A_{21}=(-1)^{2+1}\left(\tan\frac{\theta}{2}\right)=-\tan\frac{\theta}{2}$

$A_{22}=1$

Matrix of the cofactors of $I-A$

$$=\begin{bmatrix}1 & \tan\frac{\theta}{2}\\ -\tan\frac{\theta}{2} & 1\end{bmatrix}$$

$$Adj(I-A)=\begin{bmatrix}1 & -\tan\frac{\theta}{2}\\ \tan\frac{\theta}{2} & 1\end{bmatrix}$$

$$|I-A| = \begin{vmatrix} 1 & -\tan\frac{\theta}{2} \\ \tan\frac{\theta}{2} & 1 \end{vmatrix} = 1+\tan^2\frac{\theta}{2}$$

$$= \sec^2\frac{\theta}{2}$$

$$(I-A)^{-1} = \frac{1}{\sec^2\frac{\theta}{2}} \begin{bmatrix} 1 & -\tan\frac{\theta}{2} \\ \tan\frac{\theta}{2} & 1 \end{bmatrix}$$

$$\therefore (I+A)(I-A)^{-1} = \begin{bmatrix} 1 & -\tan\frac{\theta}{2} \\ \tan\frac{\theta}{2} & 1 \end{bmatrix}$$

$$\frac{1}{\sec^2\frac{\theta}{2}} \begin{bmatrix} 1 & -\tan\frac{\theta}{2} \\ \tan\frac{\theta}{2} & 1 \end{bmatrix}$$

$$\text{Det}\left[(I+A)(I-A)^{-1}\right] = \frac{1}{\sec^2\frac{\theta}{2}}$$

$$\begin{vmatrix} 1-\tan^2\frac{\theta}{2} & -2\tan\frac{\theta}{2} \\ 2\tan\frac{\theta}{2} & 1-\tan^2\frac{\theta}{2} \end{vmatrix}$$

$$= \frac{1}{\sec^2\frac{\theta}{2}}\left[\left(1-\tan^2\frac{\theta}{2}\right)^2 + 4\tan^2\frac{\theta}{2}\right]$$

$$= \frac{1}{\sec^2\frac{\theta}{2}}.\left[\left(1+\tan^2\frac{\theta}{2}\right)^2\right]$$

$$= \frac{1}{\sec^2\frac{\theta}{2}}.\sec^4\frac{\theta}{2} = \sec^2\frac{\theta}{2}$$

8. $$A = \begin{bmatrix} 1 & 2 & 0 \\ 0 & 1 & 3 \\ -2 & 5 & 3 \end{bmatrix}$$

$$\therefore A^1 = \begin{bmatrix} 1 & 0 & -2 \\ 2 & 1 & 5 \\ 0 & 3 & 3 \end{bmatrix}$$

$$\therefore A + A^1 = \begin{bmatrix} 1 & 2 & 0 \\ 0 & 1 & 3 \\ -2 & 5 & 2 \end{bmatrix} + \begin{bmatrix} 1 & 0 & -2 \\ 2 & 1 & 5 \\ 0 & 3 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 2+1 & 2+0 & 0-2 \\ 0+2 & 1+1 & 3+5 \\ -2+0 & 5+3 & 3+3 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & 2 & -2 \\ 2 & 2 & 8 \\ -2 & 8 & 6 \end{bmatrix}$$

which is a symmetric matrix.

Similarly $A - A^1$ is a skew symmetric

9. Given that

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & -2 & 1 \\ 4 & 2 & 1 \end{bmatrix}$$

$A^2 = A \times A$

$$= \begin{bmatrix} 1 & 2 & 3 \\ 3 & -2 & 1 \\ 4 & 2 & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 & 3 \\ 3 & -2 & 1 \\ 4 & 2 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1+6+12 & 2-4+6 & 3+2+3 \\ 3-6+4 & 6+4+2 & 9-2+1 \\ 4+6+14 & 8-4+2 & 12+2+1 \end{bmatrix}$$

$$= \begin{bmatrix} 19 & 4 & 8 \\ 1 & 12 & 8 \\ 14 & 6 & 15 \end{bmatrix}$$

$$A^3 = A^2 \times A = \begin{bmatrix} 19 & 4 & 8 \\ 1 & 12 & 8 \\ 14 & 6 & 15 \end{bmatrix} \times \begin{bmatrix} 1 & 2 & 3 \\ 3 & -2 & 1 \\ 4 & 2 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 63 & 46 & 69 \\ 69 & -6 & 23 \\ 92 & 46 & 63 \end{bmatrix}$$

$\therefore A^3 - 23A - 40I$

$$= \begin{bmatrix} 63 & 46 & 69 \\ 69 & -6 & 23 \\ 92 & 46 & 63 \end{bmatrix} - 23 \begin{bmatrix} 1 & 2 & 3 \\ 3 & -2 & 1 \\ 4 & 2 & 1 \end{bmatrix}$$

$$-40 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 63 & 46 & 69 \\ 69 & -6 & 23 \\ 92 & 46 & 63 \end{bmatrix} - \begin{bmatrix} 23 & 46 & 69 \\ 69 & -46 & 23 \\ 92 & 46 & 23 \end{bmatrix}$$

$$- \begin{bmatrix} 40 & 0 & 0 \\ 0 & 40 & 0 \\ 0 & 0 & 40 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = 0$$

10. Given that $A = \begin{bmatrix} \alpha & 0 \\ 1 & 1 \end{bmatrix}, B = \begin{bmatrix} 1 & 0 \\ 5 & 1 \end{bmatrix}$

$$A^2 = A \times A = \begin{bmatrix} \alpha & 0 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} \alpha & 0 \\ 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \alpha^2 + 0 & 0 \\ \alpha + 1 & 0 + 1 \end{bmatrix} = \begin{bmatrix} \alpha^2 & 0 \\ \alpha + 1 & 1 \end{bmatrix}$$

Given that $A^2 = B$

$$\Rightarrow \begin{bmatrix} \alpha^2 & 0 \\ \alpha & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 5 & 1 \end{bmatrix}$$

$\Rightarrow \alpha^2 = 1$ and $\alpha + 1 = 5$

$\Rightarrow \alpha = \pm 1$ and $\alpha = 4$

Then for no values of $\alpha, A^2 = B$

11. & 12 proced as above.

13. Given matrix is $[a_{ij}]_{2\times 3}$

$$= \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{vmatrix} \qquad \text{......(1)}$$

$\therefore a_{11} = |1-1| = 0$

$a_{12} = |1-2| = 1$

$a_{13} = |1-3| = 2$

$a_{21} = |2-1| = 1$

$a_{22} = |2-2| = 0$

$a_{23} = |2-3| = 1$

They (1) be comes $\begin{bmatrix} 0 & 1 & 2 \\ 1 & 0 & 1 \end{bmatrix}$

14. Given that $A = \begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix}$ $\quad B = \begin{bmatrix} 2 & 1 \\ 2 & 1 \end{bmatrix}$

$$AB = \begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ 2 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 2+2 & 1+1 \\ 4+4 & 2+2 \end{bmatrix} = \begin{bmatrix} 4 & 2 \\ 8 & 4 \end{bmatrix}$$

$$BA = \begin{bmatrix} 2 & 1 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 2+2 & 2+2 \\ 2+2 & 2+2 \end{bmatrix} = \begin{bmatrix} 4 & 4 \\ 4 & 4 \end{bmatrix}$$

15. Exercise to the students.

GROUP-C

Long type (questions and Answers)

1. If $A = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$ $B = \begin{bmatrix} 3 & 2 \\ 2 & 1 \end{bmatrix}$ find $(AB)^{-1}$

2. Find the inverse of the matrix $\begin{bmatrix} 1 & 1 & 2 \\ 0 & 1 & 2 \\ 1 & 2 & 1 \end{bmatrix}$

3. If $A = \begin{bmatrix} 1 & -2 & 1 \\ 0 & -1 & 1 \\ 2 & 0 & -3 \end{bmatrix}$

   then find A^{-1} and hence solve the system of equations

   $x - 2y + z = 0,\ -y + z = -2,$

   $2x - 3z = 10$

4. Find the matrix A such that

   $\begin{bmatrix} 2 & -1 \\ 1 & 0 \\ -3 & 4 \end{bmatrix} A = \begin{bmatrix} -1 & -8 & -10 \\ 1 & -2 & -5 \\ 9 & 22 & 15 \end{bmatrix}$

5. If $A = \begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}$ and

   $B = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}$

   then find BA and use this to solve the equations $x - y = 3, 2x + 3y + 4z = 17$ and $y + 2z = 7$

6. Find the inverse of the matrix $\begin{bmatrix} 1 & 1 & 2 \\ 0 & 1 & 2 \\ 1 & 2 & 1 \end{bmatrix}$

7. Find the inverse of the matrix $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 4 \\ 1 & 0 & 2 \end{bmatrix}$

8. Solve the following equations by matrix method.

   $3x - 2y + z = 1$

   $2x + y - 5z = 2$

   $x - y - 2z = 3$

9. Solve the following eqations by matrix inversion method.

   $x + y + z = 4$

   $2x - y + 3z = 1$

   $3x + 2y - z = 1$

10. By elementary operation find A^{-1} for the following $A = \begin{bmatrix} 1 & 1 & 0 \\ 1 & -1 & 1 \\ 1 & -1 & 2 \end{bmatrix}$

11. Find the inverse of the matrix $\begin{bmatrix} 4 & -2 \\ 3 & 1 \end{bmatrix}$

    using elementary row transformation.

12. Verify that $[AB]^T = B^T A^T$

    Where $A = \begin{bmatrix} 1 & 2 & 3 \\ 6 & 7 & 8 \\ 6 & -3 & 4 \end{bmatrix}, B = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 4 & 2 \\ 5 & 6 & 1 \end{bmatrix}$

<u>**Hints & Solutions**</u>

1. $A=\begin{bmatrix}2 & 1\\ 1 & 1\end{bmatrix}$ $\qquad B=\begin{bmatrix}3 & 2\\ 2 & 1\end{bmatrix}$

$AB=\begin{bmatrix}2 & 1\\ 1 & 2\end{bmatrix}\begin{bmatrix}3 & 2\\ 2 & 1\end{bmatrix}$

$=\begin{bmatrix}2.3+1.2 & 2.2+1.1\\ 1.3+2.2 & 1.2+2.1\end{bmatrix}$

$=\begin{bmatrix}8 & 5\\ 7 & 4\end{bmatrix}$

$|AB|=\begin{vmatrix}8 & 5\\ 7 & 4\end{vmatrix}=32-35=-3$

cofactor of $8=(-1)^{1+1}.4=4$

cofactor of $5=(-1)^{1+2}.7=-7$

cofactor of $7=(-1)^{2+1}.5=-5$

cofactor of $4=(-1)^{2+2}.8=8$

Matrix of the cofactors $=\begin{bmatrix}4 & -7\\ -5 & 8\end{bmatrix}$

$Adj(AB)=\begin{bmatrix}4 & -5\\ -7 & 8\end{bmatrix}$

$(AB)^{-1}=\dfrac{1}{|AB|}Adj(AB)=\dfrac{1}{-3}\begin{bmatrix}4 & -5\\ -7 & 8\end{bmatrix}$

2. Let $A=\begin{bmatrix}1 & 1 & 2\\ 0 & 1 & 2\\ 1 & 2 & 1\end{bmatrix}$

$A_{11}=$ cofactor of

$1=(-1)^{1+1}\begin{vmatrix}1 & 2\\ 2 & 1\end{vmatrix}=1-4=-3$

$A_{12}=2, A_{13}=-1$

$A_{21}=3 \quad A_{22}=-1 \quad A_{23}=-1$

$A_{31}=0 \quad A_{32}=-2 \quad A_{33}=1$

Matrix of the cofactors $=\begin{bmatrix}-3 & 2 & -1\\ 3 & -1 & -1\\ 0 & -2 & 1\end{bmatrix}$

$Adj\ A=\begin{bmatrix}-3 & 3 & 0\\ 2 & -1 & -2\\ -1 & -1 & 1\end{bmatrix}$

3. Given that $A=\begin{bmatrix}1 & -2 & 1\\ 0 & -1 & 1\\ 2 & 0 & -3\end{bmatrix}$

$|A|=\begin{vmatrix}1 & -2 & 1\\ 0 & -1 & 1\\ 2 & 0 & -3\end{vmatrix}=1$

cofactors of the elements of $|A|$ are as follows

$A_{11}=3,\ A_{12}=2,\ A_{13}=2$

$A_{21}=-6 \quad A_{22}=-5 \quad A_{23}=-4$

$A_{31}=-1 \quad A_{32}=-1 \quad A_{33}=-1$

Matrix of the cofactors $=\begin{bmatrix}3 & 2 & 2\\ -6 & -5 & -4\\ -1 & -1 & -1\end{bmatrix}$

$Adj\ A=\begin{bmatrix}3 & -6 & -1\\ 2 & -5 & -1\\ 2 & -4 & -1\end{bmatrix}$

$A^{-1}=\dfrac{1}{|A|}(adjA)=\begin{bmatrix}3 & -6 & -1\\ 2 & -5 & -1\\ 2 & -4 & -1\end{bmatrix}$

Given system of equations is

$x-2y+z=0$

$0x-y+z=-2$

$2x+0y-3z=10$

$$\Rightarrow \begin{bmatrix} 1 & -2 & 1 \\ 0 & -1 & 1 \\ 2 & 0 & -3 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ -2 \\ 10 \end{bmatrix}$$

$\Rightarrow Ax + B$

Where

$$A = \begin{bmatrix} 1 & -2 & 1 \\ 0 & -1 & 1 \\ 2 & 0 & -3 \end{bmatrix}, x = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, B = \begin{bmatrix} 0 \\ -2 \\ 10 \end{bmatrix}$$

$\Rightarrow x = A^{-1}B$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 & -6 & -1 \\ 2 & -5 & -1 \\ 2 & -4 & -1 \end{bmatrix}\begin{bmatrix} 0 \\ -2 \\ 10 \end{bmatrix}$$

$$= \begin{bmatrix} 3.0 - \ 6.\ (-2) + \ (-1).10 \\ 2.0 + (-5).(-2) + (-1)10 \\ 2.0 + (-4)(-2) + (-1).10 \end{bmatrix}$$

$$= \begin{bmatrix} -2 \\ 0 \\ -2 \end{bmatrix}$$

$\therefore x = -2, y = 0, z = -2$

4. Given that

$$\begin{bmatrix} 2 & -1 \\ 1 & 0 \\ -3 & 4 \end{bmatrix}_{3\times 2} A = \begin{bmatrix} -1 & -8 & -10 \\ 1 & -2 & -5 \\ 9 & 22 & 15 \end{bmatrix}_{3\times 3} \quad .....(1)$$

A should be of the order 2×3

Let $A = \begin{bmatrix} a & b & c \\ a & e & f \end{bmatrix}$

From (1) , we have

$$\begin{bmatrix} 2 & -1 \\ 1 & 0 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix} = \begin{bmatrix} -1 & -8 & -10 \\ 1 & -2 & -5 \\ 9 & 22 & 15 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2a - d & 2b - e & 2c - f \\ a & b & c \\ -3a + 4d & -3b\_4e & -3c + 4f \end{bmatrix}$$

$$= \begin{bmatrix} -1 & -8 & -10 \\ 1 & -2 & -5 \\ 9 & 22 & 15 \end{bmatrix}$$

$\therefore a = 1, b = -2,\ c = -5$

$2a - d = -1 \Rightarrow d = 2a + 1 = 2.1 + 1 = 3$

$2b - e = -8 \ \Rightarrow e = 2b + 8 = 2(-2) + 8 = 4$

$2c - f = -10 \Rightarrow f = 2c + 10 = 2(-5) + 10 = 0$

$$\therefore A = \begin{bmatrix} 1 & -2 & -5 \\ 3 & 4 & 0 \end{bmatrix}$$

5. Given that

$$A = \begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}, B = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}$$

$$BA = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}\begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} 2+4+0 & 2-2+0 & -4+4+0 \\ 4-12+8 & 4+6-4 & -8-12+20 \\ 0-4+4 & 0+2-2 & 0-4+10 \end{bmatrix}$$

$$= \begin{bmatrix} 6 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 6 \end{bmatrix} = 6I$$

$\Rightarrow BA = 6I$

Multiplying B^{-1}, we get

$B^{-1}B\,A = 6\,I\,B^{-1}$

$\Rightarrow A = 6B^{-1}$

$$\Rightarrow B^{-1} = \frac{1}{6}A = \frac{1}{6}\begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}$$

Given system of equations are

$$x - y = 3$$

$$2x + 3y + 4z = 17$$

$$y + 2z = 7$$

$$\Rightarrow x - y + 0z = 3$$

$$2x + 3y + 4z = 17$$

$$0x + y + 2z = 7$$

$$\Rightarrow \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 17 \\ 7 \end{bmatrix}$$

$$\Rightarrow B \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 17 \\ 7 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = B^{-1} \begin{bmatrix} 3 \\ 17 \\ 7 \end{bmatrix}$$

$$= \frac{1}{6} \begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & 1 & 5 \end{bmatrix} \begin{bmatrix} 3 \\ 17 \\ 7 \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ 4 \end{bmatrix}$$

$$\therefore x = 2, y = -1, z = 4$$

6.& 7 same as No. 3

8. The given equations are

$$3x - 2y + z = 1$$

$$2x + y - 5z = 2$$

$$x - y - 2z = 3$$

$$\Rightarrow \begin{bmatrix} 3 & -2 & 1 \\ 2 & 1 & -5 \\ 1 & 1 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

$$\Rightarrow Ax = B$$

Where

$$A = \begin{bmatrix} 3 & -2 & 1 \\ 2 & 1 & -5 \\ 1 & 2 & -2 \end{bmatrix}, x = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, B = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

$$\Rightarrow x = A^{-1}B = \frac{1}{|A|}(adj\ A)B \qquad \text{......(1)}$$

$$|A| = \begin{vmatrix} 3 & -2 & 1 \\ 2 & 1 & -5 \\ 1 & 1 & -2 \end{vmatrix} = 12$$

Matrix of the cofactors

$$= \begin{bmatrix} 3 & -1 & 1 \\ -3 & -7 & -5 \\ 9 & 17 & 7 \end{bmatrix}$$

$$adj\ A = \begin{bmatrix} 3 & -3 & 9 \\ -1 & -7 & 17 \\ 1 & -5 & 7 \end{bmatrix}$$

From (1), we get

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{12} \begin{bmatrix} 3 & -3 & 9 \\ -1 & -7 & 17 \\ 1 & -5 & 7 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} = \frac{1}{2} \begin{bmatrix} 24 \\ 36 \\ 12 \end{bmatrix}$$

$$= \begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix} \therefore x = 2, y = 3, z = 1$$

9. Same as No. 8.

10. Given that

$$A = \begin{bmatrix} 1 & 1 & 0 \\ 1 & -1 & 1 \\ 1 & -1 & 2 \end{bmatrix}$$

We know $A = IA$

$$\Rightarrow \begin{bmatrix} 1 & 1 & 0 \\ 1 & -1 & 1 \\ 1 & -1 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 0 \\ 1 & -1 & 1 \\ 1 & -1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

$$\Rightarrow \begin{bmatrix} 1 & 1 & 0 \\ 0 & -2 & 1 \\ 0 & -2 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix} A$$

$$\begin{bmatrix} R_2 \to R_2 - R_2 \\ R_3 \to R_3 - R_2 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & 1 & 0 \\ 0 & -2 & 1 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix} A$$

$$[R_2 \to R_3 - R_2]$$

$$\Rightarrow \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & -\frac{1}{2} \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ \frac{1}{2} & -\frac{1}{2} & 0 \\ 0 & -1 & 1 \end{bmatrix} A$$

$$\left[R_2 \to -\frac{1}{2} R_2\right]$$

$$\Rightarrow \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ \frac{1}{2} & -1 & \frac{1}{2} \\ 1 & -1 & 1 \end{bmatrix} A$$

$$\left[R_2 \to R_2 + \frac{1}{2} R_3\right]$$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & 1 & -\frac{1}{2} \\ \frac{1}{2} & -1 & \frac{1}{2} \\ 1 & -1 & 1 \end{bmatrix} A$$

$$(R_1 \to R_2 - R_2)$$

$$\Rightarrow A^{-1} = \begin{bmatrix} \frac{1}{2} & 1 & -\frac{1}{2} \\ \frac{1}{2} & -1 & \frac{1}{2} \\ 1 & -1 & 1 \end{bmatrix}$$

11. Let $A = \begin{bmatrix} 4 & -2 \\ 3 & 1 \end{bmatrix}$

We know $A = IA$

$$\Rightarrow \begin{bmatrix} 4 & -2 \\ 3 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$$

$$\Rightarrow \begin{bmatrix} 4 & -2 \\ 12 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 4 \end{bmatrix} A \quad [R_2 \to 4R_2]$$

$$\Rightarrow \begin{bmatrix} 4 & -2 \\ 0 & 10 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -3 & 4 \end{bmatrix} A \; [R_2 \to R_2 - 3R_1]$$

$$\Rightarrow \begin{bmatrix} 20 & -10 \\ 0 & 10 \end{bmatrix} = \begin{bmatrix} 5 & 0 \\ -3 & 4 \end{bmatrix} A$$

$$[R_1 \Rightarrow 5R_1]$$

$$\Rightarrow \begin{bmatrix} 20 & -10 \\ 0 & 10 \end{bmatrix} = \begin{bmatrix} 2 & 4 \\ -3 & 4 \end{bmatrix}$$

$$[R_1 \to R_1 + R_2]$$

$$\Rightarrow\begin{bmatrix}1&0\\0&1\end{bmatrix}=\begin{bmatrix}\frac{2}{20}&\frac{4}{20}\\-\frac{3}{10}&\frac{4}{10}\end{bmatrix}A$$

$$\left[R_1\to\frac{1}{20}R_1, R_2\to\frac{1}{10}R_1\right]$$

$$\Rightarrow\begin{bmatrix}1&0\\0&1\end{bmatrix}=\frac{1}{10}\begin{bmatrix}2&4\\-3&4\end{bmatrix}A$$

$$A^{-1}=\frac{1}{10}\begin{bmatrix}2&4\\-3&4\end{bmatrix}$$

$$=\begin{bmatrix}\frac{2}{10}&\frac{4}{10}\\-\frac{3}{10}&\frac{4}{10}\end{bmatrix}$$

12. Given that

$$A=\begin{bmatrix}1&2&3\\6&7&8\\6&-3&4\end{bmatrix}, B=\begin{bmatrix}1&2&3\\3&4&2\\5&6&1\end{bmatrix}$$

$$=\begin{bmatrix}1+6+15&2+8+18&3+4+3\\6+21+40&12+28+48&18+14+8\\6-9+20&12-12+24&18-6+4\end{bmatrix}$$

$$=\begin{bmatrix}22&28&10\\67&88&40\\17&24&16\end{bmatrix}$$

$$(AB)^T=\begin{bmatrix}22&67&17\\28&88&24\\10&40&16\end{bmatrix} \quad ....(1)$$

$$A^T=\begin{bmatrix}1&6&6\\2&7&-3\\3&8&4\end{bmatrix} \quad B^T=\begin{bmatrix}1&3&5\\2&4&6\\3&2&1\end{bmatrix}$$

$$B^TA^T=\begin{bmatrix}1&3&5\\2&4&6\\3&2&1\end{bmatrix}\begin{bmatrix}1&6&6\\2&7&-3\\3&8&4\end{bmatrix}$$

$$=\begin{bmatrix}22&67&17\\28&88&24\\10&40&16\end{bmatrix} \quad ....(2)$$

From (1) & (2) we get

$$(AB)^T=B^TA^T$$

Chapter - 5

DETERMINANTS

Multiple Choice Questions (MCQ)

A. Choose the correct answer from the given choices:

1. The value of

$$\begin{vmatrix} 102 & 18 & 36 \\ 1 & 3 & 4 \\ 17 & 3 & 6 \end{vmatrix} \text{ is } \_\_\_\_\_\_\_\_$$

(a) 2 (b) 1

(c) 0 (d) -1

2. What is the value of

$$\begin{vmatrix} a-b & b-c & c-a \\ b-c & c-a & a-b \\ c-a & a-b & b-c \end{vmatrix}$$

(a) 0 (b) 1

(c) 2 (d) 3

3. What is the value of $\begin{vmatrix} \frac{1}{a} & 1 & bc \\ \frac{1}{b} & 1 & ca \\ \frac{1}{c} & 1 & ab \end{vmatrix}$?

(a) -1 (b) 0

(c) 1 (d) 2

4. What is the value of

$$\begin{vmatrix} 1 & 0 & -5863 \\ -7361 & 2 & 7361 \\ 1 & 0 & 4137 \end{vmatrix}$$

(a) 5000 (b) 10,000

(c) 15,000 (d) 20,000

5. What is the value of

$$\begin{vmatrix} -\cos ec^2\theta & \sec^2\theta & -0.2 \\ \cot^2\theta & -\tan^2\theta & 1.2 \\ -1 & 1 & 1 \end{vmatrix}?$$

(a) -1 (b) 1

(c) 0 (d) 2

6. What is the value of

$$\begin{vmatrix} 1 & x & y \\ 0 & \sin x & \sin y \\ 0 & \cos x & \cos y \end{vmatrix}?$$

(a) $\sin(x-y)$ (b) $\cos(x-y)$

(c) $\tan(x-y)$ (d) $\cot(x-y)$

7. What is the value of

$$\begin{vmatrix} \sin^2\theta & \cos^2\theta & 1 \\ \cos^2\theta & \sin^2\theta & 1 \\ -10 & 12 & 2 \end{vmatrix}?$$

(a) 3 (b) 2

(c) 1 (d) 0

8. If ω is the cube root of unit, then

$$\begin{vmatrix} 1 & \omega & \omega^2 \\ \omega & \omega^2 & 1 \\ \omega^2 & 1 & \omega \end{vmatrix} = \_\_\_\_\_\_\_\_.$$

(a) 0 (b) 1

(c) -1 (d) 2

9. What is the value of

$$\begin{vmatrix} 0 & 8 & 0 \\ 25 & 520 & 25 \\ 1 & 410 & 0 \end{vmatrix} ?$$

(a) 150 (b) 200

(c) -250 (d) 250

10. What is the value of the determinant

$$\begin{vmatrix} 1 & a & b+c \\ 1 & b & c+a \\ 1 & c & a+b \end{vmatrix} ?$$

(a) 0 (b) 1

(c) -1 (d) -2

11. What is the value of

$$\begin{vmatrix} 2 & 3 & 4 \\ 5 & 6 & 8 \\ 6x & 9x & 12x \end{vmatrix} ?$$

(a) 2 (b) -1

(c) 1 (c) 0

12. What is the value of

$$\begin{vmatrix} 4 & a & b+c \\ 4 & b & c+a \\ 1 & c & a+b \end{vmatrix} ?$$

(a) -1 (b) 1

(c) 0 (d) 2

13. What is the value of

$$\begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix} ?$$

(a) $a+b+c$ (b) $(a+b+c)^2$

(c) $(a+b+c)^3$ (d) $(a+b+c)^4$

14. The value of $\begin{vmatrix} x^2-x+1 & x-1 \\ x+1 & x+1 \end{vmatrix}$ is_____?

(a) x^3-x^2+3 (b) x^3+x^2+3

(c) x^2+x+2 (d) x^3-x^2+2

15. What is the value of $\begin{vmatrix} 2 & 7 & 65 \\ 3 & 8 & 75 \\ 5 & 9 & 86 \end{vmatrix}$?

(a) 0 (b) 4

(c) 8 (d) 10

16. If $\begin{vmatrix} 1+x & x & x^2 \\ x & 1+x & x^2 \\ x^2 & x & 1+x \end{vmatrix}$

$= a+bx+cx^2+dx^3 +ex^4+fx^5$ then the value of a is____?

(a) 1 (b) 2

(c) 3 (d) 4

17. If $\begin{vmatrix} a & b & c \\ b & a & b \\ x & b & c \end{vmatrix} = 0$

then what is the value of x ?

(a) a (b) b

(c) c (d) $a+b+c$

18. If every element of a third order determinant of vlaue 8 is multiplied by 2 then what is the value of the new determinant?

(a) 8 (b) 16

(c) 64 (d) 128

19. Let the value of a third order determinant be A and each element is multiplied by 2, then what will be the value of the new determinant?

(a) 2A (b) 4A

(c) 8A (d) 16 A

20. What is the maximum value of $\begin{vmatrix} \cos x & \sin x \\ -\sin x & \cos x - 1 \end{vmatrix}$?

(a) 0 (b) 1

(c) 2 (d) 3

21. If $\begin{vmatrix} 2x & 5 \\ 8x & x \end{vmatrix} = \begin{vmatrix} 6 & -2 \\ 7 & 3 \end{vmatrix}$

then what is the value of x?

(a) ±4 (b) ±5

(c) ±6 (d) ±7

22. If ω is a complex root of 1 and

$$\begin{vmatrix} 1 & \omega & \omega^2 \\ \omega & \lambda & 1 \\ \omega^2 & 1 & \omega \end{vmatrix} = 0$$

(a) 1 (b) 2

(c) 3 (d) any real number

23. If $\begin{vmatrix} 3x & 7 \\ -2 & 4 \end{vmatrix} = \begin{bmatrix} 8 & 7 \\ 6 & 4 \end{bmatrix}$

then what is the value of x ?

(a) 0 (b) -1

(c) -2 (d) -3

24. If $\begin{vmatrix} x+1 & x-1 \\ x-3 & x+2 \end{vmatrix} = \begin{vmatrix} 4 & -1 \\ 1 & 3 \end{vmatrix}$

then what is the value of x ?

(a) 1 (b) 2

(c) 3 (d) 4

25. If A be a square matrix of order 3×3 and $|A| = 4$ then what is the value of $|2A|$?

(a) 4 (b) 8

(c) 16 (d) 32

26. If A_{ij} is cofactor of the elements a_{ij} of the determinant $\begin{vmatrix} 2 & -3 & 5 \\ 6 & 0 & 4 \\ 1 & 5 & -7 \end{vmatrix}$ then what is the value of $a_{32} A_{32}$

(a) 108 (b) 100

(c) 92 (d) 88

27. If $\begin{vmatrix} x & x \\ 1 & x \end{vmatrix} = \begin{vmatrix} 3 & 4 \\ 1 & 2 \end{vmatrix}$

then the positive value of x is

(a) 1 (b) 2

(c) 3 (d) 4

28. If $\begin{vmatrix} x & 4 \\ 2 & 2x \end{vmatrix} = 0$ then the value of x is?

(a) ±1 (b) ±2

(c) ±3 (d) ±4

29. What is the value of $\begin{vmatrix} a+ib & c+id \\ -c+id & a-ib \end{vmatrix}$?

(a) $a^2 + b^2$

(b) $b^2 + c^2$

(c) $c^2 + d^2$

(d) $a^2 + b^2 + c^2 + d^2$

30. What is the value of

$$\begin{vmatrix} a+b+2c & a & b \\ c & b+c+2a & b \\ c & a & c+a+2b \end{vmatrix}?$$

(a) $2(a+b+c)$ (b) $2(a+b+c)^2$

(c) $2(a+b+c)^3$ (d) $2(a+b+c)^4$

31. What is the value of $\begin{vmatrix} 1 & 2 & 1 \\ 2 & 4 & 3 \\ 3 & 6 & -9 \end{vmatrix}$?

(a) 47 (b) 27

(c) 1 (d) 0

32. The value of $\begin{vmatrix} \tan x & \sec x \\ \sec x & \tan x \end{vmatrix}$

(a) 1 (b) -1

(c) 0 (d) None of these

33. If ω is the maginary cube root of 1, then

$$\begin{vmatrix} 1 & \omega^3 & 91 \\ \omega & \omega^7 & 92 \\ \omega^2 & \omega^5 & 98 \end{vmatrix} = ?$$

(a) 91 (b) 95

(c) -1 (d) 2

34. The minimum value of

$$\begin{vmatrix} \sin x & \cos x \\ -\cos x & 1+\sin x \end{vmatrix} = ?$$

(a) 0 (b) 1

(c) -1 (d) 2

35. What is the value of

$$\begin{vmatrix} a+d & a+d+k & a+d+c \\ c & c+b & c \\ d & d+k & d+c \end{vmatrix} ?$$

(a) $4abc$ (b) $3abc$

(c) $2abc$ (d) abc

Answers

1. (c)	2. (a)	3.(b)	4. (d)
5. (c)	6. (a)	7. (d)	8. (a)
9. (b)	10. (a)	11. (d)	12. (c)
13. (c)	14. (d)	15. (a)	16. (a)
17. (a)	18. (c)	19. (c)	20. (a)
21. (c)	22. (d)	23. (c)	24. (b)
25. (d)	26. (a)	27. (b)	28. (b)
29. (d)	30. (c)	31. (d)	32. (b)
33. (c)	34. (a)	35. (d)	

B. Fill in the blanks:

1. If each constituent of a third order determinant $(\neq, 0)$ is multiplied by 2 then the determinant is multiplied by______.

2. If

$$\begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix} = \begin{vmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ x & 0 & 1 \end{vmatrix} \text{ then } x = \_\_\_\_\_$$

3. The minimum value of

$$\begin{vmatrix} \sin x & \cos x \\ -\cos x & 1+\sin x \end{vmatrix}$$

is ________.

4. The value of

$$\begin{vmatrix} 224 & 777 & 32 \\ 735 & 888 & 105 \\ 812 & 999 & 116 \end{vmatrix} = \_\_\_\_\_\_$$

5. The value of

$$\begin{vmatrix} -\cos ec^2\theta & \sec^2\theta & -0.2 \\ \cot^2\theta & -\tan^2\theta & 1.2 \\ -1 & 1 & 1 \end{vmatrix} = \_\_\_\_\_\_$$

6. The value of

$$\begin{vmatrix} a+3b & a+5b & a+7b \\ a+4b & a+6b & a+8b \\ a+5b & a+7b & a+9b \end{vmatrix} = \_\_\_\_\_\_$$

7. The value of

$$\begin{vmatrix} x^2-x+1 & x-1 \\ x+1 & x+1 \end{vmatrix} = \_\_\_\_\_\_$$

8. The value of

$$\begin{vmatrix} b+c & a-b & a \\ c+a & b-c & b \\ a+b & c-a & c \end{vmatrix} = \_\_\_\_\_\_$$

9. If a,b,c are in A.P, then the value of

$$\begin{vmatrix} x+1 & x+2 & x+a \\ x+2 & x+3 & x+b \\ x+3 & x+4 & x+c \end{vmatrix} = \_\_\_\_\_\_\_\_$$

10. The value of

$$\begin{vmatrix} 0 & \sin x & -\cos x \\ -\sin x & 0 & \sin y \\ \cos x & -\sin y & 0 \end{vmatrix} \text{ is } \_\_\_\_\_\_\_\_$$

11. The value of

$$\begin{vmatrix} 2\sin\theta & -2\cos\theta \\ \cos\theta & \sin\theta \end{vmatrix} \text{ is } \_\_\_\_\_\_\_$$

12. The value of

$$\begin{vmatrix} 0 & 5 & -8 \\ -5 & 0 & 2 \\ 8 & -2 & 0 \end{vmatrix} = \_\_\_\_\_\_\_$$

13. The value of

$$\begin{vmatrix} x+y & y+z & z+x \\ z & x & y \\ 1 & 1 & 1 \end{vmatrix} = \_\_\_\_\_\_\_\_$$

14. The value of

$$\begin{vmatrix} \frac{1}{x} & \frac{1}{y} & \frac{1}{z} \\ x^2 & y^2 & z^2 \\ yz & zx & xy \end{vmatrix} = \_\_\_\_\_\_\_\_$$

15. The value of

$$\begin{vmatrix} -6 & 0 & 0 \\ 3 & -5 & 7 \\ 2 & 8 & 11 \end{vmatrix} = \_\_\_\_\_\_\_$$

16. The value of

$$\begin{vmatrix} x & 5 & y+z \\ y & 5 & z+x \\ z & 5 & x+y \end{vmatrix} = \_\_\_\_\_\_\_\_$$

17. If $\begin{vmatrix} x+y+2z & z & z \\ x & y+z+2x & x \\ y & y & z+x+2y \end{vmatrix} =$

$k(x+y+z)^3$ then $k = \_\_\_\_\_\_\_\_\_$

18. The value of

$$\begin{vmatrix} 1 & 2 & 1 \\ 2 & 4 & 3 \\ 3 & 6 & -9 \end{vmatrix} = \_\_\_\_\_\_\_\_$$

19. The value of

$$\begin{vmatrix} -9 & 2 & 7 \\ 3 & 16 & 4 \\ 18 & -4 & -14 \end{vmatrix} = \_\_\_\_\_\_\_$$

20. The value of

$$\begin{vmatrix} 2 & 0 & 0 \\ -41 & 7 & 0 \\ 139 & -17 & 10 \end{vmatrix} = \_\_\_\_\_\_\_$$

21. The minor of the element in the first row and 2nd column is _________

22. The cofactor of element in 3rd row and 2nd column of

$$\begin{vmatrix} 1 & 2 & 3 \\ 2 & 1 & 4 \\ 1 & 0 & 2 \end{vmatrix} \text{ is } \_\_\_\_\_\_\_\_\_\_$$

23. If ω is the imaginary cube root of 1, then

$$\begin{vmatrix} 1 & \omega^3 & 91 \\ \omega & \omega^7 & 92 \\ \omega^2 & \omega^5 & 98 \end{vmatrix} = \_\_\_\_\_\_\_\_$$

24. The value of

$$\begin{vmatrix} \sin^2 x & \cos^2 x & 1 \\ \cos^2 x & \sin^2 x & 1 \\ -10 & 12 & 2 \end{vmatrix} = \_\_\_\_$$

25. If ω is the inginary cube root of unit, then

$$\begin{vmatrix} 2 & \omega^2 & \omega \\ \omega & \omega & 1 \\ \omega^2 & 1 & \omega^2 \end{vmatrix} = \_\_\_\_\_\_\_\_$$

26. If $\begin{vmatrix} aa_1 & aa_2 & aa_3 \\ ab_1 & ab_2 & ab_3 \\ ac_1 & ac_2 & ac_3 \end{vmatrix} = k \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$

then $k =$__________

27. The value of

$$\begin{vmatrix} 0 & b-a & c-a \\ a-b & 0 & c-b \\ a-c & b-c & 0 \end{vmatrix} = \_\_\_\_\_$$

28. The value of

$$\begin{vmatrix} 1 & 1 & 1 \\ 1 & -1-\omega^2 & \omega^2 \\ 1 & \omega^2 & \omega^4 \end{vmatrix} = \_\_\_\_\_\_$$

29. If x, y, z are all different from zero and

$$\begin{vmatrix} 1+x & 1 & 1 \\ 1 & 1+y & 1 \\ 1 & 1 & 1+z \end{vmatrix} = 0$$

then $\frac{1}{x} + \frac{1}{y} + \frac{1}{z} =$ ________

30. The value of

$$\begin{vmatrix} 2 & 3 & 4 \\ 5 & 6 & 8 \\ 6x & 9x & 12x \end{vmatrix} = \_\_\_\_\_\_\_$$

Answers

1. 8
2. -1
3. 0
4. 0
5. 0
6. 0
7. $x^3 - x^2 + 2$
8. $-\left(a^3 + b^3 + c^3 - 3abc\right)$
9. 0
10. 0
11. 2
12. 0
13. 0
14. 0
15. 666
16. 0
17. 2
18. 0
19. 0
20. 140
21. $\begin{vmatrix} 2 & 4 \\ 1 & 2 \end{vmatrix}$
22. $-\begin{vmatrix} 1 & 3 \\ 2 & 4 \end{vmatrix}$
23. 0
24. 0
25. 0
26. a^3
27. 0
28. $3\omega(\omega - 1)$
29. -1
30. 0

C. Answer in one sentence:

1. If $\begin{vmatrix} x+1 & w & w^2 \\ w^2 & x+w^2 & 1 \\ w^2 & 1 & x+w \end{vmatrix} = 0$ then what is the value of x.

2. If $\begin{vmatrix} x-a & 0 & 0 \\ a & x-b & 0 \\ a & b & x-c \end{vmatrix} = 0$ then what are the value of x.

3. For what value of x, the matrix $A = \begin{bmatrix} 2(x+1) & 2x \\ x & x-2 \end{bmatrix}$ is a singular matrix?

4. For what value of x, the matrix $\begin{bmatrix} 5-x & x+1 \\ 2 & 4 \end{bmatrix}$ is singular.

5. What is the value of $\begin{vmatrix} \cos 15^0 & \sin 15^0 \\ \sin 75^0 & \cos 75^0 \end{vmatrix}$?

6. What is the value of the determinant $\begin{vmatrix} 0 & 2 & 0 \\ 2 & 3 & 4 \\ 4 & 5 & 6 \end{vmatrix}$?

7. If $\begin{vmatrix} 2x+5 & 3 \\ 5x+2 & 9 \end{vmatrix} = 0$ then what is the value of x?

8. What positive value of x makes the following pair of determinants equal?
$\begin{vmatrix} 2x & 3 \\ 5 & x \end{vmatrix} = \begin{vmatrix} 16 & 3 \\ 5 & 2 \end{vmatrix}$

9. If $A = \begin{bmatrix} 3 & 1 \\ 2 & -3 \end{bmatrix}$ then find $|adjA|$.

10. If $\begin{vmatrix} x & x \\ 1 & x \end{vmatrix} = \begin{vmatrix} 3 & 4 \\ 1 & 2 \end{vmatrix}$ then write the positive value of x.

<u>**Hints & Solutions**</u>

1. $x = 0$

Hints : Given that

$$\begin{vmatrix} x+1 & w & w^2 \\ w & x+w^2 & 1 \\ w^2 & 1 & x+w \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} x+1+w+w^2 & w & w^2 \\ w+x+w^2+1 & x+w^2 & 1 \\ w^2+1+x+w & 1 & x+w \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} x & w & w^2 \\ x & x+w^2 & 1 \\ x & 1 & x+w \end{vmatrix} = 0$$

$$\Rightarrow x\begin{vmatrix} 1 & w & w^2 \\ 1 & x+w^2 & 1 \\ 1 & 1 & x+w \end{vmatrix} = 0$$

$\Rightarrow x.x^2 = 0$

$\Rightarrow x^3 = 0$

$\Rightarrow x = 0$

2. $x = a, b, c$

Hints $\begin{vmatrix} x-a & 0 & 0 \\ a & x-b & 0 \\ a & b & x-c \end{vmatrix} = 0$

$\Rightarrow (x-a)\left[(x-b)(x-c)-0\right] = 0$

$\Rightarrow (x-a)(x-b)(x-c) = 0$

$\Rightarrow x = a, b, c$.

3. $x = -2$

Reason: The matrix

$A = \begin{bmatrix} 2(x+1) & 2x \\ x & x-2 \end{bmatrix}$ is a singular matrix

$\therefore \begin{vmatrix} 2(x+1) & 2x \\ x & x-2 \end{vmatrix} = 0$

$\Rightarrow 2(x+1)(x-2)-2x^2=0$

$\Rightarrow -2x=4$

$\Rightarrow x=-2$

4. $x=3$

5. The given determinant

$$=\begin{vmatrix} \cos 15^0 & \sin 15^0 \\ \sin 75^0 & \cos 75^0 \end{vmatrix}$$

$=\cos 75^0 \cos 15^0 - \sin 75^0 \sin 15^0$

$=\cos\left(75^0+15^0\right)$

$=\cos 90^0 = 0$

6. 8

7. -13

8. $x=4$

9. -11

Hints : $A=\begin{bmatrix} 3 & 1 \\ 2 & -3 \end{bmatrix}$

$\therefore Adj\ A=\begin{bmatrix} -3 & -1 \\ -2 & 3 \end{bmatrix}$

$\therefore |Adj\ A|=\begin{vmatrix} -3 & -1 \\ -2 & 3 \end{vmatrix}=-11$

10. $x=2$

D. Answer in one sentence:

1. Define the minor of an element in a determinant.

2. Define the cofactor of an element in a determinant.

3. If the vertices of a triangle are $(x_1, y_1), (x_2, y_2)$ and (x_3, y_3), then write the area of the triangle in determinant form.

4. Determine the value of the determinant $\begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{vmatrix}$.

5. Find the value of the determinant $\begin{vmatrix} x & a & x+a \\ y & b & y+b \\ z & c & z+c \end{vmatrix}$.

6. If a_{ij} is the element is the i^{th} row and j^{th} column of a 3rd order determinant whose value is 1 and c_{ij} is the cofactor of a_{ij} then what is the value a $a_{11}(c_u + c_{21}) + a_{12}(c_{1_2} + c_{22}) + a_{13}$ $(c_{13} + c_{23})$?

7. What are the value of x and y if $\begin{vmatrix} x & y \\ 1 & 1 \end{vmatrix}=2$ and $\begin{vmatrix} x & 3 \\ y & 2 \end{vmatrix}=1$

8. What is the value of $\begin{vmatrix} 224 & 777 & 32 \\ 735 & 888 & 105 \\ 812 & 999 & 116 \end{vmatrix}$.

9. What is the value of k if

$$\begin{vmatrix} aa_1 & aa_2 & aa_3 \\ ab_1 & ab_2 & ab_3 \\ ac_1 & ac_2 & ac_3 \end{vmatrix} = k \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

10. What is the value of the determinant

$$\begin{vmatrix} 1 & 0 & -5863 \\ -7361 & 2 & 7361 \\ 1 & 0 & 4137 \end{vmatrix} ?$$

Hints & Solutions

1. The minor of am element a_{ij} in a determinant is the determinant of just lower order obtained by deleting the i^{th} row and j^{th} column.

2. The cofactor of any element a_{ij} in a determinant is defined $(-1)^{i+j}\ M_{ij}$ where M_{ij} is the minor of a_{ij}.

$\therefore C_{ij} = (-1)^{i+j} M_{ij}$ where C_{ij} is the cofactor of a_{ij}

3. The area of the triangle whose vertics are $(x_1, y_1), (x_2, y_2)$ and (x_3, y_3) is

$$\Delta = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

4. $\begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{vmatrix} = 1(45-48) - 2$

$(36-42) + 3\ (32-35)$

$= -3 - 2(-6) + 3(-3)$

$= -3 + 12 - 9$

$= 0$

5. The given determinant

$$= \begin{vmatrix} x & a & x+a \\ y & b & y+b \\ z & c & z+c \end{vmatrix}$$

$$= \begin{vmatrix} x+a & x & x+a \\ y+b & b & y+b \\ z+c & c & z+c \end{vmatrix} = 0$$

6. $a_{11}(c_{11}+c_{21}) + a_{12}(c_{12}+c_{22}) + a_{13}(c_{13}+c_{23})$

$= (a_{11}c_{11} + a_{12}c_{12} + a_{13}c_{13}) + (a_{11}c_{21} + a_{12}c_{22}$
$+ a_{13}c_{23})$

$= 1 + 0 = 1$

7. $\begin{vmatrix} x & y \\ 1 & 1 \end{vmatrix} = 2\ \&\ \begin{vmatrix} x & 3 \\ y & 2 \end{vmatrix} = 1$

$\Rightarrow x - y = 2 \qquad \& 2x - 3y = 1$

$\Rightarrow x - y = 2$

$2x - 3y = 1$

$\Rightarrow x = 5,\ y = 3$

8. $\begin{vmatrix} 224 & 777 & 32 \\ 735 & 888 & 105 \\ 812 & 999 & 116 \end{vmatrix}$

$$= \begin{vmatrix} 7\times 32 & 777 & 32 \\ 7\times 105 & 888 & 105 \\ 7\times 116 & 999 & 116 \end{vmatrix}$$

$= 7 \times 0 = 0$

9. $k = a^3$

10. 20,000

Group -B

Short type (Questions & Answers)

1. Solve $\begin{vmatrix} x+1 & w & w^2 \\ w & x+w^2 & 1 \\ w^2 & 1 & x+w \end{vmatrix} = 0$

2. Solve $\begin{vmatrix} 0 & x+a & x-b \\ x+a & 0 & x-c \\ x+b & x+c & 0 \end{vmatrix} = 0$

3. Show that

$$\begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix} = (a+b+c)^3$$

4. Prove without expanding that

$$\begin{vmatrix} bc & a & a^2 \\ ca & b & b^2 \\ ab & c & c^2 \end{vmatrix} = \begin{vmatrix} 1 & a^2 & a^3 \\ 1 & b^2 & b^3 \\ 1 & c^2 & c^3 \end{vmatrix}$$

5. Without expanding factorize the determinant

$$\begin{vmatrix} x^3-a^3 & x^2 & x \\ b^3-a^3 & b^2 & b \\ c^3-a^3 & c^2 & c \end{vmatrix}$$

6. Show that $(a+1)$ is a factor of

$$\begin{vmatrix} a+1 & 2 & 3 \\ 1 & a+1 & 3 \\ 3 & -6 & a+1 \end{vmatrix}$$

7. Prove that

$$\begin{vmatrix} a & b & c \\ x & y & z \\ p & q & r \end{vmatrix} = \begin{vmatrix} y & b & q \\ x & a & p \\ z & c & r \end{vmatrix} = \begin{vmatrix} x & y & z \\ p & q & r \\ a & b & c \end{vmatrix}$$

8. Factorise

$$\begin{vmatrix} x+a & b & c \\ b & x+c & a \\ c & a & x+b \end{vmatrix}$$

9. Show that

$$\begin{vmatrix} a^2+1 & ab & ac \\ ab & b^2+1 & bc \\ ac & bc & c^2+1 \end{vmatrix} = 1+a^2+b^2+c^2$$

10. Solve for x

$$\begin{vmatrix} 15-2x & 11 & 10 \\ 11-3x & 17 & 16 \\ 7-x & 14 & 13 \end{vmatrix} = 0$$

11. Find the value of

$$\begin{vmatrix} 17 & 58 & 97 \\ 19 & 60 & 99 \\ 18 & 59 & 98 \end{vmatrix}$$

12. Solve $\begin{vmatrix} 1+x & 1 & 1 \\ 1 & 1+x & 1 \\ 1 & 1 & 1+x \end{vmatrix} = 0$

13. Solve $\begin{vmatrix} x-a & 0 & 0 \\ a & x-b & 0 \\ a & b & x-c \end{vmatrix} = 0$

14. Prove that

$$\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^2 & b^2 & c^2 \end{vmatrix} = (a-b)(b-c)(c-a)$$

15. Without expanding prove that

$$\begin{vmatrix} 12 & 2 & 4 & -5 & 1 \\ -8 & 1 & -5 & 2 & -1 \\ 6 & 4 & a & -3 & 2 \\ -10 & 2 & 1 & 3 & 4 \\ -2 & 4 & 6 & 8 & -5 \end{vmatrix} = \begin{vmatrix} 12 & -4 & -8 & 10 & -2 \\ 4 & 1 & -5 & 2 & -1 \\ -3 & 4 & a & -3 & 2 \\ 5 & 2 & 1 & 3 & 4 \\ 1 & 4 & 6 & 8 & -5 \end{vmatrix}$$

16. Prove that

$$\begin{vmatrix} 1 & a & a^2 \\ a^2 & 1 & a \\ a & a^2 & 1 \end{vmatrix}$$ is a perfect square.

17. Without expanding find the value of the determinant

$$\begin{vmatrix} 3 & 6 & 9 \\ -2 & 4 & -6 \\ 8 & 16 & 24 \end{vmatrix}$$

18. Prove that

$$\begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix} = abc\left(1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)$$

19. Prove that

$$\begin{vmatrix} a+b+2c & a & b \\ c & b+c+2a & b \\ c & a & c+a+2b \end{vmatrix}$$

$$= 2(a+b+c)^3$$

20. If $x = cy + bz, y = az + cx$ and $z = bx + ay$ where x, y, z are not all zero, then prove that $a^2 + b^2 + c^2 + 2abc = 1$ by determinant

21. Eliminate x, y, z from

$$a = \frac{x}{y-z}, b = \frac{y}{z-x}, c = \frac{z}{x-y}$$

22. Using properties of the determinants,

Prove that

$$\begin{vmatrix} b+c & c+a & a+b \\ q+r & r+p & p+q \\ y+z & z+x & x+y \end{vmatrix} = 2\begin{vmatrix} a & b & c \\ p & q & r \\ x & y & z \end{vmatrix}$$

23. Prove that

$$\begin{vmatrix} a+x & y & z \\ x & a+y & z \\ x & y & a+z \end{vmatrix} = a^3(a+x+y+z)$$

24. Prove that

$$\begin{vmatrix} b+c & a & a \\ b & c+a & b \\ c & c & a+b \end{vmatrix} = 4abc$$

25. Prove that

$$\begin{vmatrix} a & a^2 & bc \\ b & b^2 & ca \\ c & c^2 & ab \end{vmatrix} = (a-b)(b-c)(c-a)$$

$.(ab+bc+ca)$

Hints & Solutions

1. The given equation is

$$\begin{vmatrix} x+1 & w & w^2 \\ w & x+w^2 & 1 \\ w^2 & 1 & x+w \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} x+1+w+w^2 & w & w^2 \\ w+x+w^2+1 & x+w^2 & 1 \\ w^2+1+x+w & 1 & x+w \end{vmatrix} = 0$$

$(c_1 \to c_1 + c_2 + c_3)$

$$\Rightarrow \begin{vmatrix} x+0 & w & w^2 \\ x+0 & x+w^2 & 1 \\ x+0 & 1 & x+w \end{vmatrix} = 0$$

$$\Rightarrow x\begin{vmatrix} 1 & w & w^2 \\ 1 & x+w^2 & 1 \\ 1 & 1 & x+w \end{vmatrix} = 0$$

$$\Rightarrow x.x^2 = 0$$

$$\Rightarrow x^3 = 0 \Rightarrow x = 0$$

2. The given equation is

$$\begin{vmatrix} 0 & x-a & x-b \\ x+a & 0 & x-c \\ x+b & x+c & 0 \end{vmatrix} = 0$$

$$\Rightarrow 0-(x-a)\begin{vmatrix} x+a & x-c \\ x+b & 0 \end{vmatrix} + (x+b)$$

$$\begin{vmatrix} x+a & 0 \\ x+b & x+c \end{vmatrix} = 0$$

$$\Rightarrow (x-a)(x+b)(x-c)+(x+a)(x-b)$$

$$(x+c)=0$$

$$\Rightarrow 2x\left(x^2-ab-bc+ca\right)=0$$

$$\Rightarrow x=0, x=\sqrt{ab+bc-ca}$$

3. L.H.S $= \begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix}$

$$= \begin{vmatrix} a+b+c & a+b+c & a+b+c \\ ab & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix}$$

$$\left[R_1 \to R_1+R_2+R_3\right]$$

$$=(a+b+c)\begin{bmatrix} 1 & 1 & 1 \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{bmatrix}$$

$$=(a+b+c)\begin{vmatrix} 1 & 0 & 0 \\ 2b & -a-b-c & 0 \\ 2c & 0 & -a-b-c \end{vmatrix}$$

$$\left[C_2 \to C_2-C_1 \,\&\, C_3 \to C_3-C_1\right]$$

$$=(a+b+c)^3$$

4. L.H.S $= \begin{vmatrix} bc & a & a^2 \\ ca & b & b^2 \\ ab & c & c^2 \end{vmatrix}$

$$= \begin{vmatrix} \frac{1}{a}(abc) & \frac{a^2}{a} & \frac{a^3}{a} \\ \frac{1}{b}(abc) & \frac{b^2}{b} & \frac{b^3}{b} \\ \frac{1}{c}(abc) & \frac{c^2}{c} & \frac{c^3}{x} \end{vmatrix}$$

$$= abc\begin{vmatrix} \frac{1}{a} & \frac{a^2}{a} & \frac{a^3}{a} \\ \frac{1}{b} & \frac{b^2}{b} & \frac{b^3}{b} \\ \frac{1}{c} & \frac{c^2}{c} & \frac{c^3}{c} \end{vmatrix}$$

$$=(abc)\frac{1}{abc}\begin{vmatrix} 1 & a^2 & a^3 \\ 1 & b^2 & b^3 \\ 1 & c^2 & c^3 \end{vmatrix} = \begin{vmatrix} 1 & a^2 & a^3 \\ 1 & b^2 & b^3 \\ 1 & c^2 & c^3 \end{vmatrix}$$

5. Given determinant

$$= \begin{vmatrix} x^3-a^3 & x^2 & x \\ b^3-a^3 & b^2 & b \\ c^3-a^3 & c^2 & c \end{vmatrix}$$

$$= \begin{vmatrix} x^3 & x^2 & x \\ b^3 & b^2 & b \\ c^3 & c^2 & c \end{vmatrix} - \begin{vmatrix} a^3 & x^2 & x \\ a^3 & b^2 & b \\ a^3 & c^2 & c \end{vmatrix}$$

$$= xbc\begin{vmatrix} x^2 & x & 1 \\ b^2 & b & 1 \\ c^2 & c & 1 \end{vmatrix} - a^3\begin{vmatrix} 1 & x^2 & x \\ 1 & b^2 & b \\ 1 & c^2 & c \end{vmatrix}$$

$$=(xbc-a^3)\begin{vmatrix} x^2 & x & 1 \\ b^2 & b & 1 \\ c^2 & c & 1 \end{vmatrix}$$

$$= (xbc - a^3)\begin{vmatrix} x^2 - 2 & x-b & 0 \\ b^2 - c^2 & b-c & 0 \\ c^2 & c & 1 \end{vmatrix}$$

$$[R_1 \to R_1 - R_2 \ \& \ R_2 \to R_2 - R_3]$$

$$= (xbc - a^3)(x-b)(b-c)\begin{vmatrix} x+b & 1 & 0 \\ b+c & 1 & 0 \\ c^2 & c & 1 \end{vmatrix}$$

$$= (xbc - a^3)(x+b)(b-c)(x-c)$$

6. Let $\Delta = \begin{vmatrix} a+1 & 2 & 3 \\ 1 & a+1 & 3 \\ 3 & -6 & a+1 \end{vmatrix}$

Puting $a = -1$, in the above, we get

$$\Delta = \begin{vmatrix} 0 & 2 & 3 \\ 1 & 0 & 3 \\ 3 & -6 & 0 \end{vmatrix} = 0$$

so $a+1$ is a factor.

7. Exercise to the students

8. $\begin{vmatrix} x+a & b & c \\ b & x+c & a \\ c & a & x+b \end{vmatrix}$

$$= \begin{vmatrix} x+a+b+c & b & c \\ b+x+c+a & x+c & a \\ c+a+x+b & a & x+b \end{vmatrix}$$

$$(C_1 \to C_1 + C_2 + C_3)$$

$$= (x+a+b+c)\begin{vmatrix} 1 & b & c \\ 1 & x+c & a \\ 1 & a & x+b \end{vmatrix}$$

$$= (x+a+b+c)\begin{vmatrix} 1 & b & c \\ 0 & x+c-y & a-c \\ 0 & a-b & x+b-c \end{vmatrix}$$

$$= (x+a+b)\begin{vmatrix} a+c-b & a-c \\ a-b & a+b-c \end{vmatrix}$$

$$(R_1 \to R_2 - R_1 \ R_3 \to R_3 - R_1)$$

$$= (x+a+b)(x^2 - a^2 - b^2 - c^2$$
$$+ab+bc+ca)$$

10. $\begin{vmatrix} 15-2x & 11 & 10 \\ 11-3x & 17 & 16 \\ 7-x & 14 & 13 \end{vmatrix} = 0$

$$\begin{vmatrix} 15-2x & 1 & 10 \\ 11-3x & 1 & 16 \\ 7-x & 1 & 13 \end{vmatrix} = 0 (C_2 \to C_2 - C_3)$$

$$\Rightarrow \begin{vmatrix} 15-2x & 1 & 10 \\ -4-x & 0 & 6 \\ -8+x & 0 & 3 \end{vmatrix} = 0$$

$$(R_2 \to R_2 - R_1 \ R_3 \to R_3 - R_1)$$

$$\Rightarrow 3x = 60$$

$$\Rightarrow x = 20$$

11. $\begin{vmatrix} 17 & 58 & 97 \\ 19 & 60 & 99 \\ 18 & 59 & 98 \end{vmatrix}$

$$= \begin{vmatrix} 17 & 58 & 97 \\ 2 & 2 & 2 \\ 1 & 1 & 1 \end{vmatrix}$$

$$[R_2 \to R_2 - R_1 R_3 \to R_3 - R_1]$$

$$= 0$$

12. Given equation is

$$\begin{vmatrix} 1+x & 1 & 1 \\ 1 & 1+x & 1 \\ 1 & 1 & 1+x \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} 3+x & 1 & 1 \\ 3+x & 1+x & 1 \\ 3+x & 1 & 1+x \end{vmatrix} = 0$$

$[C_1 \to C_1 + C_2 + C_3]$

$$\Rightarrow (3+x)\begin{vmatrix} 1 & 1 & 1 \\ 0 & x & 0 \\ 0 & 0 & x \end{vmatrix}$$

$$\Rightarrow (3+x)x^2 = 0$$

$$\Rightarrow x = 0, -3$$

14. $$\begin{vmatrix} 12 & 2 & 4 & -5 & 1 \\ -8 & 1 & -5 & 2 & -1 \\ 6 & 4 & a & -3 & 2 \\ -10 & 2 & 1 & 3 & 4 \\ -2 & 4 & 6 & 8 & -5 \end{vmatrix}$$

$$= \begin{vmatrix} (-2)(-6) & 2 & 4 & -5 & 1 \\ (-2)4 & 1 & -5 & 2 & -1 \\ (-2)(-3) & 4 & a & -3 & 2 \\ (-2)5 & 2 & 1 & 3 & 4 \\ (-2)1 & 4 & 6 & 8 & -5 \end{vmatrix}$$

$$= -2\begin{vmatrix} -6 & 2 & 4 & -5 & 1 \\ 4 & 1 & -5 & 2 & -1 \\ -3 & 4 & a & -3 & 2 \\ 5 & 2 & 1 & 3 & 4 \\ 1 & 4 & 6 & 8 & -5 \end{vmatrix}$$

$$= \begin{vmatrix} (-2)(-6) & (-2)2 & (-2)4 & (-2)(-5) & (-2)1 \\ 4 & 1 & -5 & 2 & -1 \\ -3 & 4 & a & -3 & 2 \\ 5 & 2 & 1 & 3 & 4 \\ 1 & 4 & 6 & 8 & -5 \end{vmatrix}$$

$$\begin{vmatrix} 12 & -4 & -8 & 10 & -2 \\ 4 & 1 & -5 & 2 & -1 \\ -3 & 4 & a & -3 & 2 \\ 5 & 2 & 1 & 3 & 4 \\ 1 & 4 & 6 & 8 & -5 \end{vmatrix}$$

19. $$\begin{vmatrix} a+b+2c & a & b \\ c & b+c+2a & b \\ c & a & c+a+2b \end{vmatrix}$$

$$= \begin{vmatrix} 2(a+b+c) & a & b \\ 2(a+b+c) & b+c+2a & b \\ 2(a+b+c) & a & c+a+2b \end{vmatrix}$$

$[C_1 \to C_1 + C_2 + C_3]$

$$= 2(a+b+c)\begin{vmatrix} 1 & a & b \\ 1 & b+c+2a & b \\ 1 & a & c+a+2b \end{vmatrix}$$

$$= 2(a+b+c)\begin{vmatrix} 1 & a & b \\ 0 & b+c+a & 0 \\ 0 & 0 & c+a+b \end{vmatrix}$$

$$= 2(a+b+c)^3$$

20. Given that $x = cy + bz$

$$y = az + cx$$

$$z = bx + ay$$

$$\Rightarrow x - cy - bz = 0$$

$$cx - y + az = 0$$

$$yx + ab - z = 0$$

Eliminating x, y, z from the above equation

$$\begin{vmatrix} 1 & -c & -b \\ c & -1 & a \\ b & a & -1 \end{vmatrix} = 0$$

$$\Rightarrow 1(1-a^2) - (-c)(-c-ab) + (-b)$$

$$(ac+b) = 0$$

$$\Rightarrow 1 - a^2 - c^2 - abc - abc - b^2 = 0$$

$$\Rightarrow a^2 + b^2 + c^2 + 2abc = 0$$

21. Given that $a=\dfrac{x}{y-z}$

$\Rightarrow x=ay-az$

$\Rightarrow x-ay+az=0$ ....(1)

similaly $bx+y-bz=0$ ....(2)

and $cx-cy-z=0$ ....(3)

Eliminating x,y,z from (1), (2) & (3), we

get $\begin{vmatrix}1 & -a & a\\ b & 1 & -b\\ c & -c & -1\end{vmatrix}=0$

$\Rightarrow ab+bc+ca+1=0$

22. Let $\Delta=\begin{vmatrix}b+c & c+a & a+b\\ q+r & r+p & p+q\\ y+z & z+x & x+y\end{vmatrix}$

$=\begin{vmatrix}2(a+b+c) & c+a & a+b\\ 2(p+q+r) & r+p & p+q\\ 2(x+y+z) & z+x & x+y\end{vmatrix}$

$[C_1\to C_1+C_2+C_3]$

$=2\begin{vmatrix}a+b+c & c+a & a+b\\ p+q+r & r+p & p+q\\ x+y+z & z+x & x+y\end{vmatrix}$

$=2\begin{vmatrix}a+b+c & -b & -c\\ p+q+r & -q & -r\\ x+y+z & -y & -z\end{vmatrix}$

$[C_2\to C_2-C_1C_3\to C_3-C_1]$

$=2\begin{vmatrix}a & -b & -c\\ p & -q & -r\\ x & -y & -z\end{vmatrix}$ $[C_1\to C_1+C_2+C_3]$

$=2\begin{vmatrix}a & b & c\\ p & q & r\\ x & y & z\end{vmatrix}$

$=2\begin{vmatrix}a & p & x\\ b & q & y\\ c & r & z\end{vmatrix}$

[changing to rows in to columns]

Group-C

Long type (Questions & Answers)

1. Prove that

$$\begin{vmatrix} (b+c)^2 & a^2 & bc \\ (c+a)^2 & b^2 & ca \\ (a+b)^2 & c^2 & ab \end{vmatrix} = (a^2+b^2c^2)(a+b+c)$$

$(b-c)(c-a)(a-b)$

2. If $x \neq y \neq z$ $\begin{vmatrix} x & x^2 & 1+x^3 \\ y & y^2 & 1+y^3 \\ z & z^2 & 1+z^3 \end{vmatrix} = 0$

then show that $1+xyz=0$

3. Show that

$$\begin{vmatrix} x & y & z \\ x^2 & y^2 & z^2 \\ yz & zx & xy \end{vmatrix} = (x-y)(y-z)(z-x)$$

$(xy+yz+zx)$

4. Prove that

$$\begin{vmatrix} a^3-x^3 & a^2 & a \\ b^3-x^3 & b^2 & b \\ c^3-x^3 & c^2 & c \end{vmatrix} = (a-b)(b-c)(a-c)$$

$(abc-x^3)$

5. Prove that

$$\frac{1}{bc+ca+ab}\begin{vmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ bc & ca & ab \end{vmatrix}$$

$= (b-c)(c-a)(a-b)$

6. Prove that

$$\begin{vmatrix} y+z & x & x \\ y & z+x & y \\ z & z & x+y \end{vmatrix} = 4xyz$$

7. Prove that

$$\begin{vmatrix} a^2+1 & ab & ac \\ ab & b^2+1 & bc \\ ac & bc & c^2+1 \end{vmatrix} = 1+a^2+b^2+c^2$$

and write its minimum value

8. Show that

$$\begin{vmatrix} (y+z)^2 & xy & zx \\ xy & (x+z)^2 & yz \\ xz & yz & (x+y)^2 \end{vmatrix}$$

$= 2xyz(x+y+z)^3$

9. Prove that

$$\begin{vmatrix} (b+c)^2 & a^2 & a^2 \\ b^2 & (c+a)^2 & b^2 \\ c^2 & c^2 & (a+b)^2 \end{vmatrix}$$

$= 2abc(a+b+c)^3$

10. If $A+B+C=\pi$,

then prove that

$$\begin{vmatrix} \sin^2 A & \cot A & 1 \\ \sin^2 B & \cot B & 1 \\ \sin^2 C & \cot C & 1 \end{vmatrix} = 0$$

Hints & Solutions

1. $$L.H.S = \begin{vmatrix} (b+c)^2 & a^2 & bc \\ (c+a)^2 & b^2 & ca \\ (a+b)^2 & c^2 & ab \end{vmatrix}$$

$$= \begin{vmatrix} (b+c)^2 - 2bc & a^2 & bc \\ (c+a)^2 - 2ca & b^2 & ca \\ (a+b)^2 - 2ab & c^2 & ab \end{vmatrix}$$

$$(C_1 \to C_1 - 2C_3)$$

$$= \begin{vmatrix} b^2+c^2 & a^2 & bc \\ c^2+a^2 & b^2 & ca \\ a^2+b^2 & c^2 & ab \end{vmatrix}$$

$$= \begin{vmatrix} a^2+b^2+c^2 & a^2 & bc \\ a^2+b^2+c^2 & b^2 & ca \\ a^2+b^2+c^2 & c^2 & ab \end{vmatrix} (C_1 \to C_1 + C_2)$$

$$= (a^2+b^2+c^2)\begin{vmatrix} 1 & a^2 & bc \\ 1 & b^2 & ca \\ 1 & c^2 & ab \end{vmatrix}$$

$$= (a^2+b^2+c^2)(a+b+c)(b-c)$$
$$(c-a)(a-b)$$

2. L.H.S $$\begin{vmatrix} x & x^2 & 1+x^3 \\ y & y^2 & 1+y^3 \\ z & z^2 & 1+z^3 \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} x & x^2 & 1 \\ y & y^2 & 1 \\ z & z^2 & 1 \end{vmatrix} + \begin{vmatrix} x & x^2 & x^3 \\ y & y^2 & y^3 \\ z & z^2 & z^3 \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix} + xyz\begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix}(1+xyz) = 0$$

$$\Rightarrow (1+xyz)(x-y)(y-z)(z-x) = 0$$

..........(1)

$$\Rightarrow 1+xyz = 0 \qquad (\because x \neq y \neq z)$$

10. Given that $A+B+C=\pi$

$$\Rightarrow A+B = \pi - C$$

$$\therefore \sin(A+B) = \sin(\pi - C) = \sin C$$

Also $\sin(B+C) = \sin A$

$$\Delta = \begin{vmatrix} \sin^2 A & \cot A & 1 \\ \sin^2 B & \cot B & 1 \\ \sin^2 C & \cot C & 1 \end{vmatrix}$$

$$= \begin{bmatrix} \sin^2 A - \sin^2 B & \cot A - \cot B & 0 \\ \sin^2 B - \sin^2 C & \cot B - \cot C & 0 \\ \sin^2 C & \cot C & 1 \end{bmatrix}$$

$$[R_1 \to R_1 - R_2\ R_2 \to R_2 - R_3]$$

$$= \begin{vmatrix} \sin^2 A - \sin^2 B & \cot A - \cot B \\ \sin^2 B - \sin^2 C & \cot B - \cot A \end{vmatrix}$$

$$= \begin{vmatrix} \sin(A+B)\sin(A-B) & \dfrac{\cos A}{\sin A} - \dfrac{\cos B}{\sin B} \\ \sin(B+C)\sin(B-C) & \dfrac{\cos B}{\sin B} - \dfrac{\cos C}{\sin C} \end{vmatrix}$$

$$= \begin{vmatrix} \sin C & \sin(A-B) & \dfrac{-\sin(A-B)}{\sin A - \sin B} \\ \sin A & \sin(B-C) & \dfrac{-\sin(B-C)}{\sin B \sin C} \end{vmatrix} = 0$$

CHAPTER - 6

PROBABILITY

Group - A

A. Choose the correct answer from the given choices:

1. If 2 P (A) = P (B) $= \frac{5}{13}$ and $P(A|B) = \frac{2}{5}$ then what is $P(A \cup B)$?

(a) $\frac{11}{15}$ (b) $\frac{11}{18}$

(c) $\frac{11}{26}$ (d) $\frac{11}{28}$

2. If E and F are events such that P(E)=0.6, P(F)=0.3 and $P(E \cap F) = 0.2$ then $P(E|F)$ is

(a) $\frac{5}{6}$ (b) $\frac{4}{5}$

(c) $\frac{3}{4}$ (d) $\frac{2}{3}$

3. One card is drawn from a pack of 52 cards. What is the probability that the card is drawn is either king or spade ?

(a) $\frac{4}{13}$ (b) $\frac{5}{13}$

(c) $\frac{6}{13}$ (d) $\frac{9}{13}$

4. A binomial distribution has mean 4 and variance 3. Write the number of trials.

(a) 12 (b) 16

(c) 20 (d) 24

5. If an event A is independent to if self, then what is P(A) ?

(a) $\frac{1}{4}$ or $\frac{1}{5}$ (b) $\frac{1}{2}$ or $\frac{1}{3}$

(c) 0 or 1 (d) $\frac{1}{6}$ or $\frac{1}{7}$

6. If P(A) = 0.6, P(B)=04, $P(A \cap B) = 0.2$ then what is the value of $P(B|A)$?

(a) $\frac{1}{2}$ (b) $\frac{1}{3}$

(c) $\frac{1}{4}$ (d) $\frac{1}{5}$

7. If P(B) = 0.5 and $P(A \cap B) = 0.32$ then what is $P(A|B)$?

(a) $\frac{10}{19}$ (b) $\frac{12}{23}$

(c) $\frac{16}{25}$ (d) $\frac{20}{33}$

8. If A and B are independent events and $P(A) = \frac{3}{5}$, $P(B) = \frac{1}{5}$ then what is $P(A \cap B)$?

(a) $\frac{3}{20}$ (b) $\frac{3}{25}$

(c) $\frac{3}{26}$ (d) $\frac{3}{29}$

9. If $P(B) = 0.5$ and $P(A \cap B) = 0.32$ then find $P(A|B)$.

(a) $\frac{13}{27}$ (b) $\frac{15}{23}$

(c) $\frac{16}{25}$ (d) $\frac{17}{26}$

10. If A and B are two events such that $P(A|B) = P(B|A)$ then what is the relation between P(A) and P (B).

(a) $P(A) = P(B)$ (b) $P(A) < P(B)$

(c) $P(A) > P(B)$ (d) None of these

B. Fill in the blanks from the given choices.

1. If $P(A) = 0.8$, $P(B) = 0.5$ and $P(B|A) = 0.4$ then $P(A|B)$ is ______.

   (i) 0.54 (ii) 0.64
   (iii) 0.74 (iv) 0.84

2. If A and B are independent events $P(A) = \frac{3}{7}$ and $P(B) = \frac{4}{7}$ then $P(A \cap B)$ is ______.

   (i) $\frac{18}{39}$ (ii) $\frac{11}{40}$
   (iii) $\frac{12}{49}$ (iv) $\frac{13}{50}$

3. If E and F are events such that $P(E) = 0.6$, $P(F) = 0.3$ and $P(E \cap F) = 0.2$, then $P(F|E) =$ ______.

   (i) $\frac{1}{2}$ (ii) $\frac{1}{3}$
   (iii) $\frac{1}{4}$ (iv) $\frac{1}{5}$

4. If $P(A) = 0.8$, $P(B)=0.5$ and $P(B|A) = 0.4$ then $P(A \cap B) =$ ______.

   (i) 0.18 (ii) 0.22
   (iii) 0.26 (iv) 0.32

5. If $P(A) = 0.8$, $P(B) = 0.5$ and $P(B|A) = 0.4$ then $P(A|B) =$ ______.

   (i) 0.42 (ii) 0.52
   (iii) 0.64 (iv) 0.82

6. If A and B are two events such that $P(A)=0.3$, $P(B)=0.4$ and $P(A \cup B) = 0.6$, then $P(A \cap B) =$ ______.

   (i) 0.1 (ii) 0.3
   (iii) 0.42 (iv) 0.5

7. If $P(A) = 0.8$ and $P(B|A) = 0.4$ then $P(A \cap B) =$ ______.

   (i) 0.25 (ii) 0.32
   (iii) 0.42 (iv) 0.52

8. If $P(B) = 0.5$, $P(A \cap B) = 0.32$, then $P(A|B) =$ ______.

   (i) 0.44 (ii) 0.54
   (iii) 0.64 (iv) 0.74

9. If $P(A)=0.8$, $P(B)=0.5$ and $P(A \cap B) = 0.32$ then $P(A \cup B) =$ ______.

   (i) 0.38 (ii) 0.58
   (iii) 0.78 (iv) 0.98

10. If A and B are events such that A and B are independent events and $P(A) = \frac{3}{5}$, $P(B) = \frac{3}{10}$ then $P(A \cap B) =$ ______.

    (i) $\frac{9}{12}$ (ii) $\frac{9}{10}$
    (iii) $\frac{9}{50}$ (iv) $\frac{9}{70}$

Group - B

Short Type (Questions & Answers)

1. If x follows a binomial distribution with parameter n=6 and p with 4p(x=4) = p(x=2). Find p. [CHSE 2016 (A)]

2. Two different digits are selected at random from the digits 1 through 9. If the sum is even, what is the probability that 3 is one of the digits selected ? [CHSE-2015 (A)]

3. Suppose that the probability of your alarm goes off in the morning is 0.9. If the alarm goes off, the probability is 0.8 that you attend your 8 A.M. class. If the alarm does not go off, the probability that you make your 8 A.M. class is 0.5. Find the probability that you make your 8 A.M. class. [CHSE-2015 (A)]

4. If A and B are two events such that $P(A) = 0.6$, $P(B) = 0.5$ and $P(A \cap B) = 0.2$ then find $P(A|B^C)$. [CHSE-2008(A)]

5. A pair of dice is thrown. Find the probablity of getting of atleast 9 if 5 appears on atleast one of dice.

6. If two dice are bassed and if A be the event that one of dice is 3 and B is the event that sum 5 occurs, then find $P(A|B)$.

7. If A and B are two events such that P(A)=0.3, P(B)=0.4 and $P(A \cup B) = 0.6$ then find (i) $P(A|B^C)$ (ii) $P(B|A)$.

9. A couple has 2 children. Find the probability that both are boys, it is known that (i) one of them is a boy (ii) the older child is a boy.

10. A box of oranges is inspected by examining three randomly selected oranges drawn without replacement. It all three oranges are good and the box is approved for sale, otherwise rejected. Find the probability that a box containing 15 oranges out of which 12 are good and 3 are bad one will be approved for sale.

10. If A and B are two independent events and P(A)=0.3, P(B)=0.6 then find (i) P (A and B) (ii) P (A and not B) (iii) P (A or B) (iv) P (neither A or B).

11. A die is tossed thrice. Find the probability of getting an odd number atleast once.

12. Find the probability of distribution of number of heads in two tosses of a coin.

13. A random variable has the following probability distribution.

x :	0	1	2	3	4	5	6	7
p(x) :	0	2p	2p	3p	p^2	$2p^2$	$7p^2$	2p

What is p ?

14. An urn contains 4 white and 6 red balls. Four balls are drawn at random form the urn. Find the probability distribution of the number of white balls.

15. If $P(A) = 0.6$, $P(B|A) = 0.5$ find $P(A \cup B)$ when A and B are independent. [CHSE-2018(A)].

Hints and Solutions

1. Given that n = 6

Also given that 4p(x = 4) = p(x = 2)

$$\Rightarrow 4n_{c_4} p^4 q^{n-4} = n_{c_2} p^2 q^{n-2}$$

$$\Rightarrow 46_{c_4} p^4 q^2 = 6_{c_2} p^2 q^4$$

$$\Rightarrow 4.\frac{6.5.4.3}{4.3.2.1} p^4 q^2 = \frac{6.5}{2.1} p^2 q^4$$

$$\Rightarrow 4p^4q^2 - p^2q^4 = 0$$

$$\Rightarrow p^2q^2(4p^2 - q^2) = 0$$

$$\Rightarrow p^2(1-p)^2[4p^2 - (1-p)^2 = 0$$

$$\Rightarrow p^2(1-p)^2(3p^2 + 2p - 1) = 0$$

$$\Rightarrow p^2(1-p)^2[3p^2 + 3p - p - 1) = 0$$

$$\Rightarrow p^2(1-p)^2[3p(p+1) - (p+1)] = 0$$

$$\Rightarrow p^2(1-p)^2(p+1)(3p-1) = 0$$

$$\Rightarrow p = 0, 1, \frac{1}{2}$$

(p = -1 is rejected)

2. There are 9 digits 1, 2, 3, 4, 5, 6, 7, 8, 9. Two different digits are selected.

Let S be the sample space.

$\therefore |S| = 9x8 = 72$

We shall find the probability that 3 is one of the numbers selected if the sum is even.

Let A be the event where 3 is one of the numbers selected and B is the event where the sum of the numbers is even.

A = {(1,3), (2,3), (4,3), (5,3), (6,3), (7,3), (8,3), (9,3), (3,1), (3,2), (3,4), (3,5), (3,6), (3,7), (3,8), (3,9)}

$|A| = 16$

B = {(1,3), (1,5), (1,7), (1,9), (2,4), (2,6), (2,8), (3,1), (3,5), (3,7), (3,9), (4,2), (4,6), (4,8), (5,1), (5,3), (5,7), (5,9), (6,2), (6,4), (6,8), (7,1), (7,3), (7,5), (7,9), (8,2), (8,4), (8,6), (9,1), (9,3), (9,5), (9,7)}

$|B| = 32$.

$A \cap B$ = {(1,3), (5,3), (7,3), (9,3), (3,1), (3,5), (3,7), (3,9)}

$\therefore \quad |A \cap B| = 8$

Required probability

$= P(A|B)$

$$= \frac{P(A \cap B)}{P(B)} = \frac{\frac{8}{72}}{\frac{32}{72}} = \frac{8}{32} = \frac{1}{4}$$

3. Let A be the event that the alarm of one person goes of and B be the event that he makes his 8 A.M. class.

According to the question

P(A) = 0.9

P (A^C) = Probability that the alarm does not goes = 1 - 0.9 = 0.1

Let B be the event that he makes 8 A.M. class = He makes 8 A.M. class and the alarm goes off or He makes 8 A.M. class and the alarm does not go off.

$= (B \cap A) \cup (B \cap A^C)$

$\therefore \quad P(B) = P[(B \cap A) \cup (B \cap A^C)]$

$= P(B \cap A) + P(B \cap A^C)$

$= P(B).P(B|A) + P(A^C).P(B|A^C)$

$= 0.9 \text{ x } 0.8 + 0.1 \text{ x } 0.5$

$= 0,72 + 0.05 = 0.77$

4. Given that $P(A) = 0.6, P(B) = 0.5$

$P(A \cap B) = 0.2$

$\therefore \quad P(A|B^C)$

$$= \frac{P(A \cap B^C)}{P(B^C)}$$

$$= \frac{P(A) - P(A \cap B)}{1 - P(B)}$$

$$= \frac{0.6 - 6.2}{1 - 0.5} = \frac{4}{5}$$

5. A pair of dice is thrown.

Let S be the sample space.

$|S| = 36$

Let A be the event of getting at least 9 and B be the event more 5 appears on atleast one of the dice.

$A = \{(3,6),(4,5),(5,4),(6,3),(4,6),$
$(5,5),(6,4),(5,6),(6,5),(6,4)\}$

$B = \{(1,5), (2,5), (3,5), (4,5), (5,5), (6,5),$
$(5,1), (5, 2), (5,3), (5,4), (5,6\}$

$A \cap B = \{(4,5),(5,4),(5,5),(5,6),6,5)\}$

$|B| = 11, |A \cap B| = 5$.

P (of getting of atleast 9 if 5 appears on a alteast one of the dice).

$$= P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{\frac{5}{36}}{\frac{11}{36}} = \frac{5}{11}.$$

6. Two dice are tossed.

Let S be the sample space.

$|S| = 36$

Let A be the event that one of the dice is 3.

$A = \{(3,1), (3,2), (3,3), (3,4), (3,5), (3,6),$
$(1,3), (2,3), (4,3), (5,3), (6,3)\}$

B is the event that the sum is 5.

$B = \{(1, 4), (4,1), (2,3), (3,2)\}$

$A \cap B = \{(2,3),(3,2)\}$

$|A \cap B| = 2, |B| = 4$

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{\frac{2}{36}}{\frac{4}{36}} = \frac{2}{4} = \frac{1}{2}.$$

7. Given that $P(A) = 0.3$

$P(B) = 0.4, \ P(A \cup B) = 0.6$

We know

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\Rightarrow 0.6 = 0.3 + 0.4 - P(A \cap B)$

$\Rightarrow P(A \cap B) = 0.3 + 0.4 - 0.6$

$= 0.7 - 0.6 = 01$

(i) $$P(A|B^C) = \frac{P(A \cap B^C)}{P(B^C)}$$

$$= \frac{P(A) - P(A \cap B)}{1 - P(B)}$$

$$= \frac{0.3 - 0.1}{1 - 0.4} = \frac{0.2}{0.3} = \frac{2}{3}$$

(ii) $$P(B|A) = \frac{P(B \cap A)}{P(A)} = \frac{0.1}{0.3} = \frac{1}{3}$$

8. Let B represents older child which is a boy and b represents younger child which is also a boy.

Also let G represents older child which is a girl and g represents younger child which is a girl.

The sample space is

$S = \{Bb, Bg, Gg, Gb\}$

$|S| = 4$

Let A be the event that both children are boys.

$A = \{Bb\}$

$\therefore |A| = 1$

(i) B be the event such that atleast one of the children is a boy.

$B = \{Bb, Bg, Gb\}$ $|B| = 3$

$$P(B) = \frac{|B|}{|S|} = \frac{3}{4}$$

$A \cap B = \{Bb\}$, $|A \cap B| = 1$

$$P(A \cap B) = \frac{|A \cap B|}{|S|} = \frac{1}{4}$$

We have to find $P(A|B)$.

$$P(A|B) = \frac{|A \cap B|}{|B|} = \frac{\frac{1}{6}}{\frac{3}{4}} = \frac{1}{3}$$

(ii) Let B be the event such that the older child is a boy.

$B = \{Bb, Bg\}$, $|B| = 2$

$$P(B) = \frac{|B|}{|S|} = \frac{2}{4} = \frac{1}{2}$$

$A \cap B = \{Bb\}$ $\therefore |A \cap B| = 1$

$$P(A \cap B) = \frac{|A \cap B|}{|S|} = \frac{1}{4}$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{\frac{1}{4}}{\frac{1}{2}} = \frac{1}{2}$$

Required Probability $= \frac{1}{2}$.

9. Let A, B and C be the respective events that the First, Second and Third drawn orange is good.

Probability that First drawn orange is good

$P(A) = \frac{12}{15}$.

Probability of getting Second orange is good

is $P(B) = \frac{11}{14}$.

Similalry probability of getting third ornage is

good $P(c) = \frac{10}{13}$.

The box is approved for sale if all three oranges are good.

Thus the probability of getting all the three

oranges good $= \frac{12}{5} \cdot \frac{11}{14} \cdot \frac{10}{13} = \frac{44}{91}$.

10. Given that P(A)=0.3 and P(B)=0.6. A and B are independent events.

(i) P (A and B)

$= P(A \cap B)$

$= P(A).P(B)$

$= 0.3 \text{ x } 0.6 = 0.18$

(ii) P (A and not B)

$= P(A \cap B^1)$

$= P(A).P(B')$

= P (A) [1-P(B)]

= 0.3 x (1 - 0.6)

= 0.3 x 0.4

= 0.12

(iii) P(A or B)

$= P(A \cup B)$

$= P(A) + P(B) - P(A \cap B)$

= 0.3 + 0.6 = 0.18

= 0.72

(iv) P (neither A nor B)

$= P(A' \text{and } B')$

$= P(A' \cap B') = P(A \cup B)' = 1 - P(P \cup B)$

= 1-0.720 = 0.28.

11. Note : The probability of getting and odd number at least once.

= 1 - probability of getting an odd number in none of the thrnos).

When a die is thrown, there are 3 odd numbers on the die out of 6 numbers. Probability of getting an odd number.

$$= \frac{\text{Number of favourable cases}}{\text{Total number of cases}} = \frac{3}{6} = \frac{1}{2}$$

Probability of getting on even number

= 1 - probability of getting an odd number

$$= 1 - \frac{1}{2} = \frac{1}{2}$$

Probability of getting an even number when the die is tossed thrice

$$= P(E) = \frac{1}{2} x \frac{1}{2} x \frac{1}{2} = \frac{1}{8}$$

Probability of getting an odd number at least once.

= 1- probability of getting an odd number in one of the thrown.

$$= 1 - P(E) = 1 - \frac{1}{8} = \frac{7}{8}.$$

12. When a coin is tossed twice, then the sample space is

$S = \{HH, HT, TH, TT\}$

$\therefore |S| = 4$

Let X denotes the number of heads in any outcome in S.

X(HH)=2, X(HT)=1, X(TH)=1, X(TT)=0

Thus X takes the value of 0, 1 or 2

It is known that

$$P(HH) = P(HT) = P(TH) = P(TT) = \frac{1}{4}$$

P(X = 0) = P (tail occurs in both tosses)

$$P(= P(\{TT\}) = \frac{1}{4}.$$

P(X=1) = P (one head and one tail occurs)

$$= P(\{TH, HT\}) = \frac{2}{4} = \frac{1}{2}$$

P(X=2) = P (head occurs in both tosses)

$$P(\{HH\}) = \frac{1}{4}$$

The required probability distribution is as follows.

X	0	1	2
P(X)	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{4}$

13. The given distribution is a probability distribution. So the sum of all probability is 1

$\Rightarrow$ $0+2p+2p+3p+p^2+2p^2+7p^2+2p = 1$

$\Rightarrow$ $10p^2 + 9p - 1= 0$

$\Rightarrow$ $(10\,p - 1)(p + 1) = 0$

$\Rightarrow$ $p = \frac{1}{10}$ ($\because$ p = -1 is rejected)

14. Let X denote the number of white balls drawn from the urn. Since there are 4 white balls, therefore x can take the values 0, 1, 2, 3 and 4.

$P(X = 0)$

= Probability of getting no white ball

= Probabilitiy that four balls drawn are red

$$= \frac{6_{C_4}}{10_{C_4}} = \frac{1}{14}$$

$P\ (X = 1)$

= Probability of getting one white ball

$$= \frac{4_{C_1} \text{ x } 6_{C_3}}{10_{C_4}} = \frac{8}{21}$$

$P\ (X = 2)$

= Probability of getting two white balls

$$= \frac{4_{C_1} \text{ x } 6_{C_2}}{10_{C_4}} = \frac{6}{14}$$

$P\ (X = 3)$

= Probability getting three white balls

$$= \frac{4_{C_3} \text{ x } 6_{C_1}}{10_{C_4}} = \frac{4}{35}$$

$P(X = 4)$

= Probability of getting four white balls

$$= \frac{4_{C_4}}{10_{C_4}} = \frac{1}{210}.$$

Thus the probability distribution of X is given by

X	0	1	2	3	4
P(X)	$\frac{1}{4}$	$\frac{8}{21}$	$\frac{6}{14}$	$\frac{4}{35}$	$\frac{1}{210}$

15. Given that $P(A) = 0.6,\ P(B|A) = 0.5$

Given that A and B are independent events.

$$P(A \cap B) = P(A).P(B) \qquad .... (1)$$

Given that

$$P(B|A) = \frac{P(A \cap B)}{P(A)}$$

$$\Rightarrow 0.5 = \frac{P(A \cap B)}{0.6}$$

$$\Rightarrow P(A \cap B) = 0.5 \text{ x } 0.6 = 0.3$$

From (1), we get

$0.3 = 0.6 \text{ x } P(B)$

$$\Rightarrow P(B) = \frac{0.3}{0.6} = \frac{3}{6} = \frac{1}{2}$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$= 0.6 + 0.5 - 0.3 = 0.8$$

Group - C

Long Type (Questions & Answers)

1. Two balls are drawn from a bag containing 5 white and 7 black balls. Find the probability of selecting 2 white balls if

   (i) the first ball is not replaced before drawing the second.

   (ii) the first ball is replaced before drawing the second.

2. It two dice are tossed and if A is the event that one of the dice is 3 and B is the event that sum 5 occurs then find $P(A|B)$.

3. Three N.C.C. cadets A, B and C took part in a shooting competition. Their probailities of hitting the targets are respectively 0.8, 0.9 and 0.7. They fire once each. What is the probability that at least two shots hit the target?

4. If A and B are independent events such that $P(A \cap B) = \frac{3}{50}$ and $P(A \cup B) = \frac{11}{25}$ then find P(A) and P(B).

5. There 3 bags B_1, B_2 and B_3 having respectively 4 white, 5 black; 3 white, 5 black and 5 white, 2 black balls. A bag is chosen at random and a ball is drawn from it. Find the probability that the ball is white.

6. There are 25 girls and 15 boys in Class-XI and 30 boys and 20 girls in Class-XII. If a student is chosen from a Class, selected at random happens to be a boy, find the probability that he has been chosen from Class-XII. [CHSE-2020]

7. Out of the adult population in a village 50% are farmers, 30% do business and 20% are service holders. It is known that 10% of the farmers, 20% of the business holders and 50% of serviceholders are above poverty line. What is the probability that a member chosen from any one of the adult population selected at random, is above poverty line ? [CHSE-2019)

8. Two cards are drawn successively with replacement from a well shuffled pack of 52 cards. Find the probability distribution of the number of kings. Also determine the mean and variance of the number of kings.

9. Four cards are drawn successively with replacement from a well shuffled pack of 52 cards. Find the probability distribution of the number of aces. Calculate the mean and variance of the number of aces. [CHSE-2020]

10. Find the probability distribution of number of heads in three tosses of a coin.

Hints & Solutions

1. A bag contains 5 white and 7 black balls. Total number of balls = 5 + 7 = 12 balls.

(i) Here the 1st ball is not replaced before the 2nd ball is drawn.

We are to get 2 white balls in each draw. Since there are 5 white balls, probability of getting a white ball in 1st draw $= \frac{5}{12}$.

Probability of getting a white ball in 2nd draw $= \frac{4}{11}$.

Probability of getting 2 white balls

$= \frac{5}{12} \cdot \frac{4}{1} = \frac{5}{33}$.

(ii) Here 2 white balls are drawn. 1st ball is replaced before drawing the 2nd ball. Probability of getting 1st white ball $= \frac{5}{12}$.

Since this ball is replaced, probability of drawing 2nd white ball $= \frac{5}{12}$.

$\therefore$ Probability of getting 2 white balls

$= \frac{5}{12} \cdot \frac{5}{12} = \frac{24}{144}$.

2. Two dice are tossed

Let S be sample space.

$\therefore |S| = 36$

Let A be the event that one of the dice is 3.

A = {(3,1), (3,2), (3,3), (3,4), (3,5), (3,6), (1,3), (2,3), (4,3), (5,3), (6,3)}

B is the event the sum is 5.

B = {(1,4), (4,1), (2,3), (3,2)}

$\therefore A \cap B = \{(2,3),(3,2)\}$

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{\frac{2}{36}}{\frac{4}{36}} = \frac{1}{2}$$

3. Probability of hitting the target by A, B and C are 0.8, 0.9 and 0.7 respectively.

$$P(A) = 0.8 = \frac{8}{10}$$

$$P(B) = 0.9 = \frac{9}{10}$$

$$P(C) = 0.7 = \frac{7}{10}$$

Probability of not hitting the target by A

$$= P(A') = 1 - P(A) = 1 - \frac{8}{10} = \frac{2}{10}$$

Similarly

$$P(B') = 1 - P(B) = 1 - \frac{9}{10} = \frac{1}{10}$$

$$P(C') = 1 - P(C) = 1 - \frac{7}{10} = \frac{3}{10}$$

$$P(A \cap B) = P(A).P(B) = \frac{8}{10} \cdot \frac{9}{10} = \frac{72}{100}$$

$$P(B \cap C) = P(B).P(C) = \frac{9}{10} \cdot \frac{7}{10} = \frac{63}{100}$$

$$P(C \cap A) = P(C).P(A) = \frac{7}{10} \cdot \frac{8}{10} = \frac{56}{100}$$

$$P(A \cap B \cap C) = P(A).P(B).P(C)$$

$$= \frac{8}{10} \cdot \frac{9}{10} \cdot \frac{7}{10} = \frac{504}{1000}$$

target $= P(A \cap B - C) + P(A \cap C - A) + P(C \cap A - B) + P(A \cap B \cap C)$

$$= \frac{72}{100} \cdot \frac{3}{10} + \frac{63}{100} \cdot \frac{2}{10} + \frac{56}{100} \cdot \frac{1}{10} + \frac{504}{1000}$$

$$= \frac{216 + 126 + 56 + 504}{1000} = \frac{902}{1000}$$

4. Given that $P(A \cap B) = \frac{3}{50}$, $P(A \cup B) = \frac{11}{25}$

Here A and B are independent events

$$P(A \cap B) = P(A) \, . P(B)$$

$$\Rightarrow \frac{3}{50} = P(A).P(B)$$

$$\Rightarrow xy = \frac{3}{50} \qquad .... (1)$$

Where P(A) = x, P(B) = y.

Again

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$\Rightarrow \frac{11}{25} = P(A) + P(B) - \frac{3}{50}$$

$$\Rightarrow P(A) + P(B) = \frac{11}{25} + \frac{3}{50} = \frac{22+3}{50} = \frac{25}{50} = \frac{1}{2}$$

$$\Rightarrow x + y = \frac{1}{2} \qquad ... (2)$$

$$(x-y)^2 = (x+y)^2 - 4xy$$

$$= \frac{1}{4} - 4.\frac{3}{50} = \frac{1}{4} - \frac{6}{25} = \frac{1}{100}$$

$$\Rightarrow x - y = \frac{1}{10} \qquad ... (3)$$

Adding (2) and (3), we get

$$2x = \frac{1}{2} + \frac{1}{10} = \frac{6}{100} = \frac{3}{5}$$

$$\Rightarrow x = \frac{3}{10}$$

Subtracting (3) from (2),

$$2y = \frac{1}{2} - \frac{1}{10} = \frac{4}{10}$$

$$y = \frac{2}{10} = \frac{1}{5}$$

$$\therefore P(A) = \frac{3}{10}, \quad P(B) = \frac{1}{5}.$$

5. Let E_1 = The selected bag is B_1

E_2 = The selected bag is B_2

E3 = The selected bag is B_3

A = The ball drawn is white.

$$P(E_1) = P(E_2) = P(E_3) = \frac{1}{3}$$

$$P(A \mid E_1) = \frac{4}{9}$$

$$P(A \mid E_2) = \frac{3}{8}$$

$$P(A \mid E_3) = \frac{5}{7}$$

By the theorem of total probability

P (white ball)

= P (A)

= $P(E_1) \, . \, P(A \mid E_1) \, . \, P(A \mid E_2)$

+. $P(A \mid E_3)$.. $P(A \mid E_3)$

$$= \frac{1}{3}.\frac{4}{9} + \frac{1}{3}.\frac{3}{8} + \frac{1}{3}.\frac{5}{7}$$

$$= \frac{1}{3}\left(\frac{4}{9} + \frac{3}{8} + \frac{5}{7}\right)$$

$$= \frac{1}{3} \, \frac{224+189+360}{504} = \frac{773}{1512}.$$

6. Let E_1 = The set of students choosen from the Class - XI.

E_2 = The set of students choosen from the Class - XII.

A = The students is a boy.

$$\therefore P(E_1) = P(E_2) = \frac{1}{2}$$

$$P(A \mid E_1) = \frac{15}{40} = \frac{3}{8}$$

$$P(A \mid E_2) = \frac{30}{50} = \frac{6}{10}.$$

We what to find $P(E_2 \mid A)$

By Baya's Theorem,

$P(E_2 | A)$

$$= \frac{P(E_2) . P(A|E_2)}{P(E_1).P(A|E_1) + P(E_2).P(A|E_2)}$$

$$= \frac{\frac{1}{2}.\frac{6}{10}}{\frac{1}{2}.\frac{3}{8} + \frac{1}{2}.\frac{6}{10}} = \frac{\frac{6}{10}}{\frac{3}{8} + \frac{6}{10}}$$

$$= \frac{\frac{6}{10}}{\frac{15+24}{40}} = \frac{24}{39} = \frac{8}{13}.$$

7. Let E_1 = The person is a farmer

E_2 = The person is a businessman

E_3 = The person is a service holder

A = The person is above the poverty line

$$\therefore P(E_1) = \frac{50}{100} = \frac{5}{10}$$

$$P(E_2) = \frac{30}{100} = \frac{3}{10}$$

$$P(E_3) = \frac{20}{100} = \frac{2}{10}$$

$$P(A|E_1) = \frac{10}{100} = \frac{1}{10}$$

$$P(A|E_2) = \frac{20}{100} = \frac{2}{10}$$

$$P(A|E_3) = \frac{50}{100} = \frac{5}{10}.$$

By the theorem of total probability P (The person is above the poverty line)

= P(A)

$= P(E_1) . P(A|E_1) + P(E_2) . P(A| E_2) + P(E_3) . P(A |E_3)$

$$= \frac{5}{10}.\frac{1}{10} + \frac{3}{10}.\frac{2}{10} + \frac{2}{10}.\frac{5}{10} = \frac{21}{100} = 21\%$$

8. Let X denote the number of Kings in a successive draw of two cards with replacement from a deck of 52 cards.

Then X is a random varriable which takes values 0 or 1 or 2.

Since we draw the cards with replacement, the two draws are independent.

P (X = 0) = P (no king and no king)

= P (no king) x P (no king)

$$= \frac{48}{52}.\frac{48}{52} = \frac{144}{169}$$

P(X=1) = P [a king and no king or no king and a king]

= P (a king and no king) + P (no king or a king)

$$= \frac{4}{52}.\frac{48}{52} + \frac{48}{52}.\frac{4}{52} = \frac{24}{169}$$

P (X=2) = P (a king and a king)

= P (a king) x (a king)

$$= \frac{4}{52} \text{x} \frac{4}{52} = \frac{1}{169}$$

Thus the required probability distribution is

X = x	0	1	2
P(x)	$\frac{144}{169}$	$\frac{24}{169}$	$\frac{1}{169}$

$$\bar{X} = 0\text{x}\frac{144}{169} + 1\text{x}\frac{24}{169} + 2\text{x}\frac{1}{169}$$

$$= 0 + \frac{24}{169} + \frac{2}{169} = \frac{26}{169} = \frac{2}{13}$$

Variance

$$\alpha^2 = \sum_{i=1}^{3} x_i^2 P(x_i) - \bar{x}^2$$

$$= 0^2 \text{x} \frac{144}{169} + 1^2 \text{x} \frac{24}{169} + 2^2 \text{x} \frac{1}{169} - \left(\frac{2}{13}\right)^2$$

$$= 0 + \frac{24}{169} + \frac{4}{169} - \frac{4}{169} = \frac{24}{169}$$

9. Let the random variable x denotes the number of aces.

Thus x can take the values 0, 1, 2, 3, 4.

Clearly the given experiment is a binomial experiemtn with n = 4.

$p = p\text{ (ace in a throw)} = \frac{4}{52} = \frac{1}{13}$

$q = p\text{ (no ace in a throw)}$

$= 1 - p = 1 - \frac{1}{13} = \frac{12}{13}$

$p(x=0) = 4_{C_0}\, p^0 q^4 = \left(\frac{12}{13}\right)^4$

$p(x=1) = 4_{C_1}\, p^1 q^3 = 4.\frac{1}{13}\left(\frac{12}{13}\right)^3 = \frac{4.12^3}{13^4}$

$p(x=2) = 4_{C_2}\, p^2 q^2$

$= 6.\left(\frac{1}{13}\right)^2\left(\frac{12}{13}\right)^2 = 6.\frac{12^3}{13^4}$

$p(x=3) = 4_{C_3}\, p^3 q$

$= 4.\left(\frac{1}{13}\right)^3\left(\frac{12}{13}\right) = \frac{48}{(13)^4}$

$p(x=4) = 4_{C_4}\, p^4 q^0 = \left(\frac{1}{13}\right)^4$

The required distribution 4

x	0	1	2	3	4
p(x)	$\left(\frac{12}{13}\right)^4$	$\frac{4.(12)^3}{13^4}$	$\frac{6.(12)^3}{12^4}$	$\frac{48}{(13)^4}$	$\left(\frac{1}{13}\right)^4$

Mean $= np = 4.\frac{1}{13} = \frac{4}{13}$

Variance $= npq = 4.\frac{1}{13}.\frac{12}{13} = \frac{48}{169}$.

10. Let the random variable X denotes the number of heads in three tosses of a coin.

$\therefore$ X can take values 0,1,2,3.

In one toss $p(H) = \frac{1}{2},\ p(T) = \frac{1}{2}$

This experiment is a bionimial experiment with

$n = 3,\ p = \frac{1}{2},\ q = \frac{1}{2}$.

$p(x=0) = 3_{C_0}\, p^0 q^3 = \left(\frac{1}{2}\right)^3 = \frac{1}{8}$

$p(x=1) = 3_{C_1}\, p^1 q^2 = 3.\frac{1}{2}.\left(\frac{1}{2}\right)^2 = \frac{3}{8}$

$p(x=2) = 3_{C_2}\, p^2 q = 3.\left(\frac{1}{2}\right)^2 - \frac{1}{2} = \frac{3}{8}$

$p(x=3) = 3_{C_3}\, p^3 q^0 = \left(\frac{1}{2}\right)^3 = \frac{1}{8}$

The probability distribution is

X = x	0	1	2	3
p(x)	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$

CHAPTER - 7

CONTINUITY & DIFFERENTIABILITY

Multiple Choice Questions (MCQ)

Group-A

A. Choose the correct answer from the given choices:

1. If $f(x)=\begin{cases}\frac{x^3-8}{x-2} & when\, x\neq 2\\ k & when\, x=2\end{cases}$

is continuous at $x=0$, then the value of k is ?

(a) 4 (b) 8

(c) 12 (d) 16

2. If the function $f(x)=\frac{\sin ax}{bx}$ is continuous at $x=0$ then $f(0)=?$

(a) b (b) a

(c) $\frac{b}{a}$ (d) $\frac{a}{b}$

3. The function

$$f(x)=\begin{cases}\frac{x^3-3x+2}{(x-1)^2} & if\ x\neq 1\\ k & if\ x=1\end{cases}$$

is continuous for all x, then the value of k is ?

(a) 1 (b) 2

(c) 3 (d) 4

4. If $f(x)=(1+2x)^{\frac{1}{x}}$ when $x\neq 0$ is continuous at $x=0$, then the value of $f(0)$ is ?

(a) e (b) e^2

(c) e^3 (d) e^4

5. If $f(x)=\frac{x^3+x^2-16x+20}{(x-2)^2}$ is $x\neq 2$

$=k$ if $x=2$

is continuous for all x, then the value of k is ______?

(a) 5 (b) 6

(c) 7 (d) 8

6. If a function $f(x)=\frac{x^2-4}{x-2}$ is continuous at $x=2$ then the value of $f(2)$ is ?

(a) 7 (b) 6

(c) 5 (d) 4

7. If the function

$$f(x)\begin{cases}\frac{\sin 3x}{2x} & when\, x\neq 0\\ k & when\, x=0\end{cases}$$

then the value of k is

(a) $\frac{4}{3}$ (b) $\frac{3}{2}$

(c) $\frac{2}{3}$ (d) $\frac{3}{4}$

8. If the function $f(x)=\begin{cases}6.5^x & for\, x\leq 0\\ 2a+x & for\, x>0\end{cases}$ is continuous at $x=0$ then the value of a is ?

(a) 2 (b) 3

(c) 4 (d) 4

9. The function $f(x)=\begin{cases}\dfrac{1-x}{\log x} & when\, x\neq 1\\ k & when\, x=1\end{cases}$ is continuous at $x=1$ then $k=?$

(a) -1 (b) 0

(c) 1 (d) 2

10. The derivative of $\sin(\cos^{-1}x)$ w.r.t $\sin^{-1}x$ is

(a) x (b) $-x$

(c) 1 (d) -1

11. The derivative of $\cos^2 x$ with respect to x is ?

(a) $\sin^2 x$ (b) $-\sin^2 x$

(c) $\sin 2x$ (d) $-\sin^2 x$

12. The derivative of $\sin x^0$ with respect to x is ____?

(a) $\cos x^0$ (b) $\cos x$

(c) $\dfrac{\pi}{180}\cos x^0$ (d) $\dfrac{\pi}{180}\cos x$

13. The derivative of x w.r.t $\tan x$ is _____?

(a) $\sec^2 x$ (b) $\cos^2 x$

(c) $-\tan^2 x$ (d) $-\cot^2 x$

14. The derivative of $\cos x$ with respect to $\sec x$ is ______?

(a) 1 (b) $-\sec^2 x$

(c) $-\cos^2 x$ (d) $-\tan^2 x$

15. The derivative $(1-2x)^3$ with respect to x is _____?

(a) $-2(1-2x)$ (b) $-4(1-2x)^2$

(c) $-4(1-2x)^3$ (d) $-6(1-2x)^2$

16. The derivative of $\sin^{-1}x+\sin^{-1}\sqrt{1-x^2}$ with respect to x is _________?

(a) -2 (b) -1

(c) 0 (d) 1

17. Derivative of $\sin^{-1}(\cos x)$ with respect to x is ________?

(a) -1 (b) 1

(c) -2 (d) 2

18. Derivative of $\sin(\cos^{-1}x)$ with respect to $\sin^{-1}x$ is ______?

(a) -1 (b) 1

(b) x (d) $-x$

19. Derivative of $\tan^{-1}x$ with respect to $\cot^{-1}\dfrac{1}{x}$ is ______?

(a) -1 (b) 1

(c) x (d) $-x$

20. Derivative $\sin^{-1}x+\cos^{-1}x$ with respect to x is _________.

(a) 1 (b) -1

(c) 2 (d) 0

21. Derivative of e^{3x} w.r.t $3e^x$ is _______

(a) 1 (b) e^{2x}

(c) $3e^{2x}$ (d) e^{-2x}

22. If the derivative of a function f is twice the function the $f(x)$ is

(a) x^2 (b) $e^{\frac{x}{2}}$

(c) $\sqrt{x}$ (d) e^{2x}

23. Derivative of e^{-x} w.r.t e^{x} is _______

(a) $-e^{-x}$ (b) e^{2x}

(c) $-e^{-2x}$ (d) e^{2x}

24. Derivative of $\sqrt{e^{x}}$ w.r.t x is

(a) $\sqrt{e^{x}}$ (b) $\frac{1}{2}\sqrt{e^{x}}$

(c) $\frac{1}{2\sqrt{e^{x}}}$ (d) $\frac{\sqrt{e^{x}}}{2\sqrt{x}}$

25. The shope of the tangent to the curue $y =$ ______ at any point is equal to the ordinate of the point.

(a) x (b) $\frac{1}{2}e^{-x}$

(c) e^{x} (d) $2e^{\frac{x}{2}}$

26. The function $f(x) = [x]$ is continuous at

(a) 1 (b) 2

(c) 2.5 (d) 3

27. If $f(x) = |x-1| + |x-2|$ then the points where the function is not differentiable is _________?

(a) 3, 4 (b) 1, 2

(c) 0, -1 (d) 4, 5

28. If a function f is defined as

$$f(x) = \begin{cases} 0 & if\ x\ is\ irrational \\ 1 & if\ x\ is\ rational \end{cases}$$

then f is _________?

(a) continuous at every point

(b) discontinuous at every point

(c) differentable at every point

(d) none of these.

29. $f(x) = \begin{cases} \frac{\sin x}{x} & when\ x \neq 0 \\ 0 & when\ x = 0 \end{cases}$

is _______ at $x = 0$.

(a) continuous

(b) discontinuous

(c) differentiable

(d) none of these

30. The function $f(x) = 2^{\frac{1}{x}}$ is not continuous at _______.

(a) 0 (b) 1

(c) -1 (d) none of these

31. $f(x) = \frac{x^2 - a^2}{x - a}$ at $x = a$ is _________

(a) continuous

(b) not continuous

(c) undecided

(d) none of these

32. $f(x) = \sin\left(\frac{1}{x}\right)$ at $x = 0$

has a ________

(a) discontinuity of first kind

(b) discontinuity of 2nd kind

(c) mixed continuous

(d) removable discontinuity

33. A function $f(x) = [x]$ is discontinuous is _______

(a) set of all real number

(b) set of all rationla number

(c) set of all irrational number

(d) set of all integral points

34. If

$$f(x)=\begin{cases} x & \text{when } 0\le x<\frac{1}{2} \\ 1 & \text{when } x=\frac{1}{2} \\ 1-x & \text{when } \frac{1}{2}<x<1 \end{cases}$$

then $f(x)=$______

(a) continuous at $x=\frac{1}{2}$

(b) discontinuous at $x=\frac{1}{2}$

(c) not defined at $x=\frac{1}{2}$

(d) none of these

35. If $f(x)=\frac{|x|}{x}, x\neq 0$ may be continuous at origin if

(a) $f(0)=0$

(b) $f(0)=-1$

(c) $f(0)=2$

(d) can not be continuous for any value of $f(0)$

36. $f(x)=\sin x$ is continuous is ________

(a) $(-\infty,\infty)$ (b) (0,1)

(c) (1,2) (d) none of these

37. $f(x)=|x|$ at $x=0$ is _______

(a) continuous and differentiable

(b) continuous but not differentiable

(c) not continuous but differentiable

(d) none of these

38. The function $f(x)=|x+2|$ is not differentiable at ________.

(a) $x=2$ (b) $x=-2$

(c) $x=-1$ (d) $x=1$

Answers

1. (c)	2. (d)	3. (c)
4. (b)	5. (c)	6. (d)
7. (b)	8. (b)	9. (a)
10. (b)	11. (d)	12. (c)
13. (b)	14. (c)	15. (d)
16. (c)	17. (a)	18. (d)
19. (b)	20. (d)	21. (b)
22. (d)	23. (c)	24. (c)
25. (c)	26. (c)	27. (b)
28. (b)	29. (b)	30. (a)
31. (b)	32. (b)	33. (d)
34. (b)	35. (d)	36. (a)
37. (b)	38. (b)	

B. Fill in the blanks:

1. If $y = \sin^{-1}\left(\frac{1}{2}x\right) + \cos^{-1}\left(\frac{1}{2}x\right)$ then $\frac{dy}{dx} =$ ________

2. If $y = \tan^{-1}\sqrt{\frac{1-\cos x}{1+\cos x}}$ then $\frac{dy}{dx} =$ _____?

3. The derivative of $\sec x$ w.r.t x is______?

4. The derivative of $\sec^{-1}\left(\frac{1}{2x^2-1}\right)$ w.r.t $\sqrt{1-x^2}$ at $x = \frac{1}{2}$ is______.

5. Derivative of $\ln \sin^{-1}\cos\left(\frac{\pi}{2} - x\right)$ is______.

6. If $x^y = e^{x-y}$ then $\frac{dy}{dx} =$ _______

7. The derivative of $\sin^2 x$ w.r.t $\cos^2 x$ is ______

8. The differential coefficient of $\tan^{-1}\left(\frac{\sin x + \cos x}{\cos x - \sin x}\right)$ w.r.t x is______

9. The derivative of $e^{3\log x}$ w.r.t x is_____

10. If $u = t^2$ and $v = \sin t^2$ then $\frac{dv}{du} =$ ______.

11. If $y = \sec^{-1}\frac{\sqrt{x}+1}{\sqrt{x}} + \sin^{-1}\left(\frac{\sqrt{x}}{\sqrt{x}+1}\right)$ then $\frac{dy}{dx} =$ __________

12. The derivative of $\sin x$ w.r.t $\cos x$ is______

13. The derivative of $\tan^{-1}\left(\frac{\sqrt{1-x^2}}{x}\right)$ w.r.t $\cos^{-1}\left[2x\sqrt{1-x^2}\right]$ is_______

14. The derivative of $\tan^{-1}\left(\frac{x}{\sqrt{1-x^2}}\right)$ w.r.t $\sin^{-1}\left(2x\sqrt{1-x^2}\right)$ is________

15. The derivative $\tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$ w.r.t $\sin^{-1}\left(\frac{2x}{1+x^2}\right)$ is_______

16. If $x = a\left(\cos t + \frac{1}{2}\log\tan\frac{t}{2}\right), y = a\sin t,$ then at $t = \frac{\pi}{3}, \frac{d^2y}{dx^2} =$ ______

17. The derivative of $\sec^{-1}\left(\frac{1}{2x^2-1}\right)$ w.r.t $\sqrt{1-x^2}$ is ______

18. If $y = \sec^{-1}\left(\frac{x+1}{x-1}\right) + \sin^{-1}\left(\frac{x-1}{x+1}\right)$ then $\frac{dy}{dx} =$ ______

19. The derivative of $\sin^{-1}(3x - 4x^3)$ w.r.t $\sin^{-1} x$ is______

20. The derivative of $\sin^{-1}\left(\frac{2x}{1+x^2}\right)$ w.r.t. $\tan^{-1}\frac{2x}{1-x^2}$ is_______

21. $\frac{d}{dx}\cot^{-1}\tan\left(\frac{\pi}{2} - x\right)$ is_______

22. If $y = \cot^{-1}\frac{\sqrt{1-x^2}}{x}$, then $\frac{dy}{dx} =$ ______

23. The derivative of $\cos^2 x$ w.r.t is_______

24. The derivative of $\sin^{-1} x + \sin^{-1}\sqrt{1-x^2}$ w.r.t x is_________

25. The derivative of $\cos x$ w.r.t $\sec x$ is _______

26. The point of discontinuity of the function $f(x) = \log|x|$ is________

27. The set of points of discontinuily of $f(x)=|\sin x|$ is _______

28. The set of points of discontinucly of $f(x)=\frac{|\sin x|}{\sin x}$ is _________

29. The set of points where the function $f(x)=|x-2|$ is differentiable is _____

30. $f(x)=|x|+|x-1|$ is not differentiable at _______.

Hints & Answers

1. 0

2. $\frac{1}{2}$

Hints

$$y=\tan^{-1}\sqrt{\frac{1-\cos x}{1+\cos x}}=\tan^{-1}\sqrt{\frac{2\sin^2\frac{x}{2}}{2\cos^2\frac{x}{2}}}$$

$$=\tan^{-1}\tan\frac{x}{2}=\frac{1}{2}x$$

$$\frac{dy}{dx}=\frac{1}{2}$$

3. $\frac{\pi}{180}\sec\frac{\pi x}{180}\tan\frac{\pi x}{180}$

Hints: $y=\sec x^0=\sec\frac{\pi}{180}$

$$\frac{dy}{dx}=\frac{\pi}{180}\sec\frac{\pi}{180}\tan\frac{\pi x}{180}$$

4. 4

5. $\frac{\ln x}{(1+\ln x)^2}$

Hints $x^y=e^{x-y}$

$$\Rightarrow \ln x^y=\ln e^{x-y}$$

$$\Rightarrow y\ln x=x-y$$

$$=y(1+\ln x)=x$$

$$\Rightarrow y=\frac{x}{1+\ln x}$$

$$\frac{dy}{dx}=\frac{d}{dx}\left(\frac{x}{1+\ln x}\right)$$

$$=\frac{(1+l\,nx)\frac{dx}{dx}-x.d\frac{(1+\ln x)}{dx}}{(1+\ln x)^2}$$

$$=\frac{\ln x}{(1+\ln x)^2}$$

7. -1

8. 1

Hints $y=\tan^{-1}\left(\frac{\sin x+\cos x}{\cos x-\sin x}\right)$

$$=\tan^{-1}\left(\frac{\tan x+1}{1-\tan x}\right)$$

$$=\tan^{-1}\tan\left(\frac{\pi}{4}+x\right)$$

$$=\frac{\pi}{4}+x$$

$$\therefore \frac{dy}{dx}=\frac{d\left(\frac{\pi}{4}+x\right)}{dx}=1$$

9. $3x^2$

Hints $y=e^{3\log x}=e^{\log x^3}=x^3$

$$\frac{dy}{dx}=\frac{dx^3}{dx}=3x^2$$

10. $\cos t^2$

Hints $\frac{dv}{du}=\frac{\frac{du}{dt}}{\frac{du}{dt}}=\frac{2t\cos t^2}{2t}=\cos t^2$

11. 0

12. $-\cot x$

13. $\frac{-1}{2}$

14. $\frac{1}{2}$

15. $\frac{1}{4}$

Hints: Let

$$y = \tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right), z = \sin^{-1}\left(\frac{2x}{1+x^2}\right)$$

Let $x = \tan\theta \qquad \Rightarrow \theta = \tan^{-1} x$

$$y = \tan^{-1}\left(\frac{\sqrt{1+\tan^2\theta}-1}{\tan\theta}\right)$$

$$= \tan^{-1}\left(\frac{\sec\theta - 1}{\tan\theta}\right) = \tan^{-1}\left(\frac{1-\cos\theta}{\sin\theta}\right)$$

$$= \tan^{-1}\left(\frac{2\sin^2\frac{\theta}{2}}{2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}\right)$$

$$= \tan^{-1}\tan\frac{\theta}{2} = \frac{\theta}{2} = \frac{1}{2}\tan^{-1} x$$

$$\frac{dy}{dx} = \frac{1}{2}\frac{d\tan^{-1} x}{dx} = \frac{1}{2}.\frac{1}{1+x^2}$$

$$2 = \sin^{-1}\left(\frac{2x}{1+x^2}\right) = 2\tan^{-1} x$$

$$\frac{dz}{dx} = 2.\frac{1}{1+x^2}$$

$$\frac{dy}{dz} = \frac{\frac{dy}{dx}}{\frac{dz}{dx}} = \frac{\frac{1}{2}\frac{1}{1+x^2}}{2.\frac{1}{1+x^2}} = \frac{1}{4}$$

16. $\frac{8\sqrt{3}}{a}$

17. $\frac{2}{x}$

18. 0

19. 3

20. 1

21. 1

22. $\frac{1}{\sqrt{1-x^2}}$

23. $-\sin 2x$

24. 0

25. $-\cos^2 x$

26. {0}

Hints: As the function $\log|x|$ is not defined at $x = 0$, therefore the set of points of discontinuity is $\{0\}$.

27. ϕ

Hints : As $f(x) = |\sin x|$ is defined for all real x, so it is continuous for all real x so the set of points of discontinuity is ϕ.

28. $\{n\pi : n \in \text{I}\}$

Hints: $\frac{|\sin x|}{\sin x}$ is not defined for $x = n\pi, n \in \text{I}$, so the set of points of discontinuity of $f(x) = \frac{|\sin x|}{\sin x}$ is $\{n\pi : n \in \text{I}\}$

29. $(-\infty, \infty) - \{2\}$

Hints: $|x-2|$ is not differentiable at $x = 2$.

So $f(x) = |x-2|$ is not differentiable at $x = 2$

The function $f(x)$ is differentiable in the set $(-\infty, \infty) - \{2\}$

30. 0,1

C. Answer in one word:

1. What is first derivative of the function $\cos^{-1}\left(\sin\sqrt{\frac{1+x}{2}}\right)+x^x$ w.r.t x at $x=1$?

2. If $f(x)=\log(\log x)$ then what is $f^1(x)$ at $x=e$?

3. What is the derivative of $\tan^{-1}\left(\frac{1+x}{1-x}\right)$?

4. If $f(x)=\tan^{-1}\left(\frac{3a^2x-x^3}{a^3-3ax^2}\right)$ then what is $f^1(x)$?

5. If $x=a\cos^3 t, y=a\sin^3 t$ then what is $\frac{dy}{dx}$ at $t=\frac{3\pi}{4}$?

6. If $x^p.y^q=(x+y)^{p+q}$, then what is $\frac{dy}{dx}$?

7. What is the derivative $\sqrt{e^x}$ w.r.t x?

8. What is the derivative $\cot\left(\frac{\pi}{4}\right)$ with respect to $\tan x$?

9. If $f(x)=\log_x(\log_e x)$ then what is $f^1(x)$ at $x=e$?

10. If $\sin y = x\cos(a+y)$ then what is $\frac{dy}{dx}$?

11. If $f(x)=|x|$ is defined on $[-2,2]$ then at which point the function is not differentiable?

Answers

1. $\frac{3}{4}$

2. $\frac{1}{2e}$

Hints: $f(x)\log_{x^2}(\log x)=\frac{\log(\log x)}{\log x^2}$

$$f^1(x)=\frac{d}{dx}\left[\frac{\log(\log x)}{\log x^2}\right]$$

$$=\frac{\log x^2.\frac{d\log(\log x)}{dx}-\log(\log x)-\frac{d}{dx}(\log x^2)}{(\log x^2)^2}$$

$$=\frac{\frac{2\log x}{x\log x}-\frac{2}{x}\log(\log x)}{y\log^2 x}$$

When $x=e,\ f^1(e)=\frac{1}{2e}$

3. $\frac{1}{1+x^2}$

4. $\frac{3a}{a^2+x^2}$

5. 1

6. $\frac{y}{x}$

7. $\frac{1}{2}\sqrt{e^x}$

8. 0

9. $\frac{1}{e}$

10. $\frac{\cos^2(a+y)}{\cos a}$

11. The function $f(x)=|x|$ is not differentiable at $x=0$.

D. Answer in one sentence.

1. Examine the continuity of $f(x)=\sin\frac{\pi(x)}{2}$ at $x=0$

2. Examine the continuity of $f(x)=\begin{cases}\frac{|x|}{x} & if\ x\neq 0\\ 0 & if\ x=0\end{cases}$ at $x=0$

3. Examine the continuity of the function $f(x)=\begin{cases}\frac{1}{x+[x]} & if\ x<0\\ -1 & if \geq 0\end{cases}\quad at\ x=0$

4. Examine the continuity of the function $f(x)=\begin{cases}\frac{e^{\frac{1}{x}}}{e^{\frac{1}{x}}} & when\ x\neq 0\\ 0 & when\ x=0\end{cases}$ at $x=0$

5. If $y=\sin^{-1}\left[x\sqrt{1-x}-\sqrt{x}\sqrt{1-x^2}\right]$ and $0<x<1$ then what is $\frac{dy}{dx}$?

6. Differentiate $\tan^{-1}\left(\frac{\sqrt{1-x^2}}{x}\right)$ w.r.t $\cos^{-1}\left(2x\sqrt{1-x^2}\right)$

7. Differentiate $a^{\ln x}$ w.r.t x.

8. What is the derivative of $\sec^{-1}x$ w.r.t x if $x<-1$

9. Differentiate $\tan^{-1}\left(\frac{\sqrt{x}-x}{1+x\sqrt{x}}\right)$

10. What is the derivative $\cos ec^{-1}x$ w.r.t x when $|x|>|1$.

11. Write a logarithmic function which is differentiable only in the open interval $(-1,1)$.

12. What is the value of the derivative of $f(x)=|x-1|+|x-3|$ at $x=2$?

13. If $f(x+y)=f(x).f(y)$ for all $x,y\in R$ and $f^1(5)=2, f^1(0)=3$ then what is the value of $f^1(5)$?

14. If $x=\int_0^y\frac{1}{\sqrt{1+4}}\,dt$ then what is the value of $\frac{d^2y}{dx^2}$?

15. Find the derivative of $\ln\sin^{-1}\cos\left(\frac{\pi-2e^x}{2}\right)$?

<u>Hints & Solutions</u>

1. The function is not continuous at $x=0$

Hints. Given $f(x)=\sin\frac{\pi[x]}{2}$

L.H.L $=\lim_{x\to -0}f(x)=\lim_{x\to -0}\sin\frac{\pi[x]}{2}$

$=\sin\frac{\pi(-1)}{2}=-1$

R.H.L $=\lim_{x\to 0^+}f(x)=\lim_{x\to 0^+}\sin\frac{\pi[x]}{2}$

$=\sin\frac{\pi.0}{2}=0$

Again $f(0)=\sin\frac{\pi[0]}{2}=0$

$\therefore \lim_{x\to 0^+}f(x)=f(0)\neq\lim_{x\to 0^-}f(x)$

So the function is discontinuous at $x=0$

2. The function is not continuous at $x=0$

Hints: Given function is

$$f(x)=\begin{cases}\frac{|x|}{x} & if\ x\neq 0\\ 0 & if\ x=0\end{cases}$$

$$\text{L.H.L}=\lim_{x\to 0^-} f(x)=\lim_{x\to 0^-}\frac{|x|}{x}=-1$$

$$\text{R.H.L}=\lim_{x\to 0^+} f(x)=\lim_{x\to 0^+}\frac{|x|}{x}=1$$

$$f(0)=0$$

$$\lim_{x\to 0^-} f(x)\neq \lim_{x\to 0^+} f(x)\neq f(0)$$

The function is not continuous at $x=0$

3. The function is continuous at $x=0$

Hints :

$$\text{L.H.L}=\lim_{x\to 0^-} f(x)=\lim_{x\to 0^-}\frac{1}{x+[x]}$$

$$=\lim_{x\to 0^{-1}}\frac{1}{x-1}=-1$$

$$\text{R.H.L}=\lim_{x\to 0^+} f(x)=\lim_{x\to 0^+}-1=-1$$

$$f(0)=-1$$

$$\therefore \lim_{x\to 0^-} f(x)=\lim_{x\to 0^+} f(x)=f(0)$$

so the function is continuous at $x=0$

4. The function is not continuous at $x=0$

Hints:

$$\text{L.H.L}=\lim_{x\to 0^-} f(x)=\lim_{h\to 0} f(0-h)$$

$$=\lim_{h\to 0}\frac{e^{\frac{1}{0-h}}-1}{e^{\frac{1}{0-h}}+1}=\lim_{h\to 0}\frac{e^{-\frac{1}{h}}-1}{e^{-\frac{1}{h}}+1}$$

$$=\lim_{h\to 0}\frac{\frac{1}{e^{1/h}}-1}{\frac{1}{e^{1/h}}+1}=\frac{0-1}{0+1}=-1$$

$$\text{R.H.L}=\lim_{x\to 0^+} f(x)=\lim_{h\to 0}\frac{e^{\frac{1}{h}}-1}{e^{\frac{1}{h}}+1}$$

$$=\lim_{h\to 0}\frac{1-\frac{1}{e^{1/h}}}{1+\frac{1}{e^{1/h}}}=\frac{1-0}{1+0}=1$$

$$f(0)=0$$

$$\therefore \lim_{x\to 0^-} f(x)\neq \lim_{x\to 0^+} f(x)=f(0)$$

So the function is not continuous at $x=0$

5. $$\frac{dy}{dx}=\frac{1}{\sqrt{1-x^2}}-\frac{1}{2\sqrt{x-x^2}}$$

Hints: $y=\sin^{-1}\left[x\sqrt{1-x}-\sqrt{x}\sqrt{1-x^2}\right]$

$$=\sin^{1}\left[x\sqrt{1-\left(\sqrt{x}\right)^2}-\sqrt{x}\sqrt{1-x^2}\right]$$

$$=\sin^{-1}x-\sin^{-1}\sqrt{x}$$

$$\therefore \frac{dy}{dx}=\frac{d\sin^{-1}x}{dx}-\frac{d\sin^{-1}\sqrt{x}}{dx}$$

$$=\frac{1}{\sqrt{1-x^2}}-\frac{1}{2\sqrt{x-x^2}}$$

6. The required derivative is $-\frac{1}{2}$

Hints: Let

$$y=\tan^{-1}\left(\frac{\sqrt{1-x^2}}{x}\right), z=\cos^{-1}\left(2x\sqrt{1-x^2}\right)$$

$$\frac{dy}{dz}=\frac{\frac{dy}{dx}}{\frac{dz}{dx}} \quad \text{......... (1)}$$

Let $x=\cos\theta \Rightarrow \theta=\cos^{-1}x$

$$y=\tan^{-1}\frac{\sqrt{1-\cos^2\theta}}{\cos\theta}=\tan^{-1}\tan\theta=\theta$$

$$=\cos^{-1}x$$

$$\frac{dy}{dx}=\frac{-1}{\sqrt{1-x^2}}$$

$$z=\cos^{-1}\left(2x\sqrt{1-x^2}\right)$$

$$=\cos^{-1}\left(2\cos\theta\sqrt{1-\cos^2\theta}\right)$$

$$=\cos^{-1}\sin 2\theta=\cos^{-1}\cos\left(\frac{\pi}{2}-2\theta\right)$$

$$=\frac{\pi}{2}-2\theta=\frac{\pi}{2}-2\cos^{-1}x$$

$$\frac{dz}{dx}=d\frac{\left(\frac{\pi}{2}-2\cos^{-1}x\right)}{dx}=\frac{2}{\sqrt{1-x^2}}$$

From (1), $\frac{dy}{dz}=-\frac{1}{2}$

7. $a^{\ln x}.\ln a.\frac{1}{x}$

Hints:

$$\frac{d\,a^{\ln x}}{dx}=\frac{d\,a^{\ln x}}{d\ln x}.\frac{d\ln x}{dx}=a^{\ln x}.\ln a.\frac{1}{x}$$

8. $\frac{d\sec^{-1}x}{dx}=\frac{1}{|x|\sqrt{x^2-1}}$

when $x<-1, |x|=-x$.

$$\therefore\frac{d\sec^{-1}x}{dx}=\frac{1}{-x\sqrt{x^2-1}}$$

9. $\frac{1}{2\sqrt{x}(1+x)}-\frac{1}{1+x^2}$

Hints: Let $y=\tan^{-1}\left(\frac{\sqrt{x}-x}{1+x\sqrt{x}}\right)$

$$=\tan^{-1}\sqrt{x}-\tan^{-1}x$$

$$\frac{dy}{dx}=\frac{d\tan^{-1}\sqrt{x}}{dx}-\frac{d\tan^{-1}x}{dx}$$

$$=\frac{1}{2\sqrt{x}(1+x)}-\frac{1}{1+x^2}$$

10. Required derivative $-\frac{1}{x\sqrt{x^2-1}}$

Hints: $y=\operatorname{cosec}^{-1}x$

$$\frac{dy}{dx}=\frac{d\operatorname{cosec}^{-1}x}{dx}=-\frac{1}{|x|\sqrt{x^2-1}} \quad ....(1)$$

Here $|x|>1$

$$\therefore|x|=x$$

From(1), we have

$$\frac{dy}{dx}=-\frac{1}{x\sqrt{x^2-1}}$$

11. The required logarithmic function is $\log(1-x^2)$.

The above function is differentiable in the interval $(-1,1)$.

12. The derivative of the function at $x=2$ is 0

i.e $f^1(2)=0$

Hints: $f(x)=|x-1|+|x-3|$

$$=\begin{cases}-2x+4 & when\,x<1\\ 2 & when\,1\le x<3\\ 2x-4 & when\,x\ge 2.\end{cases}$$

at $x=2,\quad f(x)=2$

$$\Rightarrow f^1(x)=0$$

13. $f^1(5)=6$

Hints: Given that $f^1(0)=3$

$$\Rightarrow\lim_{h\to 0}\frac{f(0+h)-f(0)}{h}=3$$

$$\Rightarrow \lim_{h\to 0}\frac{f(0).f(h)-f(0)}{h}=3$$

$$\lim_{h\to 0}\frac{f(0)[f(h)-1]}{h}=3 \quad ....(1)$$

Given that $f(x+y)=f(x).f(y)$ for $x, y\in R$

$$\Rightarrow f(0+0)=f(0)f(0)$$

$$\Rightarrow f(0)=f(0).f(0)$$

$$\Rightarrow f(0)[1-f(0)]=0$$

$$\Rightarrow f(0)=1$$

From (1), we have $\lim_{h\to 0}\frac{1.f(h)-1}{h}=3$

$$\Rightarrow \lim_{h\to 0}\frac{f(h)-1}{h}=3 \quad .... (2)$$

$$\therefore f^1(5)=\lim_{h\to 0}\frac{f(5+h)-f(5)}{h}$$

$$=\lim_{h\to 0}\frac{f(5)f(h)-f(5)}{h}$$

$$=\lim_{h\to 0}f(5)\frac{[f(h)-1]}{h}$$

$$=f(5).\lim_{h\to 0}\left[\frac{f(h)-1}{h}\right]$$

$=f(5).3$ [from (2)]

$$=2\times 3=6$$

14. $\frac{d^2y}{dx^2}=4y$

Reason: Given that

$$x=\int_0^b \frac{1}{\sqrt{1+4t^2}}dt$$

$$\Rightarrow \frac{dx}{dy}=\frac{1}{\sqrt{1+4y^2}}$$

$$\Rightarrow \frac{dy}{dx}=\sqrt{1+4y^2}$$

$$\frac{d^2y}{dx^2}=\frac{d\sqrt{1+4y^2}}{dx}$$

$$=\frac{d\left(1+4y^2\right)^{\frac{1}{2}}}{d\left(1+4y^2\right)}.\frac{d\left(1+4y^2\right)}{dy}.\frac{dy}{dx}$$

$$=\frac{1}{2\sqrt{1+4y^2}}.8y\sqrt{1+4y^2}$$

$$=4y$$

15. $\frac{dy}{dx}=1$

Hints:

$$y=\ln\sin^{-1}\cos\left(\frac{\pi-2e^x}{2}\right)$$

$$=\ln\sin^{-1}\cos\left(\frac{\pi}{2}-e^x\right)=\ln\sin^{-1}\sin e^x$$

$$=\ln e^x=x\ln e$$

$$\frac{dy}{dx}=\ln e=1$$

Group-B

Short type (Questions & Answers)

1. If $f(x)=\begin{cases} ax^2+b & if\ x<1 \\ 1 & if\ x=1 \\ 2ax-b & if\ x>1 \end{cases}$

is continuous at $x=1$, then find a and b.

2. Find the value of 'a' such that the function f defined by

$$f(x)=\begin{cases} \dfrac{\sin ax}{\sin x} & if\ x\neq 0 \\ \dfrac{1}{a} & if\ x=0 \end{cases}$$

is continuous at $x=0$

3. Show that $\sin x$ is continuous for every real x.

4. If $y=x\log\left(\dfrac{x}{a+bx}\right)$

then prove that

$$x^3\frac{d^2y}{dx^2}=\left(x\frac{dy}{dx}-y\right)^2$$

5. Find $\dfrac{dy}{dx}$ if $x^m y^n=\left(\dfrac{x}{y}\right)^{m+n}$

6. iF $x=a\sec\theta, y=b\tan\theta$

then prove that

$$\frac{d^2y}{dx^2}=-\frac{b^4}{a^2y^3}$$

7. If $\sin y=x\sin(a+y)$ then show that

$$\frac{dy}{dx}=\frac{\sin^2(a+y)}{\sin a}$$

8. If $\cos x=\sqrt{\dfrac{1}{1+t^2}}, \sin y=\dfrac{2t}{1+t^2}$

then show that $\dfrac{dy}{dx}$

is independent of t.

9. Find $\dfrac{dy}{dx}$ when $y^x=x^{\sin y}$

10. If $y^2\cot x=x^2\cot y$ then find $\dfrac{dy}{dx}$.

11. Find the derivative of $x^{\sin x}$ w.r.t x.

12. Defferentiate

$$\sin^{-1}\left(\frac{2x}{1+x^2}\right)$$

w.r.t $\cos^{-1}\left(\dfrac{1-x^2}{1+x^2}\right)$

13. Differentiate

$$y=\tan^{-1}\left[\frac{\sqrt{1-x^2}+\sqrt{1-x^2}}{\sqrt{1+x^2}-\sqrt{1-x^2}}\right]$$

14. Differentiate $y=(\sin y)^{\sin 2k}$

15. Test the differentiablily and continuity of the following function at $x=0$.

$$f(x)=\begin{cases} \dfrac{1-e^{-x}}{x} & when \neq 0 \\ 1 & when\ x=0 \end{cases}$$

16. Differentiate

$\sec^{-1}\left(\dfrac{1}{2x^2-1}\right)$ with respect to $\sqrt{1-x^2}$

17. If $y = x + \cfrac{1}{x + \cfrac{1}{x + \ldots\ldots\ldots\ldots\infty}}$,

then find $\frac{dy}{dx}$

18. Find the slope of the tangent to the curve $x = 2(\theta - \sin 2\theta), y = 2(1 - \cos\theta)$

at $\theta = \frac{\pi}{4}$

19. If $\cos y = x\cos(a + y)$ then show that

$\frac{dy}{dx} = \frac{\cos^2(a+y)}{\sin a}$

20. Write why the function

$\sin^{-1}\frac{1}{\sqrt{1-x^2}}$ not be differentiate any where.

<u>Hints & Solutions</u>

1. The given function is

$$f(x) = \begin{cases} ax^2 + b & if x < 1 \\ 1 & if\ x = 1 \\ 2ax - b & if\ x > 1 \end{cases}$$

L.H.L $= \lim_{x \to 1^-} \quad f(x)$

$= \lim_{x \to 1^-} ax^2 + b \quad = a + b$

R.H.L $= \lim_{x \to 1^+} f(x)$

$= \lim_{x \to 1^+} 2ax - b \quad = 2a - b$

$f(1) = 1$

since the function is continuous at $x = 1$

$\therefore \lim_{x \to 1^-} f(x) = f(1) = \lim_{x \to 1^+} f(x)$

$\Rightarrow a + b = 1 = 2a - b$

$\therefore a = \frac{1}{2}, b = \frac{2}{3}$

2. The given function is

$$f(x) = \begin{cases} \frac{\sin ax}{\sin x} & if\ x \neq 0 \\ \frac{1}{a} & if\ x = 0 \end{cases}$$

$\lim_{x \to 0} f(x) = \lim_{x \to 0} \frac{\sin ax}{\sin x} = a$

$f(0) = \frac{1}{a}$

Since the function is continuous at $x = 0$

$\lim_{x \to a} f(x) = f(0)$

$\Rightarrow a = \frac{1}{a} \quad \Rightarrow a^2 = 1 \quad a = \pm 1$

3. Let $x_1 \in R$ be any point and $\in$ be any arbitrary positive number $f(x) = \sin x$

$|f(x) - f(x)| = |\sin x - \sin x_1|$

$= \left|2\cos\frac{x + x_1}{2}\sin\frac{x - x_1}{2}\right|$

$= 2\left|\cos\frac{x + x_1}{2}\right|\left|\sin\frac{x - x_1}{2}\right|$ .... (1)

We know for every value of x and x_1 $\left|\cos\frac{x + x_1}{2}\right| \leq 1$

Also $\left|\sin\frac{x - x_1}{2}\right| \leq \quad \left|\frac{x - x_1}{2}\right|$

from (1), we get

$|f(x) - f(x_1)| \leq \ 2.x\left|\frac{x - x_1}{z}\right|$

$\Rightarrow |f(x) - f(x_1)| \quad \leq |x - x_1|$

When $|x - x_1| < \in$ then $| f(x) - f(x_1) \in$

so $\lim_{x \to x_1} \ \sin x = \sin x_1$

$\Rightarrow$ tne function is continuous at $x = x_1$

4. Given that $y = x\log\left(\frac{x}{a+bx}\right)$

Differentiative both sides w.r.t x, get

$$\frac{dy}{dx} = \frac{d}{dx}\left[x\log\frac{x}{a+bx}\right]$$

$$= \frac{a}{a+bx} + \log\frac{x}{a+bx} \quad \text{............(1)}$$

$$\Rightarrow x\frac{dy}{dx} = \frac{ax}{a+bx} + x\log\left(\frac{x}{a+bx}\right)$$

$$= \frac{ax}{a+bx} + y$$

$$\Rightarrow x\frac{dy}{dx} - y = \frac{ax}{a+bx} \quad \text{................. (2)}$$

Again from (1), we have

$$\frac{d^2y}{dx^2} = \frac{a(-b)}{(a+bx)^2} + \frac{\frac{1}{x}}{\frac{x}{a+bx}}\frac{d}{dx}\left(\frac{x}{a+bx}\right)$$

$$= \left(\frac{ax}{a+bx}\right)^2 \quad \text{............... (3)}$$

From (2) & (3), we have

$$x^3\frac{d^2y}{dx^2} = \left(x\frac{dy}{dx} - y\right)^2$$

5. $x^3.y^n = \left(\frac{x}{y}\right)^{m+n}$

$$\Rightarrow \log\left(x^m.y^n\right) = \log\left(\frac{x}{y}\right)^{m+n}$$

$$\Rightarrow m\log x + n\log y = m\log x - m\log y$$

$$+n\log x - n\log y$$

$$\Rightarrow (m+2n)\log y = n\log x$$

Differentiative both sides w.r.t x, we get

$$\left(m+2n\right)\frac{d\log y}{dx} = n.\frac{d\log x}{dx}$$

$$\Rightarrow \frac{dy}{dx} = \frac{ny}{(m+2n)x}$$

6. Given that $x = a\sec\theta, y = b\tan\theta$

$$\frac{dx}{d\theta} = a\sec\theta\tan\theta, \frac{dy}{d\theta} = b\sec^2\theta$$

$$\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{b}{a}\cos ec\theta$$

$$\frac{d^2y}{dx^2} = \frac{b}{a}\frac{d\cos ec\theta}{dx}$$

$$= \frac{b}{a}.\frac{d\cos ec\theta}{d\theta}.\frac{d\theta}{dx}$$

$$= -\frac{b}{a^2}.\frac{1}{\tan^3\theta} = -\frac{b^4}{a^2y^3}$$

7. Given that $\sin y = x\sin(a+y)$

Differentiating both sides w.r.t x, we get

$$\frac{d\sin y}{dx} = \frac{d}{dx}[x\sin(a+y)]$$

$$\Rightarrow \frac{dy}{dx} = \frac{\sin(a+y)}{\cos y - x\cos(a+y)}$$

$$= \frac{\sin\cot y}{\cos y - \frac{\sin y}{\sin(a+y)}.\cos(a+y)}$$

$$= \frac{\sin^2(a+y)}{\sin(a+y)\cos y - \sin y\cos(a+y)}$$

$$= \frac{\sin^2(a+y)}{\sin a}$$

8. Given that

$$\cos x = \sqrt{\frac{1}{1+t^2}}, \sin y = \frac{2t}{1+t^2}$$

$$\Rightarrow x = \cos^{-1}\frac{1}{\sqrt{1+t^2}}, y = \sin^{-1}\frac{2t}{1+t^2}$$

Let $t = \tan\theta \Rightarrow \theta = \tan^{-1}t$

$$\therefore x=\cos^{-1}\frac{1}{\sqrt{1+\tan^2\theta}}=\cos^{-1}\cos\theta$$

$$=\theta=\tan^{-1}t$$

$$\frac{dx}{dt}=\frac{1}{1+t^2} \quad .... (1)$$

$$y=\sin^{-1}\frac{2t}{1+t^2}=2\tan^{-1}t$$

$$\frac{dy}{dt}=\frac{2}{1+t^2}$$

$\therefore \dfrac{dy}{dx}=\dfrac{\frac{dy}{dt}}{\frac{dx}{dt}}=\dfrac{\frac{d(2\tan^{-1}t)}{dt}}{\frac{d\tan t}{dt}}=2.$ which is independent of t.

13. $$y=\tan^{-1}\left[\frac{\sqrt{1+x^2}+\sqrt{1-x^2}}{\sqrt{1+x^2}-\sqrt{1-x^2}}\right]$$

let $x^2=\cos\theta \quad \Rightarrow \theta=\cos^{-1}x^2$

$$\therefore y=\tan^{-1}\left[\frac{\sqrt{1+\cos\theta}+\sqrt{1-\cos\theta}}{\sqrt{1+\cos\theta}-\sqrt{1-\cos\theta}}\right]$$

$$=\tan^{-1}\left[\frac{\cos\frac{\theta}{2}+\sin\frac{\theta}{2}}{\cos\frac{\theta}{2}-\sin\frac{\theta}{2}}\right]$$

$$=\tan^{-1}\left[\frac{1-\tan\frac{\theta}{2}}{1-\tan\frac{\theta}{2}}\right]$$

$$=\tan^{-1}\tan\left(\frac{\pi}{4}+\frac{\theta}{2}\right)$$

$$=\frac{\pi}{2}+\frac{1}{2}\theta=\frac{\pi}{4}+\frac{1}{2}\cos^{-1}x^2$$

$$\frac{dy}{dx}=\frac{1}{2(1+x^2)}$$

15. Given function is

$$f(x)=\begin{cases}\dfrac{1-e^{-x}}{x} & when\, x\neq 0\\ 1 & when\, x=0\end{cases}$$

Continuity at $x=0$

$$\lim_{x\to 0}f(x)=\lim_{x\to 0}\frac{1-e^{-x}}{x}=\lim_{x\to 0}\left[\frac{1-\frac{1}{e^x}}{x}\right]$$

$$=\lim_{x\to 0}\frac{e^x-1}{xe^x}=\lim_{x\to 0}\frac{e^x-1}{x}.\lim_{x\to 0}\frac{1}{e^x}$$

$$=1.1=1$$

Given that $f(0)=1$

$$\therefore \lim_{x\to 0}f(x)=f(0)$$

$\Rightarrow$ The function is continuous at $x=0$

Differentiabrility at $x=0$

$$\text{L.H.D}=\lim_{x\to 0}\frac{f(x)-f(0)}{x-0}$$

$$=\lim_{x\to 0}\frac{f(0-h)-f(h)}{-h}$$

$$=\lim_{h\to 0}\frac{\frac{1-e^h}{-h}-1}{h}$$

$$=\lim_{h\to 0}\frac{\frac{1-e^h+h}{h^2}}{h}\quad\left(for\,\pi,\frac{0}{0}\right)$$

$$=\lim_{h\to 0}\frac{0-e^h+1}{2h}$$

(Applying L' hospital's rule)

$$\lim_{h\to 0}\frac{-e^h+0}{2}=-\lim_{-h\to 0}\frac{-e^h}{2}=\frac{-1}{2}$$

$$\text{R.H.D}=\lim_{x\to 0^+}\frac{f(x)-f(0)}{x-0}$$

$$= \lim_{h \to 0} \frac{f(0+h) - f(0)}{0+h}$$

$$= \lim_{h \to 0} \frac{f(h) - f(0)}{h}$$

$$= \lim_{h \to 0} \frac{\frac{1-e^{-h}}{h} - 1}{h}$$

$$= \lim_{x \to 0} \frac{1-e^{-h} - h}{h^2} \quad \left[form\ is \frac{0}{0} \right]$$

$$z - \frac{1}{2}$$

Here L.H.D=R.H.D

So the function is differentiable at $x = 0$

17. $$y = x + \cfrac{1}{x + \cfrac{1}{x + ...to\,\infty}}$$

$$\Rightarrow y = x + \frac{1}{y}$$

$$\Rightarrow y^2 = xy + 1$$

Differentiating both sides w.r.t x, we get

$$\frac{dy^2}{dx} = \frac{d(xy+1)}{dx}$$

$$\Rightarrow \frac{dy}{dx} = \frac{y}{2y - x}$$

20. Let $\sin^{-1} \frac{1}{\sqrt{1-x^2}} = \theta$

$$\Rightarrow \sin\theta = \frac{1}{\sqrt{1-x^2}}$$

$$\Rightarrow \cos ec\theta = \sqrt{1-x^2}$$

$$\Rightarrow \cos ec^2\theta = 1 - x^2$$

$$\Rightarrow 1 + \cot^2\theta = 1 - x^2$$

$\Rightarrow \cot^2\theta = -x^2$ which is $-ve$ and is rejected as $\cot^2\theta$ is alway $+ve$.

So the given function is not diffeentiable any where.

Group-C

Long Type (Questions & Answers)

1. If $x = \dfrac{1-\cos^2\theta}{\cos\theta}$, $y = \dfrac{1-\cos^{2n}\theta}{\cos^n\theta}$ then show that $\left(\dfrac{dy}{dx}\right)^2 = n^2\left(\dfrac{y^2+4}{x^2+4}\right)$

[CHSE-2016,2018]

2. If $y = \cos^{-1}\left[\dfrac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right]$ $\left(0 < x < \dfrac{\pi}{2}\right)$, then find $\dfrac{dy}{dx}$

3. Differentiate $\sin^{-1}\left(2ax\sqrt{1-a^2x^2}\right)$ w.r.t $\sqrt{1-a^2x^2}$

4. If $\sin(x+y) = y\cos(x+y)$ then prove that $\dfrac{dy}{dx} = -\dfrac{1+y^2}{y^2}$

5. Find $\dfrac{dy}{dx}$ when $y = \cot^{-1}(\ln\cos e^{-x}) + \dfrac{x\sin^{-1}x}{\sqrt{1-x^2}}$

6. Find the derivative of $\left(\dfrac{x-1}{x^2+5}\right)^{-4}\cos^{-1}\left(\dfrac{1-x^2}{1+x^2}\right)$

7. Find $\dfrac{dy}{dx}$ when $y = \tan^{-1}\sqrt{\dfrac{1+\sin x}{1-\sin x}}$

8. If $x^y = y^x + \tan^{-1}\left(\dfrac{\cos x}{1+\sin x}\right)$ then find $\dfrac{dy}{dx}$

9. Differentiate $\tan^{-1}\left(\dfrac{1+\sin x}{1-\sin x}\right)^{\frac{1}{2}}$ with respect to $\ln\left(\dfrac{1+\cos x}{1-\cos x}\right)$

10. Find $\dfrac{dy}{dx}$ when $x = e$ $x = e^{\tan^{-1}\left(\frac{y-x^2}{x^2}\right)}$

11. If $y = x^{\sin-1x} + x^3\dfrac{\sqrt{x^2+y}}{\sqrt{x^3+3}}$, then find $\dfrac{dy}{dx}$

Hints & Solutions

1. $x = \dfrac{1-\cos^2\theta}{\cos\theta} = \sec\theta - \cos\theta$

$\dfrac{dx}{d\theta} = \tan\theta(\sec\theta + \cos\theta)$

$y = \dfrac{1-\cos^{2n}\theta}{\cos^n\theta} = \sec^n\theta - \cos^n\theta$

$\dfrac{dy}{d\theta} = n\tan\theta\left(\sec^n\theta + \cos^n\theta\right)$

L.H.S $= \left(\dfrac{dy}{dx}\right)^2$

$= \left[\dfrac{n\tan\theta\left(\sec^n\theta+\cos^n\theta\right)}{\tan\theta\left(\sec\theta+\cos\theta\right)}\right]^2$

$= n^2\dfrac{\left(\sec^n\theta+\cos^n\theta\right)^2}{\left(\sec\theta+\cos\theta\right)^2}$ .... (1)

R.H.S $= n^2\left(\dfrac{y^2+4}{x^2+4}\right)$

$= n^2\dfrac{\left[\left(\sec^n\theta-\cos^n\theta\right)^2+4\right]}{\left(\sec\theta-\cos\theta\right)^2+4}$

$= n^2\dfrac{\left(\sin^n\theta+\cos\theta\right)^2}{\left(\sec\theta+\cos\theta\right)^2}$ .... (2)

From (1) & (2), we see that

$\left(\dfrac{dy}{dx}\right)^2 = n^2\left(\dfrac{y^2+4}{x^2+4}\right)$

2. We know

$$1+\sin x = \cos^2\frac{x}{2}+\sin^2\frac{x}{2}+2\cos\frac{x}{2}\sin\frac{x}{2}$$

$$=\left(\cos\frac{x}{2}+\sin\frac{x}{y}\right)^2$$

$$\Rightarrow \sqrt{1+\sin x}=\cos\frac{x}{2}+\sin\frac{x}{2}$$

Also $\sqrt{1-\sin x}=\cos\frac{x}{2}-\sin\frac{x}{2}$

$$\therefore y=\cot^{-1}\left[\frac{\sqrt{1+\sec x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right]$$

$$=\cot^{-1}\left[\frac{\cos\frac{x}{2}+\sin\frac{x}{2}+\cos\frac{x}{2}-\sin\frac{x}{2}}{\cos\frac{x}{2}+\sin\frac{x}{2}-\cos\frac{x}{2}+\sin\frac{x}{2}}\right]$$

$$=\cot^{-1}\left(\frac{2\cos\frac{x}{2}}{2\sin\frac{x}{2}}\right)$$

$$=\cot^{-1}\ \cot\frac{x}{2}=\frac{1}{2}x$$

$$\therefore \frac{dy}{dx}=\frac{d}{dx}\left(\frac{1}{2}x\right)=\frac{1}{2}$$

4. $\sin(x+y)=y\cos(x+y)$

Differentiating both sides w.r.t x, get

$$\frac{d\sin(x+y)}{dx}=d\frac{[y\cos(x+y)]}{dx}$$

$$\Rightarrow \cos(x+y)\left(1+\frac{dy}{dx}\right)=y\frac{d\cos(x+y)}{dx}$$

$$+\cos(x+y).\frac{dy}{dx}$$

$$\Rightarrow \frac{dy}{dx}=-\left[\frac{\cos(x+y)+y\sin(x+y)}{y\sin(x+y)}\right]$$

$$=-\left[\frac{1}{y}.\frac{1}{y}+1\right]$$

$$=-\left(\frac{1+y^2}{y^2}\right)$$

5. $y=\cot^{-1}\left(\ln\cos e^{-x}\right)+\dfrac{x\sin^{-1}x}{\sqrt{1-x^2}}$

Let $u=\cot^{-1}\left(\ln\cos e^{-x}\right)$, $v=\dfrac{x\sin^{-1}x}{\sqrt{1-x^2}}$

$\therefore y=u+v$

$$\frac{dy}{dx}=\frac{du}{dx}+\frac{dv}{dx} \qquad \text{.......... (1)}$$

$$u=\cot^{-1}\left(\ln\cos e^{-1}\right)$$

$$\therefore \frac{du}{dx}=\frac{d\cot^{-1}\left(\ln\cos e^{-x}\right)}{dx}$$

$$=\frac{d\cot^{-1}\left(\ln\cos e^{-x}\right)}{d\left(\ln\cos e^{-x}\right)}.\frac{d\ln\cos e^{-x}}{d\cos e^{-x}}.$$

$$\frac{d\cos e^{-x}}{d\,e^{-x}}.\frac{d\,e^{-x}}{dx}$$

$$=\frac{e^{-x}\sin e^{-x}}{\cos e^{-x}\left[1+\ln^2\cos\left(e^{-x}\right)\right]}$$

$$v=\frac{x\sin^{-1}x}{\sqrt{1-x^2}}$$

$$\frac{dv}{dx}=\frac{d}{dx}\left[\frac{x\sin^{-1}x}{\sqrt{1-x^2}}\right]$$

$$=\frac{x\sqrt{1-x^2}+\sin x}{1-x^2}$$

From (1), we have

$$\frac{dy}{dx}=-\frac{e^{-x}\sin e^{-x}}{\cos(e^{-x})\left[1+\ln^2\cos(e^{-x})\right]}$$

$$+\frac{x\sqrt{1-x^2}\sin^{-1}x}{1-x^2}$$

6. Let $y=\left(\frac{x-1}{x^2+5}\right)^{-4}\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$

$$=\left(\frac{x^2+5}{x-1}\right)^4 .2\tan^{-1}x$$

$$=2\left(\frac{x^2+5}{x-1}\right)^4 \tan^{-1}x$$

$$\frac{dy}{dx}=2\frac{d}{dx}\left[\left(\frac{x^2+5}{dx}\right)\tan^{-1}x\right]$$

$$=2\left[\left(\frac{x^2+5}{x-1}\right)^4\frac{d\tan^{-1}x}{dx}+\tan^{-1}x\frac{d\left(\frac{x^2+5}{x-1}\right)^4}{dx}\right]$$

$$=2\left[\left(\frac{x^2+5}{x-1}\right)^4.\frac{1}{1+x^2}+4\tan^{-1}x.\left(\frac{x^2+5}{x-1}\right)^3\right.$$

$$\left..\frac{2x(x-1)-(x^2+5)}{(x-1)^2}\right]$$

$$=2\left[\left(\frac{x^2+5}{x-1}\right)^4\frac{1}{1+x^2}+4\tan^{-1}x\left(\frac{x^2+5}{x-1}\right)^3.\right.$$

$$\frac{x^2-2x-5}{(x-1)^2}$$

7. $y=\tan^{-1}\sqrt{\frac{1+\sin x}{1-\sin x}}$

$$=\tan^{-1}\left(\frac{1+\tan\frac{x}{2}}{1-\tan\frac{x}{2}}\right)$$

$$=\tan^{-1}\left(\frac{\tan\frac{\pi}{4}+\tan\frac{\pi}{2}}{1-\tan\frac{\pi}{2}.\tan\frac{\pi}{2}}\right)$$

$$=\tan^{-1}\tan\left(\frac{\pi}{4}+\frac{\pi}{2}\right)=\frac{\pi}{4}+\frac{1}{2}x$$

$$\frac{dy}{dx}=\frac{1}{2}$$

8. Given that $x^y=y^x+\tan^{-1}\frac{\cos x}{1+\sin x}$

Let $u=x^y, v=y^x, w=\tan^{-1}\left(\frac{\cos x}{1+\sin x}\right)$

$\therefore u=v+w$ .... (1)

$\therefore \frac{du}{dx}=\frac{dv}{dx}+\frac{dw}{dx}$ .... (2)

$u=x^y\Rightarrow \ln u=\ln x^y=y\ln x$

$$\frac{d\ln u}{dx}=\frac{d\,y\ln x}{dx}$$

$$\Rightarrow\frac{1}{u}.\frac{du}{dx}=y.\frac{1}{x}+\ln x.\frac{dy}{dx}$$

$$\Rightarrow\frac{du}{dx}=u\left(\frac{y}{x}+\ln x.\frac{dy}{dx}\right)=x^y\left(\frac{y}{x}+\ln x\frac{dy}{dx}\right)$$

$=yx^{y-1}+x^y\ln x\frac{dy}{dx}$ .... (3)

$\frac{dv}{dx}=xy^{x-1}\frac{dy}{dx}+y^x\ln y$ .... (4)

$$w=\tan^{-1}\left(\frac{\cos x}{1+\sin x}\right)=\tan^{-1}\tan\left(\frac{\pi}{4}-\frac{x}{2}\right)$$

$$=\frac{\pi}{4}-\frac{1}{2}x$$

$$\frac{dw}{dx}=\frac{-1}{2}$$

From (2), we get

$$yx^{y-1}+x^y\frac{dy}{dx}=xy^{x-1}\frac{dy}{dx}+y^x\ln y-\frac{1}{2}$$

$$\Rightarrow\frac{dy}{dx}=\frac{y^x\ln y-yx^{y-1}-\frac{1}{2}}{x^y\ln x-xy^{x-1}}$$

CHAPTER - 8

APPLICATION OF DERIVATIVES

Multiple Choice Questions (MCQ)

Group-A

A. Choose the correct answer from the given choices:

1. The slope of the tangent to the curve $y = 3x^4 - 4x$ at $x = 1$ is
   (a) 2 (b) 4
   (c) 6 (d) 8

2. What is the slope of the tangent to the curve $y = 3x^2 - 4x$ at a point whose x-coordinate is 2 ?
   (a) 4 (b) 8
   (c) 10 (d) 12

3. The point on the curve $y^2 = x$, the tangent at which makes an angle of 45^0 with x-axis is?
   (a) $\left(\frac{1}{2}, \frac{1}{4}\right)$ (b) $\left(\frac{1}{2}, \frac{1}{2}\right)$
   (c) $(2, 4)$ (d) $\left(\frac{1}{4}, \frac{1}{2}\right)$

4. What is a point on the curve $x = a(\theta - \sin\theta)$. $y = a(1 - \cos\theta)$ at which the tangent is parallel to $x - axis$?
   (a) $(2a, 2\pi)$ (b) $(a\pi, 2a)$
   (c) $(2\pi, -2a)$ (d) $(2a, -2\pi,)$

5. What is the equation of the normal to the curve $y = \sin x$ at $(0,0)$?
   (a) $x + y = 1$ (b) $x + y = 0$
   (c) $x - y + 1 = 0$ (d) $x - y + 2 = 0$

6. If the tangent to the curve $x = at^2, y = 2at$ is perpendicular to $x - axis$ then what is its point of contact ?
   (a) (1,2) (b) (2,1)
   (c) (0,0) (d) (1,2)

7. What is the slope of the normal to the curve $x^{\frac{2}{3}} + y^{\frac{2}{3}} = 20$ at the point $(8, 64)$?
   (a) 1 (b) $\frac{1}{2}$
   (c) -1 (d) $-\frac{1}{2}$

8. What is the slope of the tangent to the curve $y = \sqrt{3}\sin x + \cos x$ at $\left(\frac{\pi}{3}, 2\right)$?
   (a) 2 (b) 1
   (c) 0 (d) -1

9. What is the equation of the tangent to the curve $y = |x|$ at the point $(-2, 2)$?
   (a) $x + y = 0$
   (b) $2x + 3y + 1 = 0$
   (c) $x = 0$
   (d) $y = 0$

10. What is the equation of the tangent to the curve $x = y^2 - 1$ at the point where the slope of the normal to the curve is 2?
    (a) $2x - y + 3 = 0$
    (b) $x + 2y + 2 = 0$
    (c) $3x + 2y + 2 = 0$
    (d) none of these

11. The function $f(x)=\cos x$ is decreasing on ________

(a) $\left(\pi, \frac{3\pi}{2}\right)$ (b) $\left(0, \frac{\pi}{2}\right)$

(c) $\left(\frac{3\pi}{2}, 2\pi\right)$ (d) $\left(\frac{\pi}{2}, \frac{3\pi}{2}\right)$

12. The derivative of $f(x)$ is $x(x-1)$. Then it increases on ______.

(a) $0 \le x < 1$

(b) $0 < x \le 1$

(c) $0 < x < 1$

(d) $x > 1$ and $x < 0$

13. The function $f(x)=\cos x - 2px$ is monotonically decreasing for

(a) $p < \frac{1}{2}$ (b) $p > \frac{1}{2}$

(c) $p < 2$ (d) $p > 2$

14. What is the interval in which the function $\sin^2 x - x$ is increasing?

(a) $o < x < 1$ (b) $-1 < x < 0$

(c) ϕ (d) none of these

15. What is the set of values of x for which the function $f(x)=\sin x - x$ is increasing

(a) ϕ (b) $-1 < x < 0$

(c) $0 < x < 1$ (d) $1 \le x < 2$

16. What is the intervel where the function $f(x)=\sin x + \cos x, x \in [0, 2\pi]$ is increasing?

(a) $\left[0, \frac{\pi}{4}\right] \cup \left[\frac{\pi}{4}, 2\pi\right]$

(b) $\left(\frac{\pi}{4}, \frac{\pi}{2}\right)$

(c) $\left(\frac{\pi}{2}, \frac{3\pi}{2}\right)$

(d) $\left(\frac{\pi}{2}, \frac{2\pi}{3}\right)$

17. The value of a for which the function $f(x)=a\sin x + \frac{1}{3}\sin 3x$ has an extremum at $x=\frac{\pi}{3}$ is ________

(a) -1 (b) 0

(c) 1 (d) 2

18. What is the minimum value of $\left(\frac{1}{x}\right)^x$?

(a) $e^{\frac{1}{e}}$ (b) e^e

(c) e^{-e} (d) $\left(\frac{1}{e}\right)^e$

19. The height of the cylinder of maximum volume that can be inscribed in a sphere of radius a is __________

(a) $\frac{3\pi}{4}$ (b) $\frac{5a}{4}$

(c) $\frac{a}{\sqrt{2}}$ (d) $\frac{2a}{\sqrt{3}}$

20. The maximum value of the function $f(x)=\sin x(1+\cos x)$ is _________

(a) $\frac{3\sqrt{3}}{4}$ (b) 3

(c) 4 (d) $3, \sqrt{3}$

21. If the reate of increase of the perimeter of a sphere is 3, then the rate of increase of its side us _________.

(a) $\frac{3}{4}$ (b) $\frac{4}{5}$

(c) $\frac{5}{6}$ (d) $\frac{6}{7}$

22. The rate of change of the area of the circle with respect to its raidus is _________.

(a) πr (b) 2πr

(c) 3πr (d) 4πr

Answers

1.	(d)	2.	(b)	3.	(d)	4.	(b)
5.	(b)	6.	(c)	7.	(b)	8.	(c)
9.	(a)	10.	(b)	11.	(b)	12.	(d)
13.	(b)	14.	(c)	15.	(a)	16.	(a)
17.	(d)	18.	(a)	19.	(d)	20.	(a)
21.	(a)	22.	(b)				

B. Fill in the blanks

1. The equation of the tangent to $y=\sqrt{x}$ at the point $\left(\frac{1}{4},\frac{1}{2}\right)$ is ________?
2. The equation of the normal to the curve $y=\sqrt{x}$ at (1,1) is ________?
3. The slope of the tangent to the curve $y=\sqrt{x}$ at the point (1,1) is _______
4. The intervel in which the function $f(x)=x^{\frac{1}{x}}, x>0$ is decreasing is _______
5. The intervel in which the function $y=\frac{\cos x}{x}, x>0$ is increasing is _____
6. The interval in which the function $\frac{\ln x}{x}$ is decreasing is _________
7. The slope of the normal to the curve $2y=3-x^2$ at the point (1,1) is ______
8. The set of values of x for which the function $f(x)=x^3-12x$ is increasing is _________
9. The interval in which the function $f(x)=\frac{x}{\cos x}$ is increasing is _______
10. The value of x for which the function $f(x)=4-x-x^2$ is maximum is _______
11. The minimum vlaue of $x+\frac{1}{x}$ is _____
12. The value of x for which the function $f(x)=3x^2-x+3$ is minimum is _____
13. The value of x for which the minimum value of the function $f(x)=\cos x$ is obtained in the interval $[0,2\pi]$ is ____
14. The minimum value of the function $y=2x^2$ is _______
15. The maximum value of the function $y=\sin x$ in the interval $[0,2\pi]$ is ____
16. Two positive numbers whose sum is 14 and the sum of whose squares is minimum are __________
17. The maximum value of the function $x^{\frac{1}{x}}$ is _______
18. The value of x for which the function x^x is minimum is ________
19. The minimum value of $\frac{e^{x^2}}{x^2}$ is ______
20. The value of x for which the function x^2-5x+6 is minimum is _______
21. The maximum value of $x^2e^{-x^2}$ is ________
22. If $25=t^2+4t$ then the acceleration is ________
23. If $25=3t+4$ then the velocity is _______
24. If the rate of increase of the perimeter of a square is 3, then the rate of increase of the side is ________
25. The rate of increase of the circumference of a circle and that of the area are 3 and 4 respectively then its radios is ___________
26. The rate of increase of the volume of a sphere is twice that of surface area. Then is radius is ________.

Hints & Solutions

1. $4x-4y+1=0$
2. $2x+y-3=0$
3. $\frac{1}{2}$
4. (e,∞)
5. $(0,e)$
6. (e,∞)
7. 1
8. $x>2$ and $x<-2$
9. (e,∞)
10. $\frac{-1}{2}$
11. 2
12. $\frac{1}{6}$
13. π
14. 0
15. 1
16. 7,7
17. $e^{\frac{1}{e}}$
18. $\frac{1}{e}$
19. e
20. $\frac{5}{2}$
21. $\frac{1}{e}$
22. $\frac{2}{3}$
23. $\frac{3}{2}$
24. $\frac{3}{4}$
25. $\frac{4}{3}$
26. 4

C. **Answer in one word**

1. Write the interval in which the function $\sin^2 x - x$ is increasing.

2. For the curve $y = 3x^2 + 4x$, what is the slope of the tangent to the curve at a point whose x - coordinate is -2.

3. What is the acceleration at the end of 2 seconds of the particle that moves with the rule $s = \sqrt{t} + 1$

4. Write the set of values of x for which the function $f(x) = \sin x - x$ is increasing

5. Write a function which has both relative and absolute maximum at the point (1,2)

6. Write the maximum value of the function $y = x^5$ is the interval [1,5]

7. Mention the values of x for which the function $f(x) = x^3 - 12x$ is decreasing.

8. What is the point on the curve $x = a(\theta - \sin\theta)$ $y = a(1 - \cos\theta)$ at which the tangent is parallel to the x – axis?

9. If the tangent at each point of the curve $y = x^3 - ax^2 + x + 1$ is inclined at an acute angle with the positive direction of x – axis, then find a.

10. Find the open interval in which $f(x) = x^{\frac{1}{x}}, x \geq 0$ is decreasing.

Hints & Solutions

1. Function is increasing in ϕ

Hints: Let $f(x) = \sin^2 x - x$

$f^1(x) = 2\sin x \cos x - 1$

$= \sin 2x - 1 \leq 0$ for all $x \in R$

The function is decreasing in the interval R.

$\Rightarrow$ The function is increasing in ϕ

2. Slope of the tangent $= -8$

3. $-\dfrac{1}{8\sqrt{2}}$ units

4. There is no set of values of x for which the function is increasing.

Hints: $f(x) = \sin x - x$

$f^1(x) = \cos x - 1$

The function is increasing when

$f^1(x) > 0$

$\Rightarrow \cos x - 1 > 0$

$\Rightarrow \cos x > 1$

$\Rightarrow \cos x \in \phi$

5. The required function is $y - 2 = -(x-1)^2$ This function is both relative and absolute maximum at (1,2)

6. The given function is $y = x^5$ in the interval [1,5], the maximum value of x^5 is $5^5 = 3125$.

7. $x \geq 2$ and $x \leq -2$.

Hints: The given function is

$f(x) = x^3 - 12x$

$f^1(x) = 3x^3 - 12 = 3(x^2 - 4)$

The function is increasing if $f^1(x) \geq 0$

$\Rightarrow 3(x^2 - 4) \geq 0$

$\Rightarrow (x-2)(x+2) \geq 0$

$\Rightarrow x \geq 2$ and $x \leq -2$

8. The equation of the curve is $x = a\ (\theta - \sin\theta), y = a(1-\cos\theta)$

$$\therefore \frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \cot\frac{\theta}{2}$$

The tangent in parallel to $x-$ axis

$$\Rightarrow \frac{dy}{dx} = 0 \ \Rightarrow \cot\frac{\theta}{2} = 0$$

$$\Rightarrow \theta = \pi$$

$$\therefore x = a\left(\pi - \sin\pi\right) = a\pi$$

$$y = a(1-\cos\pi) = 2a$$

The required point is $(a\pi, 2a)$

9. $-\sqrt{3} \le a \le \sqrt{3}$

10. The decreases in (e, ∞).

D. Answer in one sentence:

1. Find the interval of x in which the function $y = \frac{\ln x}{x}$, $x > 0$ is increasing.

2. For which value of x, the function $f(x) = 5 - 6x$ is increasing.

3. What is the value of a for which the function $f(x) = a\sin x + \frac{1}{3}\sin 3x$ has an extermum at $x = \frac{\pi}{3}$?

4. If $f(x) = \sin x + 2$ in the interval $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$, what can you say about the greatest value of $f(x)$?

5. Find the interval where the function $f(x) = \sin x + \cos x, x \in \left(0, \frac{\pi}{2}\right)$ is increasing or decreasing.

6. Find the exteme point of the function $f(x) = x + \frac{1}{x}$.

7. What is the equation of the normal to the curve $y = \sqrt{x}$ at the point $\left(\frac{1}{4}, \frac{1}{2}\right)$?

8. Write the equation of the tangent to the curve $y = |x|$ at the point (-2,2)

9. If the tangent of the curve $x = at^2, y = 2at$ is perpendicular to $x-$axis the what is the point of contact?

10. What is the equation of the normal to the curve $y = \sin x$ at $(0,0)$.

11. If $f(x) = x^3 + ax^2 + bx + 5\sin^2 x$ be an increasing function on the ser R, then what is the relation between a and b?

12. If f and g are two increasing fucntions, then show that fog is an increasing fucntions.

13. Mention the values of x for which the function $f(x) = x^3 - 12x$ is increasing.

14. What is the slope of the normal to the curve $2y = 3 - x^2$ at the point $(1,1)$?

15. Find the equation of the tangent to the curve $x = y^2 - 1$ at the point where the slope of the normal to the curve is 2.

Hints & Solutions

1. The function is increasing in $(0, e)$

Hints: $y = \frac{\ln x}{x}$

$$\frac{dy}{dx} = \frac{d}{x}\left(\frac{\ln x}{x}\right) = \frac{x.\frac{1}{x} - \ln x.1}{x^2}$$

$$= \frac{1-\ln x}{x^2}$$

The function increasing when $\frac{dy}{dx} > 0$

$$\Rightarrow \frac{1-\ln x}{x^2} > 0$$

$$\Rightarrow \ln x < 1$$

$$\Rightarrow \ln x < \ln e$$

$$\Rightarrow x < e$$

$\Rightarrow$ The function is increasing in $(0, e)$

2. The function is decreasing for all $x \in R$

3. If $a = 2$, then the function has an extermum at $x = \frac{\pi}{3}$.

4. The function in maximum at $x = \frac{\pi}{2}$

Hints: $f(x) = \sin x + 2$

$f^1(x) = \cos x$

$f^{11}(x) = -\sin x$

For maximum or minimum, $f^1(x) = 0$

$$\Rightarrow \cos x = 0$$

$$\Rightarrow x = \frac{\pi}{2}$$

When $x = \frac{\pi}{2}$, then $f^{11}(x) = -\sin\frac{\pi}{2} = -1$ which is -ve

so the function is maximum at $x = \frac{\pi}{2}$.

5. The function increasing then $x \in \left[0, \frac{\pi}{4}\right] \cup \left[\frac{5\pi}{4}, 2\pi\right]$ and decreasing when $x \in \left(\frac{\pi}{4}, \frac{5\pi}{4}\right)$.

6. The required extreme points are ± 1.

7. The equation of the normal is $y - \frac{1}{2} = (-1)\left(x - \frac{1}{4}\right)$

8. Equation of the tangent at $(-2, 2)$ is $x + y = 0$.

9. The required point of contact is $(0, 0)$.

10. Equation of the normal is $x + y = 0$

11. The required relation is

$$a^2 - 3b + 15 < 0$$

12. (gof) is an increasing fucntion.

Hints: Let $x_1, x_2 \in R$ such that $x_1 < x_2$

Here $x_1 < x_2 \Rightarrow f(x_1) < f(x_2)$

($\because f$ is increasing)

$$\Rightarrow g\left[f(x_1) < g[f(x_2)\right]$$

($\because g$ is increasing)

$$\Rightarrow (gof)(x_1) < (gof)(x_2)$$

$\Rightarrow gof$ is an increasing.

13. The function is increasing when $x > 2$ and $x < -2$.

14. Slope of the normal is 1.

15. Equation of the tangent is $x + 2y + 2 = 0$.

Group-B

Short type (Questions & Answers)

1. Show that the sum of intercepts on the coordinate axes of any tangent ot the curve $\sqrt{x}+\sqrt{y}=\sqrt{a}$ is constant.

2. Show that the tangent to the curve $x=a(t-\sin t), y=at(1+\cos t)$ at $t=\frac{\pi}{2}$ has the slope $1-\frac{\pi}{2}$.

3. Show that $2\sin x+3\tan x \geq 3x$ for all $x \in \left(0,\frac{\pi}{2}\right)$.

4. Find the interval in which the function $f(x)=3x^4-4x^3-12x^2+5$ is
   (i) strictly increasing
   (ii) strictly decreasing.

5. Show that the fucntion
   $f(x)=x^3-3x^2+3x, x\in R$
   is increasing on R.

6. Find the equation of the tangent to the curve $x=a\sin^3\theta$ and $y=a\cos^3\theta$ at $\theta=\frac{\pi}{4}$.

7. Find the points on the curve $y=x^3-11x+5$ at which the equation of the tangent is $y=x-11$.

8. Find the points on the curve $x^2+y^2-2x-3=0$ at which the tangent is parallel to $x-axis$.

9. Find the equation of tangent to the curve $x=\sin 3t, y=\cos 2t$ at $t=\frac{\pi}{4}$.

10. Find the equation of the tangent to the curve $x^2+3y=3$ which is parallel to $y-4x+5=0$.

11. Find the equation of the normal to the curve $y=(\log x)^2$ at $x=\frac{1}{e}$

12. Find two numbers whose sum is 24 and product is maximum.

13. Find the positive numbers whose product is 256 and whos sum is least.

14. Show that all the rectangles with a given perimeter, the square has the largest area.

15. Find the interval where the following fucntion is increasing.
    $y=\sin x+\cos x, x\in[0,2\pi]$

Hints & Solutions

1. The equation of the curve is

$\sqrt{x}+\sqrt{y}=\sqrt{a}$ ....(1)

Let P be a point on the curve (1) whose coordinaty are (x_1, y_1)

$\therefore \sqrt{x_1}+\sqrt{y_1}=\sqrt{a}$ .... (2)

Differentialing both sides of (1), we get

$$\frac{1}{2}x^{-\frac{1}{2}}+\frac{1}{2}y^{\frac{1}{2}}\frac{dy}{dx}=0$$

$$\Rightarrow \frac{1}{\sqrt{y}}\frac{dy}{dx}=-\frac{1}{\sqrt{x}}$$

$$\Rightarrow \frac{dy}{dx}=-\frac{\sqrt{y}}{\sqrt{x}}$$

At the point $(x_1, y_1), \frac{dy}{dx} = \frac{\sqrt{y_1}}{\sqrt{x_1}}$

Equation of the tangent of the curve (1) at the point (x_1, y_1) is

$$y - y_1 = \frac{dy}{dx}(x - x_1)$$

$$\Rightarrow y - y_1 = -\frac{\sqrt{y_1}}{\sqrt{x_1}} = (x - x_1)$$

$$\Rightarrow x\sqrt{y_1} + y\sqrt{x_1} = \sqrt{x_1}\sqrt{y_1}\sqrt{a}$$

$$\Rightarrow \frac{x}{\sqrt{x_1}\sqrt{a}} + \frac{y}{\sqrt{y_1}\sqrt{a}} = 1$$

Let the tangent at P intersect $x-axis$ at A and $y-axis$ at B.

$\therefore OA = \sqrt{x}, \sqrt{a}, \qquad OB = \sqrt{y_1}\sqrt{a}$

Sum of the intercepts

$$= \sqrt{x_1}\sqrt{a} + \sqrt{y_1}\sqrt{a}$$

$$= \sqrt{a}\left(\sqrt{x_1} + \sqrt{y_1}\right)$$

$= \sqrt{a}.\sqrt{a} = a$ which in constant.

2. The given curve is

$x = a(t - \sin t)$

$y = at(1 + \cos t)$

$$\therefore \frac{dx}{dt} = a(1 - \cos t)$$

$$\frac{dy}{dt} = \frac{d\, at(1 + \cos t)}{dt}$$

$$= a\left[-t\sin t + 1 + \cos t\right]$$

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{1 + \cos t - t\sin t}{1 - \cos t}$$

At $t = \frac{\pi}{2}, \frac{dy}{dx} = \frac{1 + \cos\frac{\pi}{2}\sin\frac{\pi}{2}}{1 - \cos\frac{\pi}{2}}$

$$= 1 - \frac{\pi}{2}$$

3. Let $f(x) = 2\sin x + 3\tan x - 3x$

$$f^1(x) = 2\cos x + 3\sec^2 x - 3$$

$$= 2\cos x + 3\tan^2 x > 0$$

for all $x \in \left(0, \frac{\pi}{2}\right)$

$\Rightarrow$ The function is increasing for all $x \in \left(0, \frac{\pi}{2}\right)$.

But $f(0) = 2\sin 0 + 3\tan 0 - 3.0$

$\Rightarrow f(0) = 0$

$\therefore f(x) > f(0)$

$\Rightarrow 2\sin x + 3\tan x - 3x > 0$

$\Rightarrow 2\sin x + 3\tan x > 3x$ for all $x \in \left(\pi, \frac{\pi}{2}\right)$

9. The equation of the curve is $x = \sin 3t, y = \cos 2t$

$$\frac{dx}{dy} = 3\cos 3t, \ \frac{dy}{dt} = -2\sin 2t$$

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{-2\sin 2t}{3\cos 3t}$$

When $t = \frac{\pi}{4}, \frac{dy}{dx} = \frac{-2\sin 2.\frac{\pi}{4}}{3\cos 3\frac{\pi}{4}} = \frac{2\sqrt{2}}{3}$

When $t = \frac{\pi}{4}, x = \sin 3.\frac{\pi}{4} = \frac{1}{\sqrt{2}}$

$$y = \cos 2.\frac{\pi}{4} = \cos\frac{\pi}{2} = 0$$

The point where the tangent is to be obtained is $\left(\frac{1}{\sqrt{2}}, 0\right)$

Equation of the tangent at $\left(\frac{1}{\sqrt{2}}, 0\right)$

Whose slope is $\frac{2\sqrt{2}}{3}$ is

$$y - 0 = \frac{2\sqrt{2}}{3}\left(x - \frac{1}{\sqrt{2}}\right)$$

$$\Rightarrow y = \frac{2\sqrt{2}x}{3} - \frac{2}{3}$$

10. The equation of the curve is

$$y = (\log x)^2 \quad \text{............ (1)}$$

$$\frac{dy}{dx} = \frac{d(\log x)^2}{dx} = \frac{2\log x}{x}$$

slope of the tangent to curve (1) at any point is $\frac{dy}{dx} = \frac{2\log x}{x}$

slope of the tangent $x = \frac{1}{e}$ of

$$\left(\frac{dy}{dx}\right)_{x=\frac{1}{e}} = \frac{2\log\frac{1}{e}}{\frac{1}{e}} = 2e\log\left(e^{-1}\right) = -2e$$

Slope of the normal at $x = \frac{1}{e}$

$$= \frac{1}{-\left(\frac{dy}{dx}\right)_{\frac{1}{e}}} = -\frac{1}{(-2e)} = \frac{1}{2e}$$

When $x = \frac{1}{e},\ y = \left(\log\frac{1}{e}\right)^2 = \left(-\log e\right)^2$

$$= 1$$

The point on the curve is $\left(\frac{1}{e}, 1\right)$

Equation of the normal at $\left(\frac{1}{e}, 1\right)$

$$y - 1 = \frac{1}{2e}\left(1 - \frac{1}{e}\right)$$

14. Let x and y be the length of two sides of the rectangle.

Let P be the perimeter $P = 2(x+y)$ which is constant

$$\Rightarrow x + y = \frac{p}{2}$$

$$\Rightarrow y = \frac{p}{2} - x$$

Let A be the area

$$A = xy - x\left(\frac{p}{2} - x\right) = \frac{p}{2}x - x^2$$

$$\frac{dA}{dx} = \frac{p}{2} - 2x$$

For extremum $\frac{dA}{dx} = 0 \ \Rightarrow \frac{p}{2} - 2x = 0$

$$\Rightarrow x = \frac{p}{4}$$

When $x = \frac{p}{4}$ then $\frac{d^2A}{dx^2} = -2$ which is -ve.

So A is maximum when $x = \frac{p}{4}$

$$y = \frac{p}{2} - \frac{p}{4} = \frac{p}{4}$$

When A is maximum when $x = y = \frac{p}{2}$

Group-C

Lont type (Questions & Answers)

1. Show that the semi vertical angle of a cone of given slant height is $\tan^{-1}\sqrt{2}$ when the volume is maximum.

2. Show that the radius of the right circular cylinder of greatest curved surface that can be inscribed in a given come is half of radius of the base of the cone.

3. Show that the height of a closed right circular cylinder of given surfaces and maximum volume is equal to the diameter of the base.

4. Show that the triangle of greatest area that can be inscribed in circle is equilateral.

5. Find the tangent to the curve $y = \cos(x+y)$ $0 \le x \le 2\pi$ which is parallel to the line $x + 2y = 0$

6. Find the minimum distance of a point on the curve $\frac{4}{x^2} + \frac{1}{y^2} = 1$ form the origin.

7. Determine the points of extreme values on the following curves $y^3 = (x-1)^2(x+2)$

8. Prove that the sum of the cubes of the intercepts on the coordinate axes of any tangent to the curve $x^{\frac{3}{4}} + y^{\frac{3}{4}} = a^{\frac{3}{4}}$ is a constant.

9. Show that the length of a portion of the tangent to $x^{\frac{2}{3}} + y^{\frac{2}{3}} = 4$ intercepted between the axes is a constant.

10. Find the equation of the normal to the curve given by $x = \cos^3\theta$, $y = \sin^3\theta$ at $\theta = \frac{\pi}{4}$

11. Find the altitude of the right circular cylinder of maximum volume that can be inscribed within a sphere of radius R.

12. Find the coordinates of the point on the curve $x^2y - x + y = 0$ where the slope of the tangent is maximum?

13. Prove that for all real x, $e \le \frac{e^{x^2}}{x^2}$

14. Find the maximum value of $f(x) = x^{\frac{1}{x}}, x > 0$ and show that $e^{\pi} > \pi^{e}$.

15. Discuss the extreme value of the function $y = (x+2)^4 \ (x-1)^5$

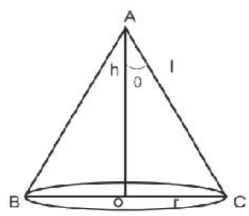
Hints & Solutions

1. Let ABC and a cone of height 'h' and a radius of the base r . let l be the length ofthe slant height.

$$\therefore r^2 + h^2 = l^2$$

$$\Rightarrow h^2 = l^2 - r^2$$

$$\Rightarrow r^2 = l^2 - h^2$$



The volume of the cone v is given by

$$v = \frac{1}{3}\pi r^2 h$$

$$= \frac{1}{3}\pi h(l^2 - h^2)$$

$$= \frac{1}{3}\pi\left(l^2 h - h^3\right)$$

$$\frac{dv}{dh} = \frac{1}{3}\left(l^2 - 3h^2\right)$$

$$\frac{d^2v}{dh^2} = \frac{1}{3}\pi(-6h) = -2\pi h$$

For maximum or minimum, $\frac{dv}{dh} = 0$

$$\Rightarrow \frac{1}{3}\pi\left(l^2 - 3h^2\right) = 0$$

$$\Rightarrow l^2 - 3h^2 = 0$$

$$\Rightarrow h^2 = \frac{l^2}{3}$$

$$h = \frac{l}{\sqrt{3}}$$

When $h = \frac{1}{\sqrt{3}}, \frac{d^2v}{dh^2} = -2\frac{l}{\sqrt{3}}$ which is -ve

So V is maximum when $h = \frac{l}{\sqrt{3}}$

$$r^2 = l^2 - h^2 = l^2 - \frac{l^2}{3} = \frac{2l^2}{3}$$

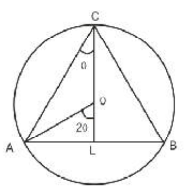
$$\Rightarrow r = \sqrt{\frac{2}{3}}l$$

Let θ be the semivertical angle of the come

$$\tan\theta = \frac{r}{h} = \sqrt{2}$$

$$\Rightarrow \theta = \tan^{-1}\sqrt{2}$$

4. Let 0 be the centre of a circle of radius a let ABC be a triangle inseribed in this circle. Let AB be the base of the triangle, of maximum are. Area of the triangle is maximum when the atitude is maximum Here C is at a maximum distance from the base AB.



C must be on the diameter perpendicular to AB. CL is the right bisector of AB.

Here $AC = BC$

ABC is isosceles.

Let θ be the semi vertical angle ACL

$$\therefore < AOL = 2\theta$$

$$AL = OA\sin 2\theta = a\sin 2\theta$$

$$OL = OA\cos 2\theta = a\cos 2\theta$$

$$CL = CO + OL = a + a\cos 2\theta$$

$$= a(1 + \cos 2\theta)$$

Let Y be the area of the tringle ABC

$$Y = \frac{1}{2}AB.CL = \frac{1}{2}.2AL.CL$$

$$= a\sin 2\theta.a(1 + \cos 2\theta)$$

$$= a^2\left(\sin 2\theta.a\sin 2\theta.\cos 2\theta\right)$$

$$= \frac{a^2}{2}\left(\sin 2\theta + \sin 4\theta\right)$$

$$\frac{dy}{d\theta} = 2a^2\left(\cos 4\theta + \cos 2\theta\right)$$

$$= 4a^2\cos 3\theta.\cos\theta$$

$$\frac{d^2y}{d\theta^2} = -4a^2\left(2\sin 4\theta + \sin 2\theta\right)$$

For extremum, $\frac{dy}{d\theta} = 0$

$$\Rightarrow \cos 3\theta.\cos\theta = 0$$

When $\cos 3\theta = 0 \Rightarrow 3\theta = \frac{\pi}{2} \Rightarrow \theta = \frac{\pi}{6}$

When $\cos\theta = 0 \Rightarrow \theta = \frac{\pi}{2}$ which is rejected.

When $\theta = \frac{\pi}{6}, \frac{d^2y}{d\theta^2} = -4a^2 \left(2\sin^2\frac{\pi}{3} + \sin\frac{\pi}{3}\right)$

$$= -4a^2\left(2.\frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2}\right)$$

$$= 4a^2.\frac{3\sqrt{3}}{2} = -6a^2\sqrt{5}$$

Which is $-ve$

So Y is minimum when $\theta = \frac{\pi}{6}$

$$\therefore \angle ACB = 2.\frac{\pi}{6} = \frac{\pi}{3}$$

$$\Rightarrow \angle A = \angle B = \frac{\pi}{3}$$

$\Rightarrow \Delta ABC$ is equilateral.

5. Equation of the curve is

$y = \cos(x+y),\ 0 \le x < 2\pi$ ....(1)

Given line is $x + 2y = 0$ .... (2)

We get

$$\frac{dy}{dx} = \frac{d\cos(x+y)}{dx}$$

$$= -\sin(x+y)\left[1 + \frac{dy}{dx}\right]$$

$$\Rightarrow \frac{dy}{dx} = -\frac{\sin(x+y)}{1+\sin(x+y)}$$

slope of the line (2) $= -\frac{1}{2}$

$$\Rightarrow \frac{-\sin(x+y)}{1+\sin(x+y)} = \frac{-1}{2}$$

$$\Rightarrow -2\sin(x+y) = -\left[1 + \sin(x+y)\right]$$

$$\Rightarrow \sin(x+y) = 1$$

$$\Rightarrow \cos(x+y) = 0 \quad \Rightarrow y = 0, x + y = \frac{\pi}{2}$$

$$\Rightarrow x = \frac{\pi}{2}, y = 0$$

The point is $\left(\frac{\pi}{2}, 0\right)$

Equation of the tangent at $\left(\frac{\pi}{2}, 0\right)$ is

$$y - 0 = \frac{-1}{2}\left(x - \frac{\pi}{2}\right)$$

$$\Rightarrow 2y = -x + \frac{\pi}{2}$$

$$\Rightarrow x + 2y = \frac{\pi}{2}$$

7. Given curve is $y^3 = (x-1)^2(x+2)$.

Differentiating both sides w.r.t x,

$$3y^2\frac{dy}{dx} = \frac{d}{dx}(x-1)^2(x+2)$$

$$= 3(x-1)(x+1)$$

$$\Rightarrow \frac{dy}{dx} = \frac{(x-1)(x+1)}{y^2}$$

For extremum, $\frac{dy}{dx} = 0$

$$\Rightarrow (x-1)(x+1) = 0$$

$$\Rightarrow x = 1, -1$$

When $x = 1, \quad y = 0$

When $x = -1$ $y^3 = (-1-1)^2(-1+2) = 4$

$$\Rightarrow y = \sqrt[3]{4} = 2^{\frac{2}{3}}$$

Points of extremum are

$(1,0)$ & $\left(-1, 2^{\frac{2}{3}}\right)$

CHAPTER - 9

INTEGRALS

Multiple Choice Questions (MCQ)

Group-A

A. Choose the correct answer from the given choices:

1. $\int b^{x+a}dx = ?$

(a) $\frac{b^{x+a}}{\log a}+C$ (b) $\frac{b^{x+a}}{\log b}+C$

(c) $b^{x+a}+C$ (d) None of these

2. $\int \frac{\cos x}{\sin^2 x}dx$ is

(a) $-\sin x + C$ (b) $\tan x + C$

(c) $\sec x + C$ (d) $-\cos ecx + C$

3. $\int \frac{\sec x}{\tan^2 x}dx$ is

(a) $-\cos x + C$ (b) $-\cot x + C$

(c) $-\cos ecx + C$ (d) none of these.

4. $\int e^{\left(\sin^{-1}x+\cos^{-1}x\right)}dx$ is

(a) $x + C$ (b) $xe^{\frac{\pi}{2}}+C$

(c) $e^{\frac{\pi}{2}}+x$ (d) None of these

5. $\int \sin\frac{x}{2}.\cos\frac{x}{2}dx =$

(a) $-\frac{1}{2}\cos x + C$ (b) $-\frac{1}{2}\sin x + C$

(c) $\frac{1}{2}\tan x + C$ (d) $\frac{1}{2}\cot x + C$

6. $\int \left(1-x^2\right)^{\frac{-1}{2}}dx =$

(a) $\cos^{-1}x + C$ (b) $\sin^{-1}x + C$

(c) $\tan^{-1}x + C$ (d) None of these

7. $\int (dx+dy) =$

(a) $xy + C$ (b) $\frac{x}{y}+C$

(c) $x+y+C$ (d) None of these

8. $\int (xdy + ydx) =$

(a) $xy + C$ (b) $\frac{y}{x}+C$

(c) $x+y+C$ (d) None of these

9. $\int \frac{ydx - xdy}{y^2} =$

(a) $\frac{x}{y}+C$ (b) $\frac{y}{x}+C$

(c) $xy + C$ (d) None of these

10. What is $\int \frac{e^{\tan^{-1}x}}{1+x^2}dx$?

(a) $-e^{\tan^{-1}x}+C$ (b) $e^{\tan^{-1}x}+C$

(c) $\frac{e^{\tan^{-1}x}}{1+x^2}+C$ (d) None of these

11. $\int \frac{xdy - ydx}{x^2} =$

(a) $\frac{y}{x} + C$ (b) $\frac{x}{y} + C$

(c) $xy + C$ (d) None of these

12. What is $\int \frac{\sec^2 x}{\cos ec^2 x} dx$?

(a) $x \tan x + C$ (b) $x - \tan x + C$

(c) $\tan x - x + C$ (d) None of these

13. What is $\int \frac{\sin 6x + \sin 4x}{\cos 6x + \cos 4x} dx$?

(a) $\frac{1}{5} \ln \sec 5x + C$

(b) $\frac{1}{5} \ln \sin 5x + C$

(c) $\frac{1}{5} \ln \tan 5x + C$

(d) None of these

14. $\int \frac{dx}{x\left[(\log x)^2 + 25\right]} =$ ______

(a) $\frac{1}{5} \tan^{-1}(\log x) + C$

(b) $\frac{1}{5} \tan^{-1}\left(\frac{1}{5} \log x\right) + C$

(c) $\tan^{-1}\left(\frac{1}{2} \log x\right) + C$

(d) None of these

15. $\int \sin^{-1} x \, dx =$ __________

(a) $x \cos^{-1} x - \sqrt{1 - x^2} + C$

(b) $x \sin^{-1} x - \sqrt{1 - x^2} + C$

(c) $x \sin^{-1} x + \sqrt{1 - x^2} + C$

(d) None of these

16. $\int \frac{3 + \cos x + \tan^2 x}{2x + \sin x + \tan x} dx =$ ________

(a) $\ln(\sin x + \tan x) + C$

(b) $\ln(2x + \sin x + \tan x) + C$

(c) $\frac{1}{2} \ln(2x + \sin x + \tan x) + C$

(d) None of these

17. $\int \frac{\sin^{-1} x - \cos x}{\sqrt{1 - \sin 2x}} e^{\sin x} \cos x \, dx = ?$

(a) $e^{\sin x} + C$ (b) $e^{\cos x} + C$

(c) $e^{-\sin x} + C$ (d) None of these

18. $\int 5^{5^{5^x}} . 5^{5^x} . 5^x \; dx =$ ______

(a) $\frac{1}{(\ln 5)^2} 5^{5^x}$ (b) $\frac{1}{(\ln 5)^3} 5^{5^{5^x}} + C$

(c) $\frac{1}{\ln 5} . 5^{5^{5^x}} + C$ (d) None of these

19. $\int \frac{\sqrt{\tan x}}{\sin x \cos x} dx =$ _______

(a) $\sqrt{\tan x} + C$ (b) $2\sqrt{\tan x} + C$

(c) $3\sqrt{\tan x} + C$ (d) None of these

20. $\int \frac{\cot x}{\ln \sin x} dx =$ __________?

(a) $\ln \ln \sin x + C$ (b) $\ln \ln \cos x + C$

(c) $\ln \sin x + C$ (d) None of these

21. What is the integral of $\int e^{x^2} . 2x \, dx$?

(a) $e^{-x^2} + C$ (b) $e^{x^2} + C$

(c) $\frac{1}{2} e^{x^2} + C$ (d) $\frac{1}{3} e^{x^2} + C$

22. $\int \frac{dx}{\cos^2 x \sin^2 x} = ?$

(a) $\tan x - \cot x + C$

(b) $\cot x - \tan x + C$

(c) $\tan x + \cot x + C$

(d) None of these

23. What is the integral of

$\int \frac{\cot^2 x - \cos ec^2 x}{x^2} dx$?

(a) $-\frac{1}{x} + C$ (b) $\frac{1}{x} + C$

(c) $\frac{2}{x} + C$ (d) None of these

24. $\int \left(\sqrt{a^2 - x^2} + \frac{x^2}{\sqrt{a^2 - x^2}} \right) dx =$ _____?

(a) $\sin^{-1} \frac{x}{a} + C$ (b) $a^2 \sin^{-1} \frac{x}{a} + C$

(c) $\frac{1}{2} \sin^{-1} \frac{x}{a} + C$ (d) None of these

25. What is the value of $\int \frac{1 + \frac{1}{x^2}}{x - \frac{1}{x} + 4} dx$?

(a) $\ln\left(x - \frac{1}{x} + 4 \right) + C$

(b) $-\ln\left(x - \frac{1}{x} + 4 \right) + C$

(c) $\frac{1}{2} \ln\left(x - \frac{1}{x} + 4 \right) + C$

(d) None of these.

26. What is the value of $\int_0^{\pi/2} \log \tan x \, dx$?

(a) $\log 2$ (b) $\log 3$

(c) $\log 4$ (d) None of these

27. What is the value of

$\int_0^{\pi/2} \frac{f(x)}{f(x) + f\left(\frac{\pi}{2} - x \right)} dx$

(a) $\frac{\pi}{8}$ (b) $\frac{\pi}{4}$

(c) $\frac{\pi}{2}$ (d) 0

28. What is the value of

$\int_0^{\pi/2} \frac{\sin x}{\sin x + \cos x} dx - \int_0^{\pi/2} \frac{\cos x}{\sin x + \cos x} dx$

(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{4}$

(c) 0 (d) π

29. What is the value of $\int_{-\pi/4}^{\pi/4} \sin^5 x \cos x \, dx$

(a) 0 (b) $\frac{\pi}{4}$

(c) $\frac{\pi}{3}$ (d) $\frac{\pi}{2}$

30. What is the value of

$\int_0^{\pi/2} \log(\tan x + \cot x) dx$?

(a) $\frac{\pi}{2} \log 2$ (b) $\pi \log 2$

(c) $2\pi \log 2$ (d) $3\pi \log 2$

Answers

1. (b)	2. (d)	3. (d)	4. (b)
5. (a)	6. (b)	7. (c)	8. (a)
9. (a)	10. (b)	11. (a)	12. (c)
13. (a)	14. (b)	15. (c)	16. (b)
17. (a)	18. (b)	19. (b)	20. (a)
21. (b)	22. (a)	23. (b)	24. (b)
25. (a)	26. (d)	27. (b)	28. (c)
29. (a)	30. (b)		

B. Fill in the blanks:

1. The definite integral which is equal to $\lim_{x\to\infty}\frac{1}{n}\sum_{r=1}^{n}\frac{r}{\sqrt{n^2+r^2}}$ is ________ [CHSE-2018]

2. $\int_0^{\pi/2}\log\tan x\,dx =$ _______.

3. The value of $\int_{-\pi/2}^{\pi/2}\log\left(\frac{2-\sin x}{2+\sin x}\right)dx$ is _______

4. The value of $\int_0^{\pi}\log\frac{1}{1+2\sin\theta}d\theta =$ ____

5. The value of $\int_0^{\sqrt[7]{2}}\frac{1}{x\left(2x^7+1\right)}dx$ is ____

6. $\int_0^{1000}e^{x-[x]}dx =$ _______

7. $\int_{\pi/6}^{\frac{5\pi}{6}}\sqrt{4-4\sin^2 t}\;dt =$ ________

8. $\int_0^{\pi/2}\frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}}dx =$ ________

9. $\int_1^3\tan^{-1}x\,dx+\int_1^3\cot^{-1}x\,dx =$ ______

10. $\int e^{\ln\left(\cos ec^2x-\cot^2x\right)}dx =$ _________

11. $\int\frac{\cos x-\cos 2x}{1-\cos x}dx =$ _________

12. $\int\sec^4\cos ec^2x\;dx =$ _________

13. $\int\frac{\cos x+x\sin x}{x\left(x+\cos x\right)}dx =$ _________

14. $\int\frac{\left(1+\log x\right)^2}{1+\log\left(x^{x+1}\right)+x\left(\log x\right)^2}dx =$ _____

15. $\int\sqrt{\frac{x}{a^3-x}}dx =$ _________

16. $\int_{-1}^{3}\left[\tan^{-1}\frac{x}{x^2+1}+\tan^{-1}\frac{x^2+1}{x}\right]dx =$ ____

17. $\int_0^{\pi/2}\log\left(\tan x+\cos x\right)dx =$ _______

18. $\int_{-1}^{1}x\,|\,x\,|\,dx =$ _________

19. The value of $\int_{-1}^{1}\frac{d}{dx}\left(\tan^{-1}\frac{1}{x}\right)dx =$ _______

20. $\int_0^{\pi}\frac{dx}{1+2\sin^2 x} =$ _________

Hints & Answers

1. $\int_0^1\frac{x}{\sqrt{1+x^2}}dx$

Hints : $\lim_{n\to\infty}\frac{1}{n}\sum_{r=1}^{n}\frac{r}{\sqrt{n^2+r^2}}$

$=\lim_{x\to\infty}\frac{1}{n}\sum_{r=1}^{n}\frac{r}{n\sqrt{1+\left(\frac{r}{n}\right)^2}}$

$=\lim_{n\to\infty}\frac{1}{n}\sum_{r=1}^{n}\frac{\frac{r}{n}}{\sqrt{1+\left(\frac{r}{n}\right)^2}}$

$=\int_0^1\frac{x}{\sqrt{1+x^2}}dx$ $\left[\begin{array}{l}\frac{r}{n}=x\\ \frac{1}{n}=dx\end{array}\right.$

$\lim_{x\to\infty}\sum_{r=1}^{n}\int_0^1$]

2. 0

3. 0

Hints: $\log\left(\frac{2-\sin x}{2+\sin x}\right)$ is an odd function.

4. $\frac{\pi}{\sqrt{3}}$

5. $\frac{1}{7}\ln\frac{6}{5}$

Hints: $\int_1^{\sqrt[7]{2}} \frac{1}{x\left(2x^7+1\right)}dx$

$=\int_1^{\sqrt[7]{2}} \frac{x^6}{x\left(2x^7+1\right)}dx$

$=\int_1^2 \frac{\frac{1}{7}dy}{y(2y+1)}$ $\quad \left[x^7 = y\right.$

$=\frac{1}{7}\int_1^2\left(\frac{1}{y}-\frac{2}{2y+1}\right)dy$

$=\frac{1}{7}\left[\ln y-\ln(2y+1)\right]_1^2$

$=\frac{1}{7}\left[\ln\left(\frac{y}{2y+1}\right)\right]_1^2$

$=\frac{1}{7}\ln\frac{6}{5}$

6. $1000(e-1)$

Hints: $\int_0^{10000} e^{x-[x]}dx$

$=1000\int_0^1 e^x dx$ $\quad \left[\because [x]=0\right]$

$=1000[e-1]$

7. 4.

Hitns: $\int_{\pi/6}^{\frac{5\pi}{6}} \sqrt{4-4\sin^2 t}\ dt$

$=\int_{\pi/6}^{5\pi/6} 2\sqrt{1-\sin^2 t}\, dt$

$=2\int_{\pi/6}^{5\pi/6} |\cot|\ dt$

$=2\left[\int_{\pi/6}^{\pi/2}|\cos|dt+\int_{\pi/2}^{\frac{5\pi}{6}}|\cot t|dt\right]$

$=2\left[\int_{\pi/6}^{\pi/2}\cos t+\int_{\pi/2}^{\frac{5\pi}{6}}-\cos t\, dt\right.$

$=2[\sin t]_{\pi/6}^{\pi/2}-2[\sin t]_{\pi/2}^{5\pi/6}$

$=2\left(1\frac{-1}{2}\right)-2\left(\frac{1}{2}-1\right)=4$

8. $\frac{\pi}{4}$

9. π

Hints: $\int_1^3 \tan^{-1}x\ dx+\int_1^3 \cot^{-1}x\ dx$

$=\int_1^3\left(\tan^{-1}x+\cot^{-1}x\right)dx$

$=\int_1^3\frac{\pi}{2}dx=\frac{\pi}{2}[x]_1^3$

$=\pi$

10. 1

Hints : $\int e^{\ln\left(\cos ec^2 x-\cot^2 x\right)}dx$

$=\int e^{\ln 1}dx$

$=\int e^0 dx=x+C$

11. $2\sin x+x+C$

Hints: $\int\frac{\cos x-\cos 2x}{1-\cos x}dx$

$=\int\frac{\cos x-\left(2\cos^2 x-1\right)}{1-\cos x}dx$

$=-\int\frac{2\cos^2 x-\cos x-1}{-(\cos x-1)}dx$

$$= \int (2\cos x + 1)\,dx$$

$$= 2\sin x + x + C$$

12. $\frac{1}{3}\tan^3 x - \cot x + 2\tan x + C$

13. $I = \int \frac{\cos x + x\sin x + x - x}{x(x+\cos x)}\,dx$

$$= \int \frac{(x+\cos x) - x(1-\sin x)}{x(x+\cos x)}\,dx$$

$$= \int \frac{1}{x}\,dx - \int \frac{1-\sin x}{x+\cos x}\,dx$$

$$= \log x - \log(x+\cos x) + C$$

14. $\log(1 + x\log x) + c$

Hints :

$$I = \int \frac{(1+\log x)^2}{1+(x+1)\log x + x(\log x)^2}\,dx$$

$$= \int \frac{(1+\log x)^2}{(1+x\log x)(1+\log x)}\,dx$$

$$= \int \frac{1+\log x}{1+x\log x}\,dx$$

$$= \int \frac{dt}{t} \quad \text{Where } t = 1 + x\log x$$

$$= \log t + C = \log(1 + x\log x) + C$$

15. $\frac{2}{3}\sin^{-1}\left(\frac{x^{3/2}}{a^{3/2}}\right) + C$

Hints : $I = \int \frac{\sqrt{x}dx}{\sqrt{\left(a^{3/2}\right)^2 - \left(x^{3/2}\right)^2}}\,dx$

$$= \frac{2}{3}\int \frac{dt}{\sqrt{\left(a^{3/2}\right)^2 - t^2}} \quad \left[whose\, t = x^{3/2}\right]$$

$$= \frac{2}{3}\sin^{-1}\frac{t}{a^{3/2}} + C = \frac{2}{3}\sin^{-1}\left(\frac{x^{3/2}}{a^{3/2}}\right) + C$$

16. 2π

Hints: $I = \int_1^3 \left[\tan^{-1}\frac{x}{x^2+1} + \cot^{-1}\frac{x}{x^2+1}\right]dx$

$$= \int_{-1}^{3} \frac{\pi}{2}\,dx = \frac{\pi}{2}[x]_{-1}^{3}$$

$$= \frac{\pi}{2}\left[3 - (-1)\right] = 2\pi$$

17. $\pi \log 2$

Hints: $I = \int_0^{\pi/2} \log\left(\frac{\sin x}{\cos x} + \frac{\cos x}{\sin x}\right)dx$

$$= \int_0^{\pi/2} \log\left(\frac{1}{\sin x \cos x}\right)dx$$

$$= -\int_0^{\pi/2} \log\sin x\,dx - \int_0^{\pi/2} \log\cos x\,dx$$

$$= -\left(-\frac{\pi}{2}\log 2\right) - \left(-\frac{\pi}{2}\log 2\right) = \pi \log 2$$

18. 0

Hints : $x|x|$ is an odd function.

$$\therefore \int_{-1}^{1} x|x|\,dx = 0$$

19. $-\frac{\pi}{2}$

Hints: $I = \int_{-1}^{1} \frac{d}{dx}\left(\tan^{-1}\frac{1}{x}\right)dx$

$$= \int_{-1}^{1} \frac{d}{dx}\left(\cot^{-1} x\right)dx$$

$$= \int_{-1}^{1} \frac{1}{1+x^2}\,dx = -2\int_0^1 \frac{1}{1+x^2}\,dx$$

$$= -2\left[\tan^{-1} x\right]_0^1 = -\frac{\pi}{2}$$

20. $\frac{\pi}{3}$

Hints: $I = \int_0^{\pi} \frac{dx}{1+\sin^2 x} = 2\int_0^{\pi/2} \frac{dx}{1+2\sin^2 x}$

$$= 2\int_0^{\pi/2} \frac{\cos ec^2 x}{\cos ec^2 + 2} dx$$

$$= 2\int_0^{\pi/2} \frac{\cos ec^2 x}{\cot^2 x + 3} dx$$

$$= -2\int_\infty^0 \frac{dt}{t^2 + \left(\sqrt{3}\right)^2} = 2\int_0^\infty \frac{dt}{t^2 + \left(\sqrt{3}\right)^2}$$

$$= 2.\frac{1}{\sqrt{3}}\left[\tan^{-1}\frac{t}{\sqrt{3}}\right]_\infty^0$$

$$= \frac{2}{\sqrt{3}}\left[\tan^{-1}\infty - \tan^{-1} 0\right]$$

$$= \frac{\pi}{3}$$

C. Answer in one word:

1. What is the value of $\int \frac{e^{\tan^{-1} x}}{1+x^2} dx$?

2. Evaluate $\int \frac{(\log x)^2}{x} dx$

3. What is $\int \frac{dx}{\sqrt{1-x^2}}$?

4. Write the value of $\int \frac{2-3\sin x}{\cos^2 x} dx$

5. What is $\int \frac{\sec^2 x}{\cos ec^2 x} dx$?

6. Evaluate $\int \frac{dx}{x^2+16}$

7. Write the value of $\int \frac{x+\cos 6x}{3x^2+\sin 6x} dx$

8. Evaluate $\int \frac{2}{1+\cos 2x} dx$

9. Given $\int e^x (\tan x + 1)\sec x \; dx =$ $e^x f(x) + C$ write $f(x)$ satisfying above.

10. Evaluate $\int (1-x)\sqrt{x} dx$

11. Evaluate $\int_{-\pi/4}^{\pi/4} \sin^3 x \; dx$

12. Evaluate $\int_0^1 \frac{2x}{1+x^2} dx$

13. Evaluate $\int_1^{\sqrt{3}} \frac{dx}{1+x^2}$

14. Evaluate $\int_0^2 \sqrt{4-x^2} dx$

15. Evaluate $\int_1^2 \frac{x^3-1}{x^2} dx$

16. Evaluate $\int_0^1 \frac{\tan^{-1} x}{1+x^2} dx$

17. Evaluate $\int_0^{e^2} \frac{dx}{x \log x}$

18. Evaluate $\int_0^{\pi/2} e^x (\sin x - \cos x) dx$

19. Evaluate $\int_0^{\pi/2} \log \sin x \, dx$

20. Evaluate $\int_0^1 \frac{x^4+1}{x^2+1} dx$

<u>Hints & Solutions</u>

1. $e^{\tan^{-1} x} + C$

2. $\frac{(\log x)^3}{3} + C$

3. $\sin^{-1} x + C$

4. $2\tan x - 3\sec x + C$

5. $\tan x - 1 + C$

Hints: $\int \frac{\sec^2 x}{\cos ec^2 x} dx = \int \tan^2 x \, dx$

$$= \int \left(\sec^2 x - 1\right) dx$$

$$= \tan x - x + C$$

6. $\frac{1}{4}\tan^{-1}\frac{x}{4}+C$

7. $\frac{1}{6}\log\left(3x^2+\sin 6x\right)+C$

Hints: Let $3x^2+\sin 6x=t$

$\Rightarrow\left(6x+6\cos 6x\right)dx=dt$

$\Rightarrow\left(x+\cos 6x\right)dx=\frac{1}{6}dt$

$I=\int\frac{1}{t}.\frac{1}{6}dt=\frac{1}{6}\int\frac{1}{t}dt$

$=\frac{1}{6}\ln t+C=\frac{1}{6}\ln\left(3x^2+\sin 6x\right)+C$

8. $\tan x+C$

9. $f(x)=\sec x$

10. $\frac{2}{3}x^{3/2}-\frac{2}{5}x^{5/2}+C$

11. 0

Hints: $\sin^3 x$ is an odd function

so $\int_{-\pi/4}^{\pi/4}\sin^3 x\,dx=0$

12. $\log 2$

Hints: $I=\int_0^1\frac{2x}{1+x^2}dx$

Let $1+x^2=t$ $\quad\Rightarrow 2x\,dx=dt$

$I=\int_1^2\frac{dt}{t}\quad=\left[\log t\right]_1^2$

$=\log 2-\log 1\quad=\log 2.$

13. $\frac{\pi}{12}$

Hints: $\int_1^{\sqrt{3}}\frac{dx}{1+x^2}\quad=\left[\tan^{-1}x\right]_1^{\sqrt{3}}$

$=\tan^{-1}\sqrt{3}-\tan^{-1}1$

$=\frac{\pi}{3}-\frac{\pi}{4}=\frac{\pi}{12}$

14. π

Hints:

$\int_0^2\sqrt{4-x^2}dx=\left[\frac{x}{2}\sqrt{4-x^2}+\frac{4}{2}\sin^{-1}\frac{x}{2}\right]_0^2$

$=0+2\sin^{-1}1=\pi$

15. 1

Hints: Let $I=\int_1^2\frac{x^3-1}{x^2}dx$

$=\int_1^2\left(x-\frac{1}{x^2}\right)dx$

$=\left[\frac{x^2}{2}-\frac{1}{x}\right]_1^2$

$=1$

16. $\frac{\pi^2}{3^2}$

Hints: $I=\int_0^1\frac{\tan^{-1}x}{1+x^2}dx$

Let $\tan^{-1}x=t$

$\Rightarrow\frac{1}{1+x^2}dx=dt$

When $x=0,t=0$

When $x=1,t=\frac{\pi}{4}$

$I=\int_0^{\pi/4}t\,dt=\left[\frac{t^2}{2}\right]_0^{\pi/4}$

$=\frac{1}{2}\left[\left(\frac{\pi}{2}^2-0\right)\right]=\frac{\pi^2}{32}$

17. $\log 2$

18. 1

19. $-\frac{\pi}{2}\log 2$

20. $\dfrac{3\pi - 4}{6}$

Hints: $I = \int_0^1 \frac{x^4+1}{x^2+1} dx = \int_0^1 \frac{\left(x^4-1\right)+2}{x^2+1} dx$

$= \int_0^1 \frac{\left(x^2-1\right)\left(x^2+1\right)+2}{x^2+1} dx$

$= \int_0^1 \left(x^2 - 1 + \frac{2}{x^2+1} \right) dx$

$= \left[\frac{x3}{3} - x + 2\tan^{-1} x \right]_0^1$

$= \dfrac{3\pi - 4}{6}$

D. <u>Answer in one sentence</u>

1. Write the area bounded by $y = -2x$, $y = 0, x = 1$ and $x = 3$.

2. What is the area bounded by $x = e^y$, $x = 0, y = 0, y = 1$?

3. Find $\int \frac{\cos 3x \cos x}{1 + \cos 2x} dx$

4. Find $\int_{-1}^{1} |x|\, dx$

5. What is the value of $\frac{d}{dx}\int_{200}^{300} \left(x^4 + 5x^3\right)^2 dx$

6. Evaluate $e^x \left[f(x) + f^1(x) \right] dx$.

7. Evaluate $\int 2^x . 4^{\frac{-x}{2}} dx.$

8. If $f(x) = \int_0^x e^{2t} \sin 3t\, dt$ then what is $f^1(x)$?

9. Find the value of m for which $\int x^m dx \neq \frac{x^{m+1}}{m+1}$

10. What is the value of $\int e^x \cos x\, dx + \int e^x \sin x\, dx$?

11. If $\int_{\frac{-1}{2}}^{\frac{1}{2}} \cos x \ln \frac{1+x}{1-x} dx = k \ln 2$, then what is the value of k.

12. $\int_{-\pi/2}^{\pi/2} \sin^5 x\, dx = ?$

13. Integrate $\int 2\sin n(\alpha - \beta) x \sin(\alpha - \beta) \;\; x\, dx$

14. Integrate $\int_{-\frac{\pi}{4}}^{\pi/4} \cos^4 x \sin^{99} x \; dx$

15. If $\int_0^1 f(t)dt = 2, \int_2^1 f(u)du = -1$ then what is $\int_0^2 f(x)dx$?

<u>Answers</u>

1. Required area $= \int_2^3 -2x\, dx = -8\ sq.$ unit.

2. The required area $= \int_0^1 e^y dx$

$= (e-1)$ sq unit.

3. $\int \frac{\cos 3x \cos x}{1 + \cos 2x} dx$

$= \int \frac{\left(4\cos^3 x - 3\cos x\right)\cos x}{2\cos^2 x} dx$

$= \int \left(\cos 2x - \frac{1}{2} \right) dx$

$= \frac{1}{2}\sin 2x - \frac{1}{2}x + C$

4. $\int_{-1}^{1}|x| = \int_{-1}^{0}|x|dx + \int_{0}^{1}|x|dx$

$= \int_{-1}^{0} -xdx + \int_{0}^{1} x\, dx$

$= -\left[\frac{x^2}{2}\right]_{-1}^{0} + \left[\frac{x^2}{2}\right]_{0}^{1} = 1$

5. $\int_{200}^{300}\left(x^4 + 5x^3\right)dx$ is a constant.

so $\frac{d}{dx}\int_{200}^{300}\left(x^4 + 5x^3\right)^2 dx = 0$

6. The required integral $= e^x f\left(x\right) + C$

7. $\int 2^x 4^{-\frac{x}{2}} dx = \int 2^x \left(2^2\right)^{\frac{-x}{2}} dx$

$= \int 2^x . 2^{-x} dx = \int dx = x + C$

8. If $f(x) = \int_0^x e^{2t}\sin 3t\, dt$

then $f^1\left(x\right) = e^{2x}\sin 3x$

9. When $m = -1$ then $\int x^m dx \neq \frac{x^{m+1}}{m+1}$

10. Required integral $= e^x \sin x + C$

11. The value of k is 0.

12. $\sin^5 x$ is an odd function

so $\int_{-\pi/2}^{\pi/2}\sin^5 x\, dx = 0$

13. $\int 2\sin\left(\alpha - \beta\right)x \sin\left(\alpha + \beta\right)x dx$

$= -\frac{\cos 2\beta x}{2\beta} + \frac{\cos 2\alpha x}{2\alpha} + C$

14. The above integral is 0 as $\cos^4 x \sin^{99} x$ is an odd function.

15. $\int_0^2 f(x)dx = \int_0^1 f(x)dx + \int_1^2 f(x)dx$

$= 2 + 1 = 3$

GROUP-B

Short type (Questions & Answers)

1. Show that $\int_0^1 \frac{\ln x}{\sqrt{1-x^2}}dx = -\frac{\pi}{2}\ln 2$.

2. If $f^1\left(x\right) = e^x + \frac{1}{1+x^2}$ and $f\left(0\right) = 1$

3. Evaluate $\int\left(\log x\right)^2 dx$

4. Evaluate $\int\frac{2x+9}{\left(x+3\right)^2}dx$

5. Evaluate $\int_0^1 \frac{x^5\left(4-x^2\right)}{\sqrt{1-x^2}}dx$

6. Evaluate $\int\frac{\sin x \cos x}{\sin^2 x - 2\sin x + 3}dx$

7. Evaluate $\int_0^1 x^7\sqrt{\frac{1+x^2}{1-x^2}}dx$

8. Evaluate $\int x^2 \tan^{-1} x - dx$

9. Evaluate $\int\frac{1}{\left(1-x\right)\sqrt{1-x^2}}dx$

10. Prove that $\int_0^{\pi/2}\frac{\sin^n x}{\sin^n x + \cos^n x}dx = \frac{\pi}{4}$.

11. Integrate $\int \sec x \tan x\sqrt{\tan^2 x - 3}\, dx$

12. Evaluate $\int\frac{x\cos^{-1} x}{\sqrt{1-x^2}}dx$

13. Evaluate $\int\frac{3x+1}{\left(x+1\right)^2\left(x+3\right)}dx$

14. Evaluate $\int\frac{\sin\left(x-\alpha\right)}{\sin\left(x+\alpha\right)}dx$

15. Evaluate $\int\frac{x^2}{\left(x^2+4\right)\left(x^2+a\right)}dx$

16. Evaluate $\int e^x\left(\frac{1-\sin 2x}{1-\cos 2x}\right)dx$

17. Evaluate $\int \frac{dx}{x\ln x\sqrt{(\ln x)^2-4}}$

18. Integrate $\int \frac{dx}{x^{1/2}+x^{1/3}}$

19. Integrate $\int \frac{xe^x}{(1+x)^2}dx$

20. Integrate $\int \frac{a}{b+Ce^x}dx$

21. Integrate $\int \frac{e^{x-1}+x^{e-1}}{e^x+x^e}dx$

22. Evaluate $\int \sin^4 x\cos^3 dx$

23. Evaluate $\int \frac{3\sin x+28\cos x}{5\sin x+6\cos x}dx$

24. Evaluate $\int_0^{\pi/2}\log\left(\frac{4+3\sin x}{4+3\cos x}\right)dx$

25. $\int \frac{\sec x\ \cos ecx}{\ln\tan x}dx=?$

Hints & Solutions

1. Let $I=\int_0^1 \frac{\ln x}{\sqrt{1-x^2}}dx$

Let $x=\sin\theta$

$\Rightarrow dx=\cos\theta\, d\theta$

When $x=0$, then $\theta=0$

When $x=1$ then $\theta=\frac{\pi}{2}$

$I=\int_0^{\pi/2}\frac{\ln\sin\theta}{\sqrt{1-\sin^2\theta}}\cos\theta\, d\theta$

$=\int_0^{\pi/2}\ln\sin\theta\, d\theta$ .... (1)

Also $I=\int_0^{\pi/2}\ln\sin\left(\frac{\pi}{2}-\theta\right)d\theta$

$=\int_0^{\pi/2}\ln\cos\theta\, d\theta$ .... (2)

Adding (1) & (2) we get

$2I=\int_0^{\pi/2}\ln\sin\theta\ d\theta+\int_0^{\pi/2}\ln\cos\theta\ d0$

$=\int_0^{\pi/2}[\ln\sin\theta\cos\theta]d\theta$

$=\int_0^{\pi/2}\ln\frac{\sin 2\theta}{2}d\theta$

$=\int_0^{\pi/2}\ln\sin 2\theta\, d\theta-\frac{\pi}{2}\ln 2$ ....(3)

Let $2\theta=t\ \Rightarrow 2d\theta=dt$

$\Rightarrow d\theta=\frac{1}{2}dt$

When $\theta=0,\ t=0$

When $\theta=\frac{\pi}{2},\ t=\pi$

$\therefore \int_0^{\pi/2}\ln\sin 2\theta\, d\theta=\int_0^{\pi}\ln\sin t.\frac{1}{2}dt$

$=\frac{1}{2}\int_0^{\pi}\ln\sin t\ dt$

$=\frac{1}{2}\left[\int_0^{\pi/2}\ln\sin t\, dt+\int_0^{\pi/2}\ln\sin(\pi-t)\, dx\right]$

$=\frac{1}{2}\left[2.\int_0^{\pi/2}\ln\sin t\, dt\right.$

$=\int_0^{\pi/2}\ln\sin t\ dt=I$

From (3), we get

$2I=I-\frac{\pi}{2}\ln 2$

$\Rightarrow I=-\frac{\pi}{2}\ln 2$

2. Given that $f^1(x) = e^x + \frac{1}{1+x^2}$

Integrating both sids, we get

$$\int f^1(x)\,dx = \int e^x dx + \int \frac{1}{1+x^2}dx$$

$\Rightarrow f(x) = e^x + \tan^{-1}x + C$ ....(1)

Putting $x = 0,$ we get

$f(0) = e^0 + \tan^{-1}0 + C$

$1 = 1 + 0 + C \quad \Rightarrow C = 0$

From (1), we have $f(x) = e^x + \tan^{-1}x$

5. $\int_0^1 \frac{x^5(4-x^2)}{\sqrt{1-x^2}}dx$

$= \int \frac{x^5(3+1-x^2)}{\sqrt{1-x^2}}dx$

$= 3\int_0^1 \frac{x^5}{\sqrt{1-x^2}}dx + \int_0^1 x^5\sqrt{1-x^2}\,dx$

Let $x = \sin\theta$

$dx = \cos\theta\, d\theta$

$I = 3\int_0^{\pi/2} \frac{\sin^5\theta}{\sqrt{1-\sin^2\theta}}\cos\theta\; d\theta$

$+\int_0^{\pi/2} \sin^5\theta\sqrt{1-\sin^2\theta}\cos\theta\; d\theta$

$= 3\int_0^{\pi/2}\sin^5\theta\, d\theta + \int_0^{\pi/2}\sin^5\theta.\cos^2\theta\, d\theta$

$= \frac{64}{105}$

6. $\int \frac{\sin\theta\ \cos\theta}{\sin^2\theta - 2\sin\theta + 3}d\theta$

Let $\sin\theta = x$

$\cos\theta\, d\theta = dx$

$I = \int \frac{x\,dx}{x^2 - 2x + 3}$

$= \frac{1}{2}\int \frac{(2x-2)+2}{x^2-2x+y}dx$

$= \frac{1}{2}\int \frac{2x-2}{x^2-2x+3}dx + \int \frac{1}{x^2-2x+3}dx$

$= \frac{1}{2}\ln(x^2-2x+3) + \int \frac{1}{(x-1)^2+2}dx$

$= \frac{1}{2}\ln(x^2-2x+3) + \frac{1}{\sqrt{2}}\tan^{-1}\frac{x-1}{\sqrt{2}} + C$

$x = \sin\theta$

7. $\int_0^1 x^7\sqrt{\frac{1+x^2}{1-x^2}}dx$

$= \int_0^1 x^7\sqrt{\frac{(1+x^2)^2}{(1-x^2)(1+x^2)}}dx$

$= \int_0^1 x^7\frac{1+x^2}{\sqrt{1-x^4}}dx$

$= \int_0^1 \frac{x^7}{\sqrt{1-x^y}}dx + \int_0^1 \frac{x^9}{1-x^4}dx = I_1 + I_2$

$I_1 = \int_0^1 \frac{x^7}{\sqrt{1-x^4}}dx$

$= \int_0^1 \frac{x^4.x^3dx}{\sqrt{1-x^4}} \left[x^4 = t\, 4x^3dx = dt\, x^3dx = \frac{1}{4}dt\right]$

$= \frac{1}{4}\int_0^1 \frac{tdt}{\sqrt{1-t}}$

$[1-t = z^2 - dt \quad dt = -2zdz \quad t = 0$,

$z = 1, t = 1, z = 0]$

$= \frac{1}{4}\int_1^0 \frac{(1-z^2).(-2zdz)}{z}$

$= \frac{-1}{2}\int_1^0 (1-z^2)dz$

$= \frac{1}{2}\int_0^1 (1-z^2)dz$

$$= \frac{1}{2}\int_0^1 \left(1-\frac{1}{3}\right) = \frac{1}{3}$$

$$I_2 = \int_0^1 \frac{x^9}{\sqrt{1-x^4}}dx$$

$$= \int_0^1 \frac{x^8 .xdx}{\sqrt{1-\left(x^2\right)^2}}$$

$$= \int \frac{t^4 \frac{1}{2}dt}{\sqrt{1-t^2}}$$

$[x^2 = t \quad 2xdx = dt \quad xdx = \frac{1}{2}dt$ when $x = 0, t = 0$, $x = 1, t = 1]$

$$= \frac{3\pi}{32}$$

$$I = I_1 + I_2 = \frac{1}{3} + \frac{3\pi}{32}$$

GROUP-C

Long Type (Questions & Answers)

Integrate the following

1. $\int \frac{1}{\cos x\left(1+2\sin x\right)}dx$

2. Evaluate $\int_0^\pi \frac{x}{1+\sin x}dx$

3. $\int \frac{12\sin x - 2\cos x + 3}{\sin x + \cos x}dx$

4. $\int \frac{1+x^2}{x\sqrt{x^4+1}}dx$

5. $\int \left(\sqrt{\tan x} + \sqrt{\cot x}\right)dx$

6. $\int \frac{\sin x}{\sin 4x}dx$

7. $\int x^2\left(\sin^4 x + \cos^4 x\right)dx$

8. $\int \frac{2\sin x + 3\cos x}{3\sin x + 4\cos x}dx$

9. $\int \frac{1}{\left(x-2\right)\sqrt{3x^2 - 16x + 24}}dx$

10. $\int \frac{\cos x}{\sin 2x + \cos x}dx$

11. $\int \frac{1}{2\sin x + \cos x + 3}dx$

12. Show that $\int_0^\pi \frac{x\sin x}{1+\cos^2 x}dx = \frac{\pi^2}{4}$

13. Show that $\int_0^1 \frac{\log\left(1+x\right)}{1+x^2}dx = \frac{\pi}{8}\log 2$

14. Evaluate $\int_0^\pi \frac{x}{1+\sin x}dx$.

15. Evaluate $\int_0^{\pi/2} \frac{\cos x}{\left(1+\sin x\right)\left(3+\sin x\right)}dx$

16. Evaluate $\int_2^7 \frac{dx}{\sqrt{x+2}+\sqrt{x-3}}$

18. Evaluate $\int_0^{\pi/2} \frac{\sqrt{\tan x}}{\sqrt{\tan x}+\sqrt{\cot x}}dx$

19. Prove that

$$\int_0^{\pi/2} \frac{dx}{a^2\cos^2 x + b^2\sin^2 x} = \frac{\pi}{2ab}$$

$a > 0, b > 0$

20. Evaluate $\int_0^{\pi/2} x\tan^{-1} x\,dx$

21. Evaluate $\int \frac{2\cos x + 7}{4 - \sin x}dx$

22. Evaluate $\int \frac{1}{\cos\left(1+2\sin x\right)}dx$

Hints & Solutions

1. $I = \int \frac{1}{\cos x(1+2\sin x)} dx$

$= \int \frac{\cos x}{\cos^2 x(1+2\sin x)} dx$

$= \int \frac{\cos x}{(1-\sin x)(1+\sin x)(1+2\sin x)} dx$

$= \int \frac{dt}{(1-t)(1+t)(1+2t)}$

$\sin x = t$

$\cos x\, dx = dt]$

Let $\frac{1}{(1-t)(1+t)(1+2t)} = \frac{A}{1-t} + \frac{B}{1+t} + \frac{C}{1+2t}$

$\therefore A = \frac{1}{6}, B = \frac{-1}{2}, C = \frac{4}{3}$

$\therefore \frac{1}{(1-t)(1+t)(1+2t)} = \frac{1}{6} . \frac{1}{1-t} - \frac{1}{2}$

$\frac{1}{1+t} + \frac{4}{3} . \frac{1}{1+2t}$

$I = \frac{1}{6}\int \frac{1}{1-t} dt - \frac{1}{2}\int \frac{1}{1+t} dt + \frac{4}{3} \int \frac{1}{1+2t} dt$

$= -\frac{1}{6} \ln(1-t) - \frac{1}{2} \ln(1+t) + \frac{2}{3} \ln(1+2t) + C$

$= -\frac{1}{6} \ln(1-\sin x) - \frac{1}{2} \ln(1+\sin x) + \frac{2}{3} \ln(1+2\sin x) + C$

2. $I = \int_0^{\pi} \frac{x}{1+\sin x} dx$

$= \int_0^{\pi/2} \frac{x}{1+\sin x} dx + \int_0^{\pi/2} \frac{(\pi - x)}{1+\sin(\pi - x)} dx$

$= \int_0^{\pi/2} \frac{x}{1+\sin x} dx + \pi \int_0^{\pi/2} \frac{1}{1+\sin x} dx$

$- \int_0^{\pi/2} \frac{x}{1+\sin x} dx$

$= \pi \int_0^{\pi/2} \frac{1}{1+\sin x} dx$

$= \pi \int_0^{\pi/2} \frac{1}{1 + \frac{2\tan\frac{x}{2}}{1+\tan^2\frac{x}{2}}} dx$

$= \pi \int_0^{\pi/2} \frac{\sec^2\frac{\pi}{2}}{\left(1+\tan\frac{x}{2}\right)^2} dx \qquad 1+\tan\frac{x}{2} = t$

$= \pi \int_0^2 2.\frac{dt}{t^2}$

$= 2\pi \left[-\frac{1}{t} \right)_1^2 = \pi$

3. Let $I = \int \frac{12\sin x - 2\cos x + 3}{\sin x + \cos x} dx$

Let $12\sin x - 2\cos x + 3 = l(\sin x + \cos x)$

$+m(\cos x - \sin x) + n$

Equating the coefficients of similar terms we get

$l - m = 12$ .... (1)

$l + m = -2$ .... (2)

$n = 3$

Solving (1) & (2), we get $l + 5, m = -7, n = 3$

$\therefore I = \int \frac{5(\sin x + \cos x) - 7(\cos x - \sin x) + 3}{\sin x + \cos x} dx$

$= 5x - 7\ln(\sin x + \cos x) + \frac{3}{\sqrt{2}}$

$\int \frac{1}{\frac{1}{\sqrt{2}}\sin x + \frac{1}{\sqrt{2}}\cos x}$

$= 5x - 7\ln(\sin x + \cos x) + \frac{3}{\sqrt{2}}$

$$\int \frac{1}{\cos x\cos\frac{\pi}{4}+\sin x\sin\frac{\pi}{4}}dx$$

$$= 5x-7\ln\left(\sin x+\cos x\right)+\frac{3}{\sqrt{2}}\int\sec\left(x-\frac{\pi}{4}\right)dx$$

$$= 5x-7\ln\left(\sin x+\cos x\right)+\frac{3}{\sqrt{2}}$$

$$\left[\ln\left\{\sec\left(x-\frac{\pi}{4}\right)+\tan\left(x-\frac{\pi}{4}\right)\right]+C$$

4. $I=\int\frac{1+x^2}{x\sqrt{x^4+1}}dx$

$$=\int\frac{1+x^2}{x\sqrt{x^2\left(x^2+\frac{1}{x^2}\right)}}dx$$

$$=\int\frac{1+x^2}{x^2\sqrt{x^2+\frac{1}{x^2}}}dx$$

$$=\int\frac{1+\frac{1}{x^2}}{\sqrt{\left(x-\frac{1}{x}\right)^2+2}}dx$$

$$x-\frac{1}{x}=t$$

$$\left(1+\frac{1}{x^2}\right)dx=dt$$

$$=\int\frac{dt}{\sqrt{t^2+\left(\sqrt{2}\right)^2}}dx$$

$$=\ln\left[t+\sqrt{t^2+2}\right]+C$$

$$=\ln\left[x-\frac{1}{x}+\sqrt{\left(x-\frac{1}{x}\right)^2+2}\right]+C$$

5. $\int\left(\sqrt{\tan x}+\sqrt{\cot x}\right)dx$

$$=\int\left(\frac{\sqrt{\sin x}}{\sqrt{\cos x}}+\frac{\sqrt{\cos x}}{\sqrt{\sin x}}\right)dx$$

$$=\int\frac{\sin x+\cos x}{\sqrt{2\sin x+\cos x}}dx$$

$$=\sqrt{2}\int\frac{\sin x+\cos x}{\sqrt{1-1+2\sin x\cos x}}dx$$

$$=\sqrt{2}\int\frac{\sin x+\cos x}{\sqrt{1-\left(\sin x-\cos x\right)^2}}dx$$

$$=\sqrt{2}\int\frac{dt}{\sqrt{1-t^2}}dx$$

$$=\sqrt{2}\sin^{-1}t+C=\sqrt{2}\sin^{-1}\left(\sin x-\cos x\right)+C$$

6. $\int\frac{\sin x}{\sin 4x}dx$

$$=\int\frac{\sin x}{2\sin 2x\cos 2x}dx$$

$$=\frac{1}{2}\int\frac{\sin x}{2\sin x\cos x.\cos 2x}$$

$$=\frac{1}{4}\int\frac{\cos x}{\cos^2 x\cos 2x}dx$$

$$=\frac{1}{4}\int\frac{\cos x}{\left(1-\sin^2 x\right)\left(1-2\sin^2 x\right)}dx$$

$$[\sin x=t$$
$$\cos x\,dx=dt]$$

$$=\frac{1}{4}\int\frac{dt}{\left(1-t^2\right)\left(1-2t^2\right)}$$

$$=\frac{1}{4}\int\left(\frac{2}{1-2t^2}-\frac{1}{1-t^2}\right)dt$$

$$=\frac{1}{2}\int\frac{1}{1-2t^2}dt-\frac{1}{4}\int\frac{1}{1-t^2}dt$$

$$=\frac{1}{2}.\frac{1}{2\sqrt{2}}\ln\left(\frac{1+\sqrt{2}t}{1-\sqrt{2}t}\right)-\frac{1}{8}\ln\left(\frac{1+t}{1-t}\right)+C$$

$$= \frac{1}{4\sqrt{2}} \ln\left(\frac{1+\sqrt{2}\sin x}{1-\sqrt{2}\sin x}\right) - \frac{1}{8}\ln\left(\frac{1+\sin x}{1-\sin x}\right) + C$$

7. $I = \int x^2\left(\sin^4 x + \cos^4 x\right)dx$

$$= \int x^2\left[\left(\sin^2 x + \cos^2 x\right)^2 - 2\sin^2 x\cos^2 x\right]dx$$

$$= \int x^2\left[1 - \frac{1}{2}\sin^2 2x\right]dx$$

$$= \int x^2\left[1 - \frac{1}{4}.2\sin^2 2x\right]dx$$

$$= \int x^2\left[1 - \frac{1}{4}(1-\cos 4x)\right]dx$$

$$= \int x^2\left[\frac{3}{4} + \frac{1}{4}\cos 4x\right]dx$$

$$= \frac{3}{4}\int x^2 dx + \frac{1}{4}| x^2 \cos 4x\, dx$$

$$= \frac{3}{4}.\frac{x^3}{3} + \frac{1}{4}\left[x^2\frac{\sin 4x}{4} - \int 2x.\frac{\sin 4x}{4}dx\right]$$

$$= \frac{x^3}{4} + \frac{1}{16}x^2\sin 4x - \frac{1}{8}\int x\sin 4x\, dx$$

$$= \frac{x^3}{4} + \frac{1}{16}x^2\sin 4x + \frac{1}{32}x\cos 4x - \frac{1}{128}$$

$\sin 4x + C$

8. Let $I\int\frac{2\sin x + 3\cos x}{3\sin x + 4\cos x}dx$

Let $2\sin x + 3\cos x = l\left(3\sin x + 4\cos x\right)$

$+m\left(3\cos x - 4\sin x\right)$

Equating the coefficients, we get

$3l - 4m = 2$
$4l + 3m = 3$

Solving, $l = \frac{18}{25},\ m = -\frac{1}{25}$

$$\therefore 2\sin x + 3\cos x = \frac{28}{25}\left(3\sin x + 4\cos x\right)$$

$$-\frac{1}{25}\left(3\cos x - 4\sin x\right)$$

$$I = \int\frac{\frac{18}{25}\left(2\sin x + 4\cos x\right) - \frac{1}{25}\left(3\cos x - 4\sin x\right)}{3\sin x + 4\cos x}$$

$$= \frac{18}{25}\int dx - \frac{1}{25}\ln\int\frac{3\cos x - 4\sin x}{2\sin x + 4\cos x}dx$$

$$= \frac{18}{25}x - \frac{1}{25}\ln\left(3\sin x + 4\cos x\right) + C$$

9. $\int\frac{1}{(x-2)\sqrt{3x^2-16x+24}}dx$

Let $x - 2 = \frac{1}{t}$

$$dx = -\frac{1}{t^2}dt$$

$$\therefore 3x^2 - 16x + 24 = \frac{4t^2 - 4t + 3}{t^2}$$

$$\sqrt{3x^2 - 16x + 24} = \frac{\sqrt{4t^2 - 4t + 3}}{t}$$

$$I = \int\frac{-\frac{1}{t^2}dt}{\frac{1}{t}\frac{\sqrt{4t^2-4t+3}}{t}}$$

$$= -\frac{1}{2}\int\frac{1}{\sqrt{\left(1-\frac{1}{2}\right)^2 + \left(\frac{\sqrt{2}}{2}\right)^2}}dt$$

$$= \frac{1}{2}\left[\left(t - \frac{1}{2}\right) + \sqrt{\left(t-\frac{1}{2}\right)^2 + \left(\frac{\sqrt{2}}{2}\right)^2}\right] + C$$

$$= -\frac{1}{2}\left[t - \frac{1}{2} + \sqrt{t^2 - t + \frac{3}{4}}\right] + C$$

Note : Students should work out the other problems.

CHAPTER - 10

APPLICATION OF INTEGRALS

Multiple Choice Questions (MCQ)

Group-A

A. Choose the correct answer from the given choices:

1. The area of the trapezium bounded by the sides $y = x, x = 0, y = 3$ and $y = 4$ is

(a) $\frac{5}{2} sq\,unit$ (b) $\frac{7}{2} sq\,unit$

(c) $\frac{9}{2} sq\,unit$ (d) $\frac{11}{2} sq\,unit$

2. The area enclosed by the curve $y^2 = x$ and straight live $x = 0, y = 1$ is_______ sq. unit.

(a) $\frac{1}{7}$ (b) $\frac{1}{5}$

(c) $\frac{1}{3}$ (d) $\frac{1}{2}$

3. The area bounded by $y = \sin x, y = 0$ and $x = 0, x = \frac{\pi}{2}$ is________sq. unit

(a) 1 (b) 2

(c) 3 (d) 4

4. The area of the circle $x^2 + y^2 = 2ax$ is ________ sq. unit.

(a) $4\pi a^2$ (b) $3\pi a^2$

(c) $2\pi a^2$ (d) πa^2

5. The area bounded by the line $y = 2x, x - axis$ and the ordinate $x = 3$ is________sq. unit.

(a) 9 (b) 8

(c) 7 (d) 6

6. The area bounded by the curve $y = 2x, x - axis$ and the ordinate $x = 3$ is __________sq. unit.

(a) 1 (b) 2

(c) 3 (d) 4

7. The area of the region bounded by $y = \sin x, x = \frac{\pi}{2}$ and $y = 0$ is_______ sq. unit

(a) 4 (b) 3

(c) 2 (d) 1

8. The area bounded by the curve $y = 3x^2 + 5, y = 0$ and two ordinates $x = 1$ and $x = 2$ is_________sq unit

(a) 4 (b) 8

(c) 12 (d) 16

9. The area bounded by $y = e^x, y = 0$, $x = 2$ and $x = 4$ is______sq. unit.

(a) $e^4 - e^2$ (b) $e^4 + e^2$

(c) $e^3 + e$ (d) $e^3 - e$

10. The area of region enclosed by $y^2 = 4ax$ and $x^2 = 4ay$ is _________ sq unit

(a) $\frac{10}{3}$ (b) $\frac{13}{3}$

(c) $\frac{16}{3}$ (d) $\frac{20}{3}$

Answers

1. (b)	2. (c)	3. (a)	4. (d)
5. (a)	6. (b)	7. (d)	8. (c)
9. (a)	10. (c)		

B. Fill in the blanks

1. The area of smaller portion of the circle $x^2 + y^2 = 4$ cut off by the line $x = 1$ is __________ sq. unit.

2. The area included between the line $y = x$ and the parabola $x^2 = 4y$ is _______ sq. unit.

3. The area bounded by the curve $x = 2 - y - y^2$ and $y - axis$ is ________ sq. unit.

4. The area enclosed by two curves given by $y^2 = x + 1$ and $y^2 = -x + 1$ is________ sq. unit.

5. The area of the region bounded by the curves $y^2 = 4a^2(x-1)$ and the lines $x = 1$ and $y = 4a$ is _______ sq. unit.

6. The area bounded by the parabola $y^2 = 8x$ and its latus rectum is _____ sq. unit.

7. The area bounded by the curves $y^2 = 8x$ and $x^2 = 8y$ is _________

8. The area enclosed between the curves $y = x^3$ and $y = \sqrt{x}$ is _______sq. unit.

9. The area bounded by the curve $x = 4 - y^2$ and $y - axis$ is _______sq. unit.

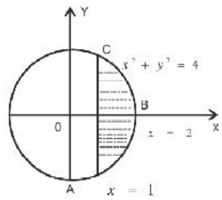
Hints & Answers

1. $\frac{4\pi - 3\sqrt{3}}{3}$

Hints : Required area

= Area of ABC

$= 2\int_1^2 y\,dx$

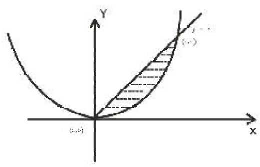


$= 2\int_1^2 \sqrt{4 - x^2}\,dx$

$= \left[\frac{x\sqrt{4-x^2}}{2} + \frac{4}{2}\sin^{-1}\frac{x}{2}\right]_1^2$

$= \frac{4\pi - 3\sqrt{3}}{3}$ sq unit.

2. $\frac{8}{3}$ sq. unit



Hints:

The given parabola is

$x^2 = 4y$ .....(1)

The given line $y = x$ ......(2)

Required area $= \int_0^4 \left(x - \frac{x^2}{4}\right)dx$

$= \left[\frac{x^2}{2} - \frac{x^3}{12}\right]_0^4$

$= \frac{8}{3}$ sq. unit.

3. $\frac{9}{2}$ sq. unit

Hints: on $y-axis, x=0$

$$\Rightarrow 2-y-y^2=0$$
$$\Rightarrow y^2+y-2=0$$
$$\Rightarrow y^2+2y-y-2=0$$
$$\Rightarrow (y-1)(y+2)=0$$
$$\Rightarrow y=1,-2$$

Required area $=\int_{-2}^{1} x\,dy$

$$=\int_{-2}^{1}\left(2-y-y^2\right)dy$$

$$=\frac{9}{2}\text{ sq. unit.}$$

4. $\frac{8}{3}$ sq. units

Hitns: Given curves are

$$y^2=x+1 \quad ....(1)$$
$$y^2=-x+1 \quad ....(2)$$

Curve (1) a parabola having axis $y=0$ and vertex $(-1,0)$.

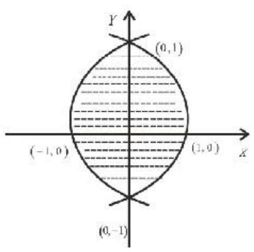
Curve (2) is a parabola having axis $y=0$ and vertex $(1,0)$

Required area

$$=\int_{-1}^{1}\left[\left(1-y^2\right)-\left(y^2-1\right)\right]dy$$

$$=\int_{-1}^{1}\left(2-2y^2\right)dy$$

$$=2\int_{-1}^{1}\left(1-y^2\right)dy$$



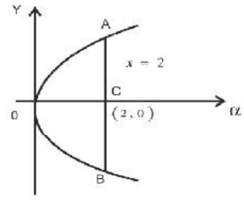
$$=2\left[y-\frac{y^3}{3}\right]_{-1}^{1}$$

$$=\frac{8}{3}\text{ sq. unit.}$$

5. $\frac{169}{3}$ sq. unit.

6. $\frac{32}{3}$ sq. unit.

Hints :



Required area= Area of AOB

=2area of AOC

$$=2\int_0^2\sqrt{8x}\,dx$$

$$=2.2\sqrt{2}\int_0^2 x^{\frac{1}{2}}dx$$

$$=4\sqrt{2}\left[\frac{x^{3/2}}{\frac{3}{2}}\right]_0^2$$

$$=\frac{8\sqrt{2}}{3}\times 2\sqrt{2}=\frac{32}{3}\text{ sq. unit.}$$

7. $\frac{64}{3}$ sq. unit

Hints: Two curves are $y^2=8x$ ....(1)

$$x^2=8y \quad ....(2)$$

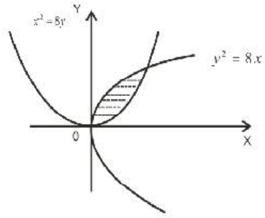
From (1) we get $y=2\sqrt{2}\sqrt{x}$ ....(3)

From (2), we get $y=\frac{x^2}{8}$ ....(4)

Solving (1) & (2) we get $x=0$, $x=8$

Required area

$$=\int_0^8\left(\sqrt{8x}-\frac{x^2}{8}\right)dx$$



$$=\frac{64}{3}\text{ sq. unit.}$$

8. $\frac{5}{12}$ sq. unit.

Hints : The points of intersection are $(0,0)$ and $(1,1)$

Required area $= \int_0^1 \left(\sqrt{x} - x^3\right) dx$

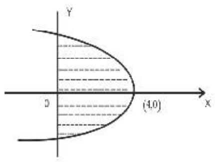
$= \frac{5}{12}$ sq. unit.

9. $\frac{32}{3}$ sq. units.

Hints: The given curve is $x = 4 - y^2$

$\Rightarrow y^2 = 4 - x$

$\Rightarrow y = \sqrt{4 - x}$



Required area $= 2\int_0^4 \sqrt{4 - x} dx$

$= \frac{32}{3}$ sq. units.

C. Answer in one sentence:

1. What is the area bounded by the curve $y = f(x), x - axis$ and two ordinates $x = a$ and $x = b$?

2. What is the area bounded by the curve $x = f(y), y - axis$ and two abscissa $x = a$ and $x = b$?

3. What is the area between two curves $y = f_1(x)$ and $y = f_2(x)$ and two odinates $x = a$ and $x = b$?

4. What is the area of an ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$?

5. What is the area of the circle radius a ?

6. What is the area bounded by the parabola $y^2 = 4ax$ and the latus rectum?

7. What is the area of the ellipse $\frac{x^2}{25} + \frac{y^2}{16} = 1$?

8. What is the area of the portion of the parabola $y^2 = 4x$ bounded by the double ordinate through $(3,0)$?

9. What is the area of the region between the curve $y = \cos x$ and $y = \sin x$ and bounded by $x = 0$

Answers

1. $\int_a^b f(x)dx$

2. $\int_a^b f(y)dy$

3. $\int_a^b \left[f_1(x) - f_2(x)\right] dx$

4. πab

5. πa^2

6. $\frac{8}{3}a^2$

7. 20π

8. $8\sqrt{3}$ sq. unit

9. $\left(\sqrt{2} - 1\right)$ sq. unit

GROUP-B

Short questions (Answers & Solutions)

1. Find the area enclosed by the parabola $y^2 = 4x$ and the line $y = 2x$.
2. Find the area bounded by the curve $y = x, x-axis$ and $x = -2$ and $x = 2$
3. Find the area enclosed by $y = 4x - 1$ and $y^2 = 2x$
4. Find the area of the region in the first quadrant enclosed by the $x-axis$, the line $y = x$ and the circle $x^2 + y^2 = 32$
5. Find the area of the region between the curves $y = \cos x$ and $y = \sin x, x \in \left[0, \frac{\pi}{4}\right]$
6. Find the area bounded by $y = e^x, y = 0, x = 2$ and $x = 4$.
7. Find the area bounded by $y = x^2, y = 0$ and $x = 1$
8. Find the area bounded by $y = \sin x, y = 0, x = \alpha, x = \beta, (\beta > \alpha > 0)$
9. Find the area bounded by $y = \sin x, y = 0, x = \frac{\pi}{2}$.
10. Determine the area within the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

Answers

1. Given that $y^2 = 4x$ .... (1)

$y = 2x$ ....(2)

From (1) & (2), we have

$$4x^2 = 4x$$

$$\Rightarrow 4x(x-1) = 0$$

$$\Rightarrow x = 0$$

When $x = 0, y = 0$

When $x = 1, y = 2$

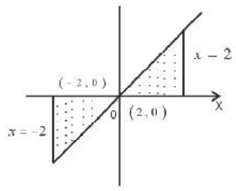
Area enclosed by the parabola and the line

$$= \int_0^1 \left(\sqrt{4x} - 2x\right) dx$$

$$= \int_0^1 2\sqrt{x}dx - \int_0^1 2xdx = \frac{1}{3}$$

2. Required area

$$= \int_0^2 y\,dx + \left|\int_{-2}^0 ydx\right|$$

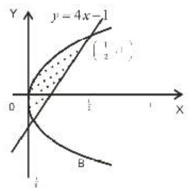


$$= \int_0^2 x\,dx + \left|\int_{-2}^0 x\,dx\right|$$

$$= \left[\frac{x^2}{2}\right]_0^2 + \left|\left[\frac{x^2}{2}\right]_{-2}^0\right|$$

$= 2 + 2 = 4$ sq unit.

3. Two curves are



$y^2 = 2x$ ....(1)

$y = 4x - 1$ ....(2)

Their points of intersection are $\left(\frac{1}{8}, \frac{1}{2}\right)$ and $\left(\frac{1}{2}, 1\right)$

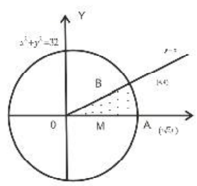
Required area

$$= \int_{1/8}^{1/2} \sqrt{2x}dx - \int_{1/8}^{1/2} (4x-1)\,dx$$

$$= \sqrt{2}\int_{\frac{1}{8}}^{\frac{1}{2}} \sqrt{x}dx - 4\int_{\frac{1}{8}}^{\frac{1}{2}} x\,dx + \int_{\frac{1}{8}}^{\frac{1}{2}} dx$$

$= \frac{1}{4}$ sq. units

4. The equations of the curves are



$y = x$ .... (1)

$x^2 + y^2 = 32$ .....(2)

The line (1) intersect the circle(2)

at $B(4,4)$ and the circle intersect $x-axis$ at $A(4\sqrt{2},0)$.

Let as draw BM perpendicular to $x-axis$.

Required area = area of the region $OBAO$

$=$ Area of the region $OBMO$

$+$ Area of the region $BMAB$

Area of the region $OBMO$

$$=\int_0^y ydx=\int_0^4 x\,dx=\frac{1}{2}\left[x^2\right]_0^4=8$$

Area of the rigion $BMAB$

$$=\int_4^{4\sqrt{2}} ydx=\int_4^{4\sqrt{2}}\sqrt{32-x^2\,dx}$$

$$=\int_4^{4\sqrt{2}}\sqrt{\left(4\sqrt{2}\right)^2-x^2\,dx}$$

$$=\left[\frac{1}{2}x\sqrt{32-x^2}+\frac{1}{2}x32\sin^{-1}\frac{x}{4\sqrt{2}}\right]_4^{4\sqrt{2}}$$

$=4\pi-8$

Required area $=(8+4\pi-8)$ sq. unit

$=4\pi$ sq. unit

5. $\sqrt{2}-1$ sq. unit

6. e^4-e^2 sq. unit

7. $\frac{1}{3}$ sq. unit

8. $a^2\ln\left(\frac{\beta}{\alpha}\right)$ sq. unit

9. 1 sq. unit

10. $\pi ab.$ sq. unit

GROUP-C

Long Questions

1. Find the area of the region bounded by the curve $y^4=x^3$ and the double ordinate through $(2,0)$

2. Determine the area included between the parabola $y^2=x$ and the circle $x^2+y^2=2x$.

3. Find the area of the portion of the ellipse $\frac{x^2}{12}+\frac{y^2}{16}=1$ bounded by the major axis and the double ordinate $x=3$

4. Find the area of the ellipse $\frac{x^2}{9}+\frac{y^2}{16}=1$

5. Show that the area of the smaller region bounded by the ellipse $\frac{x^2}{9}+\frac{y^2}{4}=1$ and the line $\frac{x}{3}+\frac{y}{2}=1$ is $\frac{3}{2}(\pi-2)$ sq. unit.

6. Find the area of the portion of the parabola $y^2=4x$ bounded by the double ordinate through $(3,0)$.

Hints & Solutions

1. The given curve is

$$y^2=x^3$$

$$\Rightarrow y=x^{\frac{3}{4}}\quad ....(1)$$

The area is bounded the curve (1) and the double ordinate through (2,0).

Required area $=\int_0^2 y\,dx$

$$= \int_0^2 x^{3/4} dx = \left[\frac{x^{\frac{7}{4}}}{\frac{7}{4}} \right]_0^2$$

$$= \frac{8}{7}\sqrt{8} \text{ sq. unit.}$$

2. The equation of the

parabola is $y^2 = x$ .....(1)

The equation of the circle is

$$x^2 + y^2 = 2x \quad ....(2)$$

Solving (1) and (2), we get

$x^2 + x = 2x$

$\Rightarrow x^2 - x = 0$

$\Rightarrow x(x-1) = 0$

Then the point of intersection of the parabola (1) and the circle (2) are (0,0), (1,1)and (1,-1)

Requirtedare $= 2 \times Area$ of OAB

$$= 2\left[\int_0^1 \sqrt{2x - x^2}\, dx - \int_0^1 \sqrt{x}\, dx \right]$$

$$= 2\left[\int_0^1 \sqrt{1 - (1-x)^2}\, dx - \frac{2}{3} \right]$$

$$= \frac{\pi}{4} + \frac{\pi}{3} \text{ sq. unit}$$

3. $\frac{2}{\sqrt{3}} = \left[\frac{3\sqrt{3}}{2} + 5\pi \right]$ sq. unit

4. 12π sq. unit

5. The equation of the ellipse is

$$\frac{x^2}{9} + \frac{y^2}{4} = 1$$

$$\Rightarrow \frac{x^2}{3^2} + \frac{y^2}{2^2} = 1 \quad ......(1)$$

Equation of the line is

$$\frac{x}{3} + \frac{y}{2} = 1 \quad ....(2)$$

The line (2) intersect the ellipse (1) at $A(3,0)$ and $B(0,2)$

Required area $=$ Area of $O\ BCAO$

- Area of O AB

$$= \int_0^3 2\sqrt{1 - \frac{x^2}{9}}\, dx - \int_0^3 2\left(1 - \frac{x}{3}\right) dx$$

$$= \frac{2}{3}\int_0^3 \sqrt{3^2 - x^2}\, dx - \frac{2}{3}\int_0^3 (3-x)\, dx$$

$$= \frac{2}{3}\left[\frac{x}{2}\sqrt{3^2 - x^2} + \frac{9}{2}\sin^{-1}\frac{x}{3} \right]_0^3 - \frac{2}{3}\left[3x - \frac{x^2}{2} \right]_0^3$$

$$= \frac{3}{2}(\pi - 2) \text{ sq. unit}$$

6. $8\sqrt{3}$ sq. unit.

CHAPTER - 11

DIFFERENTIAL EQUATION

Multiple Choice Questions (MCQ)

Group-A

A. Choose the correct answer from the given choices:

1. Order and degree of the differential equation $\frac{d^2y}{dx^2} = \frac{2y^3 + \left(\frac{dy}{dx}\right)^2}{\sqrt{\frac{d^2y}{dx^2}}}$ are

   (a) 1st order, 2nd degree

   (b) 2nd order, 3rd degree

   (c) 2nd order & 2nd degree

   (d) 3rd order, 3rd degree

2. Order and degree of the differential equation $\ln\left(\frac{d^2y}{dx^2}\right) = y$ are

   (a) 2 & 1 (b) 3 & 4

   (c) 3 & 5 (d) 2 & 2

3. The order and degree of the differential equation $\frac{d^2y}{dx^2} = \frac{3y + \frac{dy}{dx}}{\sqrt{\frac{d^2y}{dx^2}}}$ are

   (a) 3 & 3 (b) 2 & 3

   (c) 4 & 1 (d) 4 & 3

4. If $\frac{d^2s}{dt^2} = 0$ then s is a _______ function of t.

   (a) constant (b) linear

   (c) quadratics (d) cubic

5. What is the solution of $\frac{dy}{dx} = 8x$ given $y = 2$ when $x = 1$?

   (a) $y = 4x^2$ (b) $y = 4x^2 - 2$

   (c) $y = 4x^2 + 5$ (d) none of these

6. Write the order and degree of the differential equation $x\frac{d^2y}{dx^2} = \left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}$

   (a) 1,2 (b) 2,2

   (c) 2,3 (d) 3,3

7. What is the solution of the differential equation $\frac{dy}{dx} = y + 2$?

   (a) $y + 2 = Ce^x$

   (b) $\frac{1}{2}(y+2) = Ce^{2x}$

   (c) $y + 3 = Ce^x$

   (d) $y + 4 = Ce^x$

8. What is the general solution of $\frac{dy}{dx} = (x^2+1)\ (y^2+1)$?

   (a) $\tan^{-1} y = \frac{1}{2}x^2 + x + C$

   (b) $\tan^{-1}\left(\frac{4}{2}\right) = \frac{1}{4}x^4 + x + C$

   (c) $\tan^{-1} y = \frac{1}{3}x^3 + x + C$

   (d) none of these

9. Write the order and degree of the differential equation $\frac{dy}{dx}+3y\frac{d^2y}{dx^2}=0$

(a) 1st order, 1st degree

(b) 2nd order, 1st degree

(c) 2nd order, 2nd degree

(d) none of these

10. Write the degree of the differential equation

$$x^3\left(\frac{d^2y}{dx^2}\right)^2+x\left(\frac{dy}{dx}\right)^4=0$$

(a) 1st degree (b) 2nd degree

(c) 3rd degree (d) 4th degree

11. What is the differential equation representing the family of curves $y=mx$ whese m is an arbitrary constant?

(a) $y=\frac{dy}{dx}$ (b) $xy=\frac{dy}{dx}$

(c) $y=x\frac{dy}{dx}$ (d) none of these

12. What is the solution of the differential equation

$$\left(\frac{dy}{dx}\right)^2-\frac{dy}{dx}\left(e^x+e^x\right)+1=0\ ?$$

(a) $y+e^{-x}=c$

(b) $y-e^x=C$

(c) $\left(y+e^x\right)\left(y-e^x\right)=C$

(d) None of these

13. What is the solution of $\frac{dy}{dx}=\cos\left(x-y\right)$?

(a) $\cot 2\left(x-y\right)=x+C$

(b) $\cot\left(x-y\right)=x+C$

(c) $\cot\frac{1}{2}\left(x-y\right)=x+C$

(d) none of these

14. What is the solution of the differential equation

$$\left(x+2y^3\right)\frac{dy}{dx}=y$$

(a) $\frac{y}{x}=x^2+C$ (b) $2\frac{x}{y}=y^2+C$

(c) $\frac{x}{y}=y^2+C$ (d) none of these

15. What is the solution of the differential equation

$$\frac{dy}{dx}=\frac{1}{x\left(x+1\right)}\ ?$$

(a) $y=\ln\left(\frac{x+1}{x}\right)+C$

(b) $y=\ln\left(\frac{x}{x+1}\right)+C$

(c) $y=\ln\left(x+1\right)+C$

(d) $y=\ln x+C$

16. What is the solution of $\frac{dy}{dx}=\frac{1+y^2}{\sqrt{1-x^2}}$ if $y=1$ when =1?

(a) $\sin^{-1}x-\tan^{-1}y=0$

(b) $\tan^{-1}y+\sin^{-1}x=\pi$

(c) $\tan^{-1}y=\sin^{-1}x-\frac{\pi}{2}$

(d) none of these.

17. What is the particular solution of $\frac{dy}{dx}=\frac{1}{1+x^2}$ when $x=0, y=1$?

(a) $y=2\tan^{-1}x+C$

(b) $y=\tan^{-1}x+1$

(c) $y=\frac{1}{2}\tan^{-1}x+3$

(d) $3y=\tan^{-1}x+2$

18. What is the solution of the equation $\frac{d^2y}{dx^2} = e^{-2x}$?

(a) $y = \frac{1}{4}e^{-2x} + Cx + d$

(b) $y = \frac{1}{5}e^{-2x} + Cx + d$

(c) $y = \frac{1}{6}e^{-2x} + Cx + d$

(d) $y = \frac{1}{7}e^{-2x} + Cx + d$

19. What is the general solution of $\frac{dy}{dx} = x + xy$?

(a) $1 + 2y = Ce^{x^2}$ (b) $1 + y = Ce^{\frac{x^2}{2}}$

(c) $1 + 3y = Ce^{\frac{x2}{2}}$ (d) none of these

20. What is the differential equation representing the family of curves $y = A\cos[x + B]$?

(a) $\frac{d^2y}{dx^2} - y = 0$ (b) $\frac{d^2y}{dx^2} + y = 0$

(c) $\frac{d^2y}{dx^2} + 2y = 0$ (d) $\frac{d^2y}{dx^2} - 2y = 0$

21. What is the differential equation from $y = c\sec x$ by eliminating the arbitrary constant?

(a) $y^2\frac{dy}{dx} = \tan x$

(b) $y\frac{dy}{dx} = \tan x$

(c) $\frac{dy}{dx} = y\tan x$

(d) none of these

22. What is the general solution of the differential equation $\frac{dy}{dx} = \frac{x^2}{y^2}$?

(a) $x^3 + y^3 = C$ (b) $x^3 - y^3 = C$

(c) $x^2 + y^2 = C$ (d) $x^2 - y^2 = C$

23. What is the solution of $xdx + ydy = 0$?

(a) $x^2 - y^2 = C$ (b) $x^2 + y^2 = C$

(c) $x^2y^2 = C$ (d) $\frac{x^2}{y^2} = C$

24. What is the differential equation whose solution is $y = mx + C$?

(a) $\frac{d^2y}{dx^2} = 0$ (b) $\frac{d^2y}{dx^2} = C$

(c) $\frac{dy}{dx} = mx$ (d) $\frac{d^2y}{dx^2} = y$

<u>Answers</u>

1. (b)	2. (a)	3. (b)	4. (c)
5. (b)	6. (b)	7. (a)	8. (c)
9. (b)	10. (b)	11. (c)	12. (c)
13. (c)	14. (c)	15. (b)	16. (c)
17. (b)	18. (a)	19. (b)	20. (b)
21. (c)	22. (b)	23. (b)	24. (a)

B. Fill in the blanks:

1. The particular solution of the equation $\frac{dy}{dx} = \sin x$ where $y = 2$ when $x = \pi$ is _____.

2. The particular solution of $\frac{dy}{dx} = (1+x)^4$, $y = 0$ when $x = -1$ is _______

3. The order of the differential equation $\frac{d^3y}{dx^3} = \left(\frac{d^2y}{dx^2}\right)^2 + \left(\frac{dy}{dx}\right)^4 + y$

4. The differential equation of all straight lines passing through origin is ______

5. The general solution of the differential equation $\frac{d^3y}{dx^2} = \sin x + \cos x$ contans _____ number of arbitrary constants.

6. The differential equation of the parabola $y^2 = 4x + 12$ is ______

7. The order of the differential equation $\left(\frac{dy}{dx}\right)^8 + \frac{d^2y}{dx^2} = 0$ is _______

8. The degree of the differential equation $\left(\frac{dy}{dx}\right)^4 + y^5 = \frac{d^3y}{dx^3}$ is _______

9. The differential equation whose general solution is $y = 3x + k$ is ______

10. The particular solution of the differential equation $\frac{dy}{dx} = \frac{1+y^2}{1+x^2}, y = \sqrt{3}$ when $x = 1$.

11. The differential equatio of all non horizonatal lines in a plane is _______

12. The solution of $\ln\left(\frac{dy}{dx}\right) = 3x + 4y$ is _____

13. The integrating factor of the differential equation $(x - \ln y)\frac{dy}{dx} = -y \ln y$ is _____

14. The differential equation whose primotive is $y = a\cos x + b\sin x$ is ____

15. The differential equation whose solution is $y = e^{x+a}$ is _______

16. The solution of $(1+x^2)\tan^{-1} y = (1+y^2)\tan^{-1} x\, dx$ is _____?

17. If p and q order of the differential equation $y = e^{\frac{dy}{dx}}$, then the relation between p and q is _______?

Hints & Solutions

1. $y = -\cos x + C$

2. $y = \frac{1}{5}(1+x)^5 + C$

3. 3^{rd} order.

4. $x\,d\,y - y\,d\,x = 0$

5. Two

6. $y\,dy - 2dx = 0$

7. Two

8. One

   Hints : The degree of a differential equation is the power of the highest derivative. Here the highest derivative is $\frac{d^3y}{dx^3}$. Its power is one.

9. $\frac{dy}{dx} = 3$

10. $\tan^{-1} y = \tan^{-1} x + \frac{\pi}{2}$

11. $\frac{dy}{dx} = a$

12. $\frac{1}{3}e^{3y}+\frac{1}{4}e^{-4y}=C$

Hints: $\ln\left(\frac{dy}{dx}\right)=3x+4y$

$$\Rightarrow \frac{dy}{dx}=e^{3x+4y}$$

$$\Rightarrow \frac{dy}{dx}=e^{3x}.e^{4y}dx$$

$$\Rightarrow e^{-4y}dy=e^{3x}dx$$

$$\int e^{3x}dx-\int e^{-4y}dy=C$$

$$\Rightarrow \frac{1}{3}e^{3x}dx+\frac{1}{4}e^{-4y}=C$$

13. $\ln y$

14. $\frac{d^2y}{dx^2}+y=0$

15. $\frac{dy}{dx}-y=0$

16. $\left(\tan^{-1}x\right)^2-\left(\tan^{-1}y\right)^2=C$

17. $p=q=1$

Hints : The general solutions of the differential

equation is $y=a\,\text{sint}+\text{b e}^t$ ......(1)

$$\frac{dy}{dx}=a\cos t+be^t \quad ....(2)$$

$$\frac{d^2y}{dx^2}=-a\sin t+be^t \quad ......(3)$$

Adding (1) and (3) from (1), we get

$$y-\frac{d^2y}{dx^2}=2a\sin t$$

$$\Rightarrow a=\frac{1}{2\sin t}\left(y-\frac{d^2y}{dx^2}\right)$$

From (2), we get

$$\frac{dy}{dt}=\frac{1}{2\sin t}\left(y-\frac{d^2y}{dx^2}\right)\cos t+\frac{1}{2}\left(y+\frac{d^2y}{dt^2}\right)$$

C. Answer in one word:

1. What is the degree of the differential equation $\left(\frac{dy}{dx}\right)^4+y^5=\frac{d^3y}{dx^3}$,

2. If p and q are respectively degree and order of the differential equation $y=e^{\frac{dy}{dx}}$.

3. From the differential equation whose general solution is $y=a\sin \text{t}+\text{b e}^t$

4. How many arbitrary constants does the general solution of the differential equation $\frac{d^2y}{dx^2}=\sin x+\cos x$ contain?

5. What is the solution of $\frac{dy}{dx}=8x$ given $y=2$, when $x=1$?

6. Write the order of the differential eqation $\left(\frac{dy}{dx}\right)^8+\frac{d^2y}{dx^2}=0$

7. Write the solution of the equation $\frac{d^2y}{dx^2}=0$.

8. What is the differential equation whose general solution is $y=3x+K$

9. Write the differential equation of the parabola $y^2=4x+12$

10. Given the general solution as $y=\left(x^2+c\right)e^{-x}$ of a differential eqation, what is the particular solution if $y=0$ when $x=1$

11. What is the particular solution of $\frac{dy}{dx}=\frac{1}{1+x^2}$ given that when $x=0, y=1$

12. What is the differential equation whose solution is $y = e^{x+a}$?

13. Obtain a differential equation that should be satisfied by the family of concentric circles $x^2 + y^2 = a^2$

14. Find the differential equation of all straight lines passing through origin.

15. Write the particular solution of $\frac{dy}{dx} = (1+x)^4$, $y = 0$ when $x = -1$.

Answers

1. First degree

2. $p = q = 1$

3. $\frac{dy}{dt} = \frac{1}{2\sin t}\left(y - \frac{d^2y}{dt^2}\right) + \frac{1}{2}\left(y + \frac{d^2y}{dt^2}\right)$

4. Two arbitrary constants

5. $y = 4x^2 - 2$

6. 2nd order.

7. $y = cx + d$

8. $\frac{dy}{dx} = 3$

9. $y\,dx - 2dx = 0$

10. $y = (x^2 + 1)e^{-x}$

11. $y = \tan^{-1} x + 1$

12. $\frac{dy}{dx} = y$

13. $x + y\frac{dy}{dx} = 0$

14. $x\,dy = y\,dx$

15. $y = \frac{1}{5}(1+x)^5$

D. Answer in one sentence:

1. Write the order and degree of the differential equation

$$\frac{d^3y}{dx^3} = \left(\frac{d^2y}{dx^2}\right)^2 + \left(\frac{dy}{dx}\right)^4 + y$$

2. Write the particular solution of the equation $\frac{dy}{dx} = \sin x$ for which $y = 2$

When $x = \pi$

3. Write the order of the differential equation whose general solution is $y = ax^2 + b$ where a and b are arbitrary constants.

4. What is the order of the differential equation of all conics whose centre is at origin?

5. What is the differential equation whose solution is $y = mx + c$.

6. What is the solution of $x\,dx + y\,dy = 0$?

7. Write the order and degree of the differential equation $\ln\left(\frac{d^2y}{dx^2}\right) = y$

8. Write the differential equation of the family of straight lines parallel to $y - axis$.

9. What is the general solution of the differential equation is $\frac{dy}{dx} = \frac{x^2}{y^2}$?

10. For the differential equation from $y = c\sec x$ by eliminating the arbitrary constants.

11. For m the differential equation representing the family of curves $y = A\cos(x + B)$.

12. Write general solution of $\frac{dy}{dx} = x + xy$.

13. Write the general solution of $\frac{d^2y}{dx^2} = e^{-2x}$?

14. What is the general solution of the differential equation

$$\left(1+y^2\right)+\left(x-e^{\tan^{-1}y}\right)\frac{dy}{dx}=0.$$

15. Write differential equation of circles passing through the origin and having their centres on the $x-axis$.

Answers

1. The order and degree of the given differential equation three and one respectively.

2. Required particular solution is $y=-\cos x+1$

3. As in the solution, there are two arbitrary constants, the differential equation is of 2nd order.

4. The general equation of all conic whose centre is $(0,0)$ is $ax^2+2h\,xy+by^2=0$.

   As it has three arbitrary constants, so the order of the differential equation is 3.

5. The required differential equation is $\frac{d^2y}{dx^2}=0$

6. The solution of the given differential equation is $x^2+y^2=C$.

7. As the given differential equation is reduced to $\frac{d^2y}{dx^2}=e^y$, its order is 2 and degree is 1.

8. The family of straight lines parallel to $y-axis$ is $x=c$ where c is a constant, So the required differential equation is $\frac{dx}{dy}=0$

9. The general solution of the given differential equation is $x^3-y^3=C$.

10. The required differential equation is $\frac{dy}{dx}=y\tan x$.

11. The required differential equation is $\frac{d^2y}{dx^2}+y=0$

12. The required general solution is $1+y=Ce^{\frac{x^2}{2}}$.

13. The required general solution is $y=\frac{1}{4}e^{-2x}+cx+d$.

14. The required general solution is $2x\,e^{\tan^{-1}y}=e^{2\tan^{-1}y}+K$.

15. The required differential equation is $y^2=x^2+2xy\frac{dy}{dx}$.

GROUP-B

Short Type (Questions & Answers)

1. Solve $\left(1+x^2\right)\frac{dy}{dx}+2xy=\cos x$

2. Solve $\frac{dy}{dx}=\frac{1}{x^2-7x+12}$

3. Find the integrating factor of the differential equation.

   $\left(1+y^2\right)dx+\left(x-e^{-\tan^{-1}y}\right)dy=0$

4. Solve $(x+y)dy+(x-y)dx=0$

5. Find the particular solution of the differential equation $\frac{d^2y}{dx^2}=6x$ given that $y=1$ and $\frac{dy}{dx}=2$ when $x=0$

6. Solve

   $x^2(y-1)dx+y^2(x-1)dx=0$

7. Solve $\left(x+2y^3\right)\frac{dy}{dx}=y$

8. Solve $\left(x^2-1\right)\frac{dy}{dx}+2xy=1$

9. Solve $x\,dy+e^{-y}x\sin x\,dx=0$

10. Find the differential equation representing family of curves given by $(x-a)^2+2y^2=a^2$ where a is an arbitrary constant.

11. Solve $\cos ec.\frac{d^2y}{dx^2}=x$

12. Find the particular solution of the following differential equation $\frac{dy}{dx}=\frac{1+y^2}{1+x^2}$ given that $y=\sqrt{3}$ when $x=1$.

13. Solve

    $(x+\tan y)dy=\sin 2y\,dx$

14. Solve $\frac{dy}{dx}=e^{2t+3y}$

15. Solve $\frac{dy}{dx}+y=e^{-x}$

Answers

1. $y\left(1+x^3\right)=\sin x+C$

2. $y=-\ln(x-3)+\ln(x-4)+C$

3. $e^{\tan^{-1}y}$

4. $\frac{1}{2}\ln\left(y^2+x^2\right)+\tan^{-1}\frac{y}{x}=C$

5. $y=x^3+2x+1$

6. $\frac{x^2}{2}+xt+\ln(x-1)+\frac{y^2}{2}+y+\ln(y-1)=C$

7. $\frac{x}{y}=y^2+C$

8. $y\left(x^2-1\right)=x+C$

9. $ye^y-e^y-x\cos x+\sin x=C$

10. $y^2=2y\frac{dy}{dx}+2y^2\left(\frac{dy}{dx}\right)^2$

11. $y=-x\sin x-2\cos x+Cx+d$

12. $\tan^{-1}y=\tan^{-1}x+\frac{\pi}{2}$

13. $x\sqrt{\cot y}=\sqrt{\tan y}+C$

14. $-\frac{1}{3}e^{-3y}=\frac{1}{2}e^{2t}+C$

15. $ye^x=x+C$

GROUP-C

Long type (Questions & Answers)

Solve the following differential equation

1. $x\,dy - y\,dx = \sqrt{x^2 + y^2}\,dx$
2. $\dfrac{dy}{dx} = \dfrac{3x-7y+7}{3y-7x-3}$
3. $\dfrac{dy}{dx} = \dfrac{1}{2}\left(\dfrac{y}{x} + \dfrac{y^2}{x^2}\right)$
4. $(x-y+1)dx - (x+y+5)dy = 0$
5. $(2x+y+1)dx + (4x+2y-1)dy = 0$
6. $\dfrac{d^2y}{dx^2} = \dfrac{1}{x(x+1)} + \cos ec^2 x$
7. $x\dfrac{dy}{dx} + y = y^2 \ln x$
8. $\dfrac{dy}{dx} - y\cot x = xy^4$
9. $\dfrac{dy}{dx} = \dfrac{y^2 + xy}{x^2 - xy}$
10. $\dfrac{d^2y}{dx^2} = 4e^x + x\cos x + \sec^2 x$
11. $\dfrac{dy}{dx} + 2y\tan x = \sin x, y\left(\dfrac{\pi}{3}\right) = 0$
12. $\dfrac{dy}{dx} = \dfrac{y-x+1}{y+x+5}$
13. $y\,dy + e^{-y}x\,dy = 0$
14. $\dfrac{d^2y}{dx^2} = \sin 3x$
15. $\dfrac{dy}{dx} + y = \dfrac{1}{1+e^x}$
16. $(1+y^2)x\,dx + (1-x^2)y\,dy = 0$

Answers

1. $\ln\left[\dfrac{y}{x} + \sqrt{1 + \dfrac{y^2}{x^2}}\right] = \ln x + C$
2. $\ln\left[\dfrac{Y^2 - X^2}{X^2}\right]^{3/2} \times \left(\dfrac{Y-X}{Y+X}\right)^{\frac{7}{2}} = CX^{-3}$

   Where $X = x, Y = y-1$
3. $\dfrac{1}{y}\sqrt{x} = \dfrac{2}{\sqrt{x}} + C$
4. $(x+3)^2 - 2(x+3)(y+2) - (y+2)^2 = C$
5. $2(2x+y-1) + \ln(2x+y-1) = 3x + C$
6. $\ln x - 2\ln(v-1) - 2\left(-\dfrac{1}{v-1}\right) = C$

   where $v = \dfrac{y}{x}$
7. $\dfrac{1}{y} = \ln x + 1 + Cx$
8. $\dfrac{1}{y^3}.\sin^3 x = \dfrac{9}{4}x\cos x - \dfrac{9}{4}\sin x - \dfrac{9}{12}$

   $x\cos 3x + \dfrac{1}{4}\cos 3x + C$
9. $-\dfrac{x}{y} - \ln\dfrac{y}{x} = 2\ln x + C$
10. $y = 4e^x$ - $x\cos x + 2\sin x + \ln\sec x - x - 2$
11. $y\sec^2 x = \sec x - 2$
12. $\dfrac{1}{2}\ln\left[(x+2)^2 + (y+3)^2\right] + \tan^{-1}\left|\dfrac{y+3}{x+3}\right| = C$
13. $(y-1)e^y = x\cos x - \sin x + C$
14. $y = -\dfrac{\sin 3x}{9} + (x+1)$
15. $ye^x = \log(1+e^x) + C$
16. $\ln(1+y^2) = \ln(1-x^2) + C$

Chapter - 12

VECTORS

Multiple Choice Questions (MCQ)

Group-A

A. Choose the correct answer from the given choices:

1. If $\vec{a}=\hat{i}+2\hat{i}+2\hat{k},\vec{b}=2\hat{i}-2\hat{j}+2\hat{k}$ and $\vec{c}=-\hat{i}+2\hat{j}+\hat{k}$ then
   (a) $\vec{a}$ and $\vec{b}$ have the same direction
   (b) $\vec{a}$ and $\vec{c}$ have the opposite direction
   (c) $\vec{b}$ and $\vec{c}$ have opposite direction
   (d) no pair of vectors have the same direction.

2. If the vectors $\vec{a}=2\hat{i}+3\hat{j}-6\hat{k}$ and $\vec{b}=\alpha\hat{i}+\hat{j}-2\hat{k}$ are parallel then $\alpha=$
   (a) 2 (b) $\frac{2}{3}$
   (c) $-\frac{2}{3}$ (d) $\frac{1}{3}$

3. If the position vectors of two points A and B are $3\hat{i}+\hat{k}$ and $2\hat{i}+\hat{j}-\hat{k}$ then the vector $\overrightarrow{BA}$ is
   (a) $-\hat{i}+\hat{j}-2\hat{k}$ (b) $\hat{i}+\hat{j}$
   (c) $\hat{i}-\hat{j}+2\hat{k}$ (d) $\hat{i}-\hat{j}-2\hat{k}$

4. If $|k\vec{a}|=1$ then
   (a) $\vec{a}=\frac{1}{k}$ (b) $\vec{a}=\frac{1}{|k|}$
   (c) $k=\frac{1}{|\vec{a}|}$ (d) $k=\frac{\pm 1}{|\vec{a}|}$

5. The direction cosines of the vector $\overrightarrow{PQ}$ where $\overrightarrow{OP}=(1,0,-2)$ and $\overrightarrow{OQ}=(3,-2,0)$ are
   (a) $2,-2,2$
   (b) $4,-2,-2$
   (c) $\frac{1}{\sqrt{3}},-\frac{1}{\sqrt{3}},\frac{1}{\sqrt{3}}$
   (d) $\frac{2}{\sqrt{6}},-\frac{-1}{\sqrt{6}},\frac{-1}{\sqrt{6}}$

6. $(2\hat{i}-4\hat{j}).(\hat{i}+\hat{j}+\hat{k})=$ _______
   (a) -3 (b) 2
   (c) -1 (d) -2

7. If $\vec{a}=\hat{i}+2\hat{j}-\hat{k},\vec{b}=\hat{i}+\hat{j}+2\hat{k}$ and $\vec{i}=2\hat{i}-\hat{j}$ then
   (a) $\vec{a}\perp\vec{b}$
   (b) $\vec{b}\perp\vec{c}$
   (c) $\vec{a}\perp\vec{c}$
   (d) no pair of vectors are perpendicular

8. $(\hat{i}+\hat{k})\times(\hat{i}+\hat{j}+\hat{k})=$ _______
   (a) $\hat{i}-\hat{k}$ (b) $\hat{k}-\hat{i}$
   (c) $\hat{k}-2\hat{i}-\hat{j}$ (d) 2

9. A vecter perpendicular to the vectors $\hat{i}+\hat{j}$ and $\hat{i}+\hat{k}$ is_______

(a) $\hat{i}-\hat{j}-\hat{k}$ (b) $\hat{j}-\hat{k}+\hat{i}$

(c) $\hat{k}-\hat{j}-\hat{i}$ (d) $\hat{i}+\hat{j}+\hat{k}$

10. The area of the triangle with vertices $(1,0,0),(0,1,0)$ and $(0,0,1)$ is________

(a) $\frac{1}{2}$ (b) 1

(c) $\frac{\sqrt{3}}{2}$ (d) 2

11. If $\hat{a}$ and $\hat{b}$ are unit vectors such that $\hat{a}\times\hat{b}$ is a unit vectors, then the angle between $\hat{a}\times\hat{b}$ is a unit vectors, then the angle between $\hat{a}$ and $\hat{b}$ is_____

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{2}$

(c) π (d) none of these

12. $\vec{a}.(\vec{b}\times\vec{a})=$_______

(a) 1 (b) 0

(c) $\vec{0}$ (d) none of these

13. $(\vec{a}).\vec{b}\times(-\vec{c})=$_________

(a) $(\vec{a}\times\vec{b}).\vec{c}$ (b) $-\vec{a}.(\vec{b}\times\vec{c})$

(c) $(\vec{a}\times\vec{c}).\vec{b}$ (d) $\vec{a}.(\vec{c}\times\vec{b})$

14. For the non zero vectors $\vec{a},\vec{b}$ and $\vec{c}$, $\vec{a}.(\vec{b}\times\vec{c})=0$ if________

(a) $\vec{b}\perp\vec{c}$ (b) $\vec{a}\perp\vec{b}$

(c) $\vec{a}\,||\,\vec{c}$ (d) $\vec{a}\perp\vec{c}$

15. If $\vec{a}=\hat{i}+\hat{j},\vec{b}=\hat{i}-\hat{j}$ and $\vec{c}=5\hat{i}+2\hat{j}+3\hat{k}$, then $\vec{b}.(\vec{c}\times\vec{a})=$______

(a) -2 (b) -4

(c) -6 (d) -8

16. If ABCD is rhombus whose diagonals cut at the origin O, then

$\overrightarrow{OA}+\overrightarrow{OB}+\overrightarrow{OC}+\overrightarrow{OD}=$_____

(a) $\vec{O}$ (b) $\overrightarrow{AB}+\overrightarrow{AC}$

(c) $2(\overrightarrow{AB}+\overrightarrow{AC})$ (d) $\overrightarrow{AC}+\overrightarrow{BD}$

17. The unit vector along $\hat{i}+\hat{j}+\hat{k}$ is

(a) $\frac{\hat{i}-\hat{j}+\hat{k}}{\sqrt{3}}$ (b) $\frac{\hat{i}+\hat{j}-\hat{k}}{\sqrt{3}}$

(c) $\frac{\hat{i}+\hat{j}+\hat{k}}{\sqrt{3}}$ (d) none of these

18. If the position vectors A and B are $3\hat{i}+2\hat{j}-3\hat{k}$ and $2\hat{i}+5\hat{j}-\hat{k}$ then $\overrightarrow{AB}=$______.

(a) $\hat{i}-3\hat{j}+2\hat{k}$ (b) $-\hat{i}+3\hat{j}+2\hat{k}$

(c) $\hat{i}+3\hat{j}-2\hat{k}$ (d) none of these

19. If ABCD be a parallelogram whose diagonals intersect at P and o be the origin, then $\overrightarrow{OA}+\overrightarrow{OB}+\overrightarrow{OC}+\overrightarrow{OD}$ equals______.

(a) $\overrightarrow{OP}$ (b) $20\vec{P}$

(c) $30\vec{P}$ (d) $4\overrightarrow{OP}$

20. If two forces $4\hat{i}+\hat{j}-3\hat{k}$ and $3\hat{i}+\hat{j}-\hat{k}$ act an a particle at a point. Then theor resultant is _______

(a) $7\hat{i}+2\hat{j}-4\hat{k}$ (b) $7\hat{i}-2\hat{j}+4\hat{k}$

(c) $-7\hat{i}+2\hat{j}+4\hat{k}$ (d) none of these

21. If $\left|3\hat{i}+\hat{j}-a\hat{k}\right|=5$ then the value of a is

(a) $\pm\sqrt{5}$ (b) $\pm\sqrt{10}$

(c) $\pm\sqrt{15}$ (d) $\pm\sqrt{20}$

22. If $\vec{a}=2\hat{i}-5\hat{j}+8\hat{k}$, $\vec{b}=\hat{i}-3\hat{j}-7\hat{k}$ and $\vec{c}=-3\hat{j}+2\hat{j}-\hat{k}$ then $\left|\vec{a}+\vec{b}+\vec{c}\right|$ is______

(a) 2 (b) 4

(c) 6 (d) 8

23. If $\vec{a}=\hat{i}+\sqrt{3}\hat{j}$ and $\vec{b}=3\hat{j}+4\hat{k}$ then the value of $\left|\vec{a}\right|+\left|\vec{b}\right|$ is

(a) 5 (b) 6

(c) 7 (d) 8

24. If $\vec{a}=3\hat{i}+3\hat{j}+\hat{k}$ and $\vec{b}=-2\hat{i}+\hat{j}-2\hat{k}$ then the unit vector parallel to $\vec{a}+\vec{b}$ is

(a) $\frac{\hat{i}+4\hat{j}-\hat{k}}{3\sqrt{2}}$ (b) $\frac{\hat{i}+\hat{j}+\hat{k}}{3\sqrt{2}}$

(c) $\frac{-\hat{i}+4\hat{j}-\hat{k}}{3\sqrt{2}}$ (d) none of these

25. If $\vec{a},\vec{b},\vec{c}$ are the position vectors of the vertices of ΔABC then $\overrightarrow{AB}+\overrightarrow{BC}+\overrightarrow{CA}$ is equal to

(a) $\vec{O}$ (b) $2\vec{a}$

(c) $2\vec{b}$ (d) $\vec{a}+\vec{b}+\vec{c}$

26. If D,E,F be the middle point of the sides BC,CA and AB of ΔABC then $\overrightarrow{AD}+\overrightarrow{BE}+\overrightarrow{CF}$ is

(a) $2\vec{a}$ (b) $2\vec{b}$

(c) $2\vec{c}$ (d) $\vec{O}$

27. If $\vec{a},\vec{b}$ are the position vectors A and B respectively then the position vector of a point C on AB produced such that $\overrightarrow{AC}=3\overrightarrow{AB}$ is ________

(a) $3\vec{a}-\vec{b}$ (b) $3\vec{b}-\vec{a}$

(c) $3\vec{a}-2\vec{b}$ (d) $3\vec{b}-2\vec{a}$

28. If ABCD is a parallelogram and $\overrightarrow{AB}=\vec{a}$ and $\overrightarrow{AD}=\vec{b}$ the $\overrightarrow{BD}=$________

(a) $\vec{b}-\vec{a}$ (b) $\vec{a}-\vec{b}$

(c) $\vec{a}+\vec{b}$ (d) none of these

29. In a parallelogram ABCD if $AB=\hat{i}+2\hat{j}+3\hat{k}$ and $\overrightarrow{BC}=2\hat{i}+4\hat{j}-5\hat{k}$ then the unit vector along $\overrightarrow{AC}$ is

(a) $\frac{1}{7}\left(-3\hat{i}+6\hat{j}+2\hat{k}\right)$

(b) $\frac{1}{7}\left(3\hat{i}-6\hat{j}+2\hat{k}\right)$

(c) $\frac{1}{7}\left(3\hat{i}+6\hat{j}-2\hat{k}\right)$

(d) none of these

30. If $\vec{a},\vec{b},\vec{c}$ are position vectors of the points A,B,C respectively then $\overrightarrow{AB}+\overrightarrow{BC}+\overrightarrow{AC}$ is

(a) $\vec{O}$ (b) $2\left(\vec{b}-\vec{a}\right)$

(c) $2\left(\vec{c}-\vec{a}\right)$ (d) $\vec{a}+\vec{b}+\vec{c}$

31. If the position vectors of A and B are $3\hat{i}-2\hat{j}+\hat{k}$ and $2\hat{i}+4\hat{j}-3\hat{k}$ then the length of $\overrightarrow{AB}$ is______

(a) $\sqrt{15}$ (b) $\sqrt{25}$

(c) $\sqrt{35}$ (d) $\sqrt{53}$

32. If $\vec{a}$ and $\vec{b}$ represent the sides $\overrightarrow{AB}$ and $\overrightarrow{BC}$ of a regular hexagone ABCDEF, then $\overrightarrow{FA}$ is ______

(a) $\vec{b}-\vec{a}$ (b) $\vec{a}-\vec{b}$

(c) $\vec{a}+\vec{b}$ (d) none of these

33. The position vector of a point which divides externally the join of $2\vec{a}-3\vec{b}$ and $3\vec{a}-2\vec{b}$ in the ratio 2:3 is _________.

(a) $\frac{1}{5}\left(12\vec{a}-13\vec{b}\right)$ (b) $\frac{1}{5}\left(13\vec{a}-12\vec{b}\right)$

(c) $\frac{1}{5}\left(\vec{a}-\vec{b}\right)$ (d) none of these

34. If $\left|\vec{a}+\vec{b}\right|=\left|\vec{a}-\vec{b}\right|$ then

(a) $\vec{a}$ is parallel to $\vec{b}$

(b) $\vec{a}$ is perpendicular to $\vec{b}$

(c) $\left|\vec{a}\right|=\left|\vec{b}\right|$

(d) none of these

35. If $\vec{a},\vec{b},\vec{c}$ are three mutually, perpendicular vectors each of magnitude unity then $\left|\vec{a}+\vec{b}+\vec{c}\right|$ is equal to _________

(a) 3 (b) 1

(c) $\sqrt{3}$ (d) none of these

36. The angle between the vectors $2\hat{i}+3\hat{j}+\hat{k}$ and $2\hat{i}-\hat{j}-\hat{k}$ is _____

(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{4}$

(c) $\frac{\pi}{3}$ (d) none of these

37. If $\left|\vec{a}.\vec{b}\right|=\left|\vec{a}\times\vec{b}\right|$ then the angle between $\vec{a}$ and $\vec{b}$ is ________

(a) 0^0 (b) 45^0

(c) 135^0 (d) 180^0

38. If the vectors $\hat{i}-2x\hat{j}-3y\hat{k}$ and $\hat{i}+3x\hat{j}+2y\hat{k}$ are orthogonal to each othe then the locus of the point (x, y) is

(a) a circle (b) an ellipse

(c) parabola (d) a straight line

39. If $\left|\vec{a}\right|=2,\left|\vec{b}\right|=5$ and $\left|\vec{a}\times\vec{b}\right|=8$ then $\vec{a}\,.\,\vec{b}$ is equal to

(a) 0 (b) 2

(c) 4 (d) 6

40. If $\vec{a}$ be any vector then

$$\left|\vec{a}\times\hat{i}\right|^2+\left|\vec{a}\times\hat{j}\right|^2+\left|\vec{a}\times\hat{k}\right|^2=\_\_\_\_$$

(a) $\left|\vec{a}\right|^2$ (b) $2\left|\vec{a}\right|^2$

(c) $3\left|\vec{a}\right|^2$ (d) $4\left|\vec{a}\right|^2$

Answers

1. (d)	2. (c)	3. (c)	4. (d)
5. (c)	6. (d)	7. (c)	8. (b)
9. (a)	10. (c)	11. (b)	12. (b)
13. (a)	14. (c)	15. (c)	16. (a)
17. (c)	18. (b)	19. (d)	20. (a)
21. (c)	22. (c)	23. (c)	24. (a)
25. (a)	26. (d)	27. (d)	28. (a)
29. (c)	30. (c)	31. (d)	32. (b)
33. (a)	34. (b)	35. (c)	36. (a)
37. (b)	38. (a)	39. (d)	40. (b)

B. Fill in the blanks:

1. If $\left|\vec{a}.\vec{b}\right| = \left[\vec{a} \times \vec{b}\right]$ then the angle between $\vec{a}$ and $\vec{b}$ is _______

2. If $\vec{a}$ and $\vec{b}$ are two vectors such that $\vec{a}\,.\,\vec{b} = 0$ and $\vec{a} \times \vec{b} = \vec{0}$ then _______

3. If $\vec{a}$ lies in the plane of $\vec{b}$ and $\vec{c}$ then the value of $\left[\vec{a}\ \vec{b}\ \vec{c}\right]$ is __________

4. If $\left|\vec{a}\right| = 1, \left|\vec{b}\right| = 5$ and $\left|\vec{c}\right| = 3$ then the value of $\left[\vec{a}-\vec{b}\ \ \vec{b}-\vec{c}\ \ \vec{c}-\vec{a}\right]$ is ________

5. If $\vec{a} = 2\hat{i} + 3\hat{j} + \hat{k}$, $\vec{b} = 2\hat{i} + p\hat{j} + 3\hat{k}$ and $\vec{c} = 2\hat{i} + 17\hat{j} - 3\hat{k}$ are coplanar vectors then the value of p is _______

6. If $\vec{a}$ and $\vec{b}$ are two vectors of magnitudes $\sqrt{3}$ and 2 respectively and $\vec{a}.\vec{b} = \sqrt{6}$ then the angle between $\vec{a}$ and $\vec{b}$ is _________

7. If $3\hat{i} + 2\hat{j} + 9\hat{k}$ and $\hat{i} + p\hat{j} + 3\hat{k}$ are perpendicular then the value of p is _______

8. $\vec{a}$ and $\vec{b}$ are two vectors having the same length $\sqrt{2}$ and their scalar product is -1. Then the angle between two vectors is _______

9. The projection of the vector $7\hat{i} + \hat{j} - 4\hat{k}$ on $2\hat{i} + 6\hat{j} + 3\hat{k}$ is _______

10. If $\vec{a}$ and $\vec{b}$ are unit vectors inclined at an angle θ, then the value of $\sin\dfrac{\theta}{2}$ is _____

11. For a unit vector $\vec{a}, \left(\vec{x}-\vec{a}\right).\left(\vec{x}+\vec{a}\right) = 15$ then $\left|\vec{x}\right|$ = _________

12. If $\left(\vec{a}-\vec{b}\right).\left(\vec{a}+\vec{b}\right) = 27$ and $\left|\vec{a}\right| = 2\left|\vec{b}\right|$ then the value of $\left|\vec{a}\right|$ is ________

13. If $\left|\vec{a}\right| = 3, \left|\vec{b}\right| = 2$ and $\vec{a}\,.\,\vec{b} = 6$ then $\left|\vec{a}+\vec{b}\right| =$ _______

14. If $\vec{a}$ and $\vec{b}$ be two vectors of the same magnitude such that the angle between them is 60^0 and $\vec{a}.\vec{b} = 8$ then the value of $\left|\vec{a}\right| =$ _______

15. If $\left|\vec{a}\right| = 4, \left|\vec{b}\right| = 3$ and $\vec{a}\,.\,\vec{b} = 6$, then the angle between $\vec{a}$ and $\vec{b}$ is ________

16. If $\vec{a}, \vec{b}, \vec{c}$ are three mutually perpendicular vectors each of magnitude unit then $\left|\vec{a}+\vec{b}+\vec{c}\right| =$ ________

17. If $\vec{a}$ and $\vec{b}$ are two vectors such that $\left|\vec{a}.\vec{b}\right| = \left|\vec{a} \times \vec{b}\right|$ then the angle between $\vec{a}$ and $\vec{b}$ is ______

Choose the correct answer

18. For any three vectors $\vec{a}, \vec{b}, \vec{c}$, $[\vec{a}\ \ \vec{b}+\vec{c}\ \ \vec{a}+\vec{b}+\vec{c}\,] =$ _______.

    (i) 0 (ii) 2

    (iii) 3 (iv) 4

19. The value of $[\hat{i}\ \hat{j}\ \hat{k}]$ is _______.

    (i) 0 (ii) -1

    (iii) 1 (iv) 2

20. If the vectors $\vec{a} = 2\hat{i} - 3\hat{j} + \hat{k}$, $\vec{b} = 3\hat{i} - \hat{j} + \hat{k}$ and $\vec{c} = \hat{i} + 2\hat{j} - \hat{k}$ are coplanar, then the value of t is _______.

    (i) 1 (ii) 2

    (iii) 3 (iv) 4

21. If $|\vec{a}| = 1$, $|\vec{b}| = 5$ and $|\vec{c}| = 3$, then the value of $[\vec{a}-\vec{b}\ \ \vec{b}-\vec{c}\ \ \vec{c}-\vec{a}] =$ _______

Answers

1. 45^0
2. Either $\vec{a}$ or $\vec{b}$ is a null vector
3. 0
4. 0
5. -4
6. $\frac{\pi}{4}$
7. -15
8. $\frac{2\pi}{3}$
9. $\frac{8}{7}$
10. $\frac{1}{2}\left|\vec{a}-\vec{b}\right|$
11. 4
12. 6
13. 5
14. 4
15. $\frac{\pi}{3}$
16. $\sqrt{3}$
17. $\frac{\pi}{4}$
18. 0
19. 1
20. 3
21. 0

C. Answer in one word.

1. What is the angle between $2\hat{i}+3\hat{j}+\hat{k}$ and $2\hat{i}-\hat{j}-\hat{k}$?
2. What is the scalar component of the vector $\vec{b}=8\vec{i}+\vec{j}$ in the direction of the vector $\vec{a}=\hat{i}+2\hat{j}-2\hat{k}$?
3. If $6\hat{i}+2\hat{j}-3\hat{k}$ and $\hat{i}-4\hat{j}-m\hat{k}$ are perpendicular to each other, then what is the value of m?
4. If $\left|\vec{\alpha}+\vec{\beta}\right|=\left|\vec{\alpha}-\vec{\beta}\right|$ then what is the angle between $\vec{\alpha}$ and $\vec{\beta}$?
5. What is the projection of $\hat{i}+\hat{j}+\hat{k}$ on the vector $\hat{i}$?
6. What is the angle between $3\hat{i}+4\hat{j}$ and $2\hat{i}+\hat{j}+\sqrt{3}\hat{k}$?
7. What is the angle between two vectors $\vec{a}$ and $\vec{b}$ with magnitude 2 and 1 respectively such that $\vec{a}.\vec{b}=\sqrt{3}$?
8. If $\vec{a}=2\hat{i}+2\hat{j}-\hat{k}$ and $\vec{b}=6\hat{i}-3\hat{j}+2\hat{k}$ then what is $\vec{a}.\vec{b}$?
9. If $\vec{a}=2\hat{i}+2\hat{j}+3\hat{k}$, $\vec{b}=-\vec{i}+2\vec{j}+\hat{k}$ and $\vec{c}=3\hat{i}+\hat{j}$ and $\vec{a}+t\vec{b}$ is perpendicular to $\vec{c}$, then what is the value of t?
10. If $\left|\vec{a}.\vec{b}\right|=\left|\vec{a}\times\vec{b}\right|$ then what is the angle between $\vec{a}$ and $\vec{b}$?
11. If $\left(\vec{a}\times\vec{b}\right)+\left(\vec{a}\,.\,\vec{b}\right)=144$ and $\left|\vec{a}\right|=4$ then what is $\left|\vec{b}\right|$?
12. What is the unit vector which is in the direction of the sum of the vectors $\vec{a}=2\vec{i}+2\vec{j}-5\vec{k}$ and $\vec{b}=2\vec{i}+\vec{j}-7\vec{k}$?
13. What are the values m and n for which the vectors $(m-1)\hat{i}+(n+2)\hat{j}+4\hat{k}$ and $(m+1)\hat{i}+(n-2)+8\hat{k}$ are parallel?

14. The position vectors of A and B are $3\hat{i}-2\hat{j}+\hat{k}$ and $2\hat{i}+4\hat{j}-3\hat{k}$ then what is the length of $\overline{AB}$?

15. What is the magnitude of $2\vec{a}\times 3\vec{a}$?

16. If $\vec{a},\vec{b}$ and $\vec{c}$ be three vectors such that $\vec{a}+\vec{b}+\vec{c}=\vec{0}$ and $|\vec{a}|=1, |\vec{b}|=4$ and $|\vec{c}|=2$ then what is the value of $\vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a}$?

17. What is the value of 'a' such that the vectors $6\hat{i}+2\hat{j}-3\hat{k}$ and $\hat{i}+6\hat{j}+a\hat{k}$ are perpendicular?

18. If $\vec{a}$ and $\vec{b}$ are unit vectors and $\vec{a}-\vec{b}$ is also a unit vector, then what is measure of the angle between $\vec{a}$ and $\vec{b}$?

19. What is the measure of the angle between two main diagonals of the cube?

Answers

1. $\frac{\pi}{2}$
2. $\frac{10}{3}$
3. $-\frac{2}{3}$
4. $\frac{\pi}{3}$
5. 1
6. $\frac{\pi}{4}$
7. $\frac{\pi}{6}$
8. 4
9. 8
10. $\frac{\pi}{4}$
11. 3
12. $\frac{1}{13}\left(4\hat{i}+3\hat{j}-12\hat{k}\right)$
13. $m=3, n=-6$
14. $\sqrt{53}$
15. 0
16. $\frac{-21}{2}$
17. 6
18. 120^0
19. $\cos^{-1}\frac{1}{3}$

D. Answer in one sentence:

1. Write a vector normal to $\hat{i}+\hat{k}$ and $\hat{i}+\hat{j}$?

2. If $\vec{a}$ and $\vec{b}$ are unit vectors such that $\vec{a}\times\vec{b}$ is unit vectors, then what is the angle between $\vec{a}$ and $\vec{b}$?

3. What is the unit vector perpendicular to the vectors $\hat{i}-\hat{j}$ and $2\hat{i}-3\hat{j}$?

4. Find the value of 'a' such that the vector $6\hat{i}+2\hat{j}-3\hat{k}$ is perpendicular to $\hat{i}+6\hat{j}+a\hat{k}$.

5. Write a vector in the direction of the vector $\hat{i}-2\hat{j}+3\hat{k}$ what has magnitude 9 units.

6. Write the value of the cosine of the angle which the vector $\vec{a}=\hat{i}+\hat{j}+\hat{k}$ makes with $y-axis$.

7. If $\vec{a}=x\vec{i}+2\vec{j}-z\vec{k}$ and $\vec{b}=3\vec{i}+y\vec{j}-2\vec{k}$ are two equal vectors then $x+y+z$?

8. Find a vector in the direction of vector $2\hat{i}-3\hat{j}+6\hat{k}$ which has magnitude 21 units.

9. Find the unit vector in the direction of the sum of the vectors $\vec{a} = 2\vec{i} + 2\vec{j} - 5\vec{k}$ and $\vec{b} = 2\vec{i} + 2\vec{j} - 7\vec{k}$

10. If $\vec{a}.\vec{b} = \frac{1}{2}$ then what is angle between $\hat{a}$ and $\hat{b}$?

11. If $\left(\vec{a}\times\vec{b}\right)^2 + \left(\vec{a}.\vec{b}\right) = 144$ then what is the value of ab?

12. What is the projection of $\hat{i} + \hat{j} + \hat{k}$ upon the vector $\hat{i}$?

13. If A,B,C,D,E are the vertices of a regular pentagon, find the vector sum $\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CD} + \overrightarrow{DE} + \overrightarrow{EA}$.

14. If $\vec{a}, \vec{b}$ and $\vec{c}$ are three mutually perpendicular vectors, each of magnitude unity, then what will be magnitude of $\vec{a} + \vec{b} + \vec{c}$?

15. How many directions of a null vector has?

Answers

1. A vector normal to $\hat{i} + \hat{k}$ and $\hat{i} + \hat{j}$ is $-\hat{i} + \hat{j} + \hat{k}$.

2. The angle between $\vec{a}$ and $\vec{b}$ is $\frac{\pi}{2}$

3. The unit vector perpendicular to $\hat{i} - \hat{j}$ and $2\hat{i} - 3\hat{j}$ is $-\hat{k}$

4. The required value of a is 6.

5. The required vector is $3\left(\hat{i} - 2\hat{j} + 2\hat{k}\right)$.

6. The required cosine of the angle is $\frac{1}{\sqrt{3}}$

7. The required value of $x + y + z$ is 0.

8. The required vector is $6\hat{i} - 9\hat{j} + 18\hat{k}$

9. The required unit vector is $\frac{4}{13}\hat{i} + \frac{3}{13}\hat{j} - \frac{12}{13}\hat{k}$.

10. The required angle between $\hat{a}$ and $\hat{b}$ is $\frac{\pi}{3}$.

11. The value of ab is 12.

12. The required projection is 1.

13. $\overrightarrow{Ab} + \overrightarrow{BC} + \overrightarrow{CD} + \overrightarrow{DE} + \overrightarrow{EA} = \vec{0}$

14. The magnitude of $\vec{a} + \vec{b} + \vec{c}$ is $\sqrt{3}$

15. The null vector has arbitrary direction.

GROUP-B

Short type (Questions & Answers)

1. Let $\vec{a}=\hat{i}+\hat{j}+\hat{k},\vec{b}=4\hat{i}-2\hat{j}+3\hat{k}$ and $\vec{c}=\hat{i}-2\hat{j}+\hat{k}$. Find a vector of magnitude 6 units which is parallel to the vector $2\vec{a}-\vec{b}+3\vec{c}$.

2. Prove that for any vector $\vec{a},\vec{b}$ and $\vec{c}$ $\left[\vec{a}+\vec{b}\;\;\vec{b}+\vec{c}\;\;\vec{c}+\vec{a}\right]=2\left[\vec{a}\;\vec{b}\;\vec{c}\right]$

3. If $\vec{a},\vec{b},\vec{c}$ are three mutually perpendicular then prove that $\left[\vec{a}.\left(\vec{b}\times\vec{c}\right)\right]^2=a^2b^2c^2$

4. If $\hat{i}+\hat{j}+\hat{k}$ and $2\hat{j}-\alpha\hat{j}+3\hat{k}$ are orthogonal to each other then find α.

5. Using vector method prove that an angle inscribed in a semi circle is a right angle.

6. If $\vec{a}=\vec{i}-2\vec{j}+3\vec{k},\vec{b}=2\vec{i}+\vec{j}-\vec{k}$ and $\vec{c}=\hat{j}+\hat{k}$ then find $\left[\vec{a}\;\vec{b}\;\vec{c}\right]$.

7. Find a vector $\vec{b}$ such that $\vec{a}\times\vec{b}=\vec{c}$ and $\vec{a}.\vec{b}=3$ where $\vec{a}=\hat{i}+\hat{j}+\hat{k},\vec{c}=\hat{j}-\hat{k}$.

8. Prove by vector method that in a trangle ABC, $c^2=a^2+b^2-2ab\cos C$

9. Determine the area of the parallelogram whose sides are the vectors $2\hat{i}+\hat{j}$ and $\hat{i}-\hat{k}$.

10. If the position vecotrs of the points A,B,C are $2\hat{i}+\hat{j}-\hat{k}$, $3\hat{i}-2\hat{j}+\hat{k}$ and $\hat{i}+4\hat{j}-3\hat{k}$ respectively the prove that A,B,C are collinear.

11. Find the scalar projection of the vector $\vec{a}=3\vec{i}+6\vec{j}+9\vec{k}$ on $\vec{b}=2\vec{i}+2\vec{j}-\vec{k}$.

12. Find the value of λ such that the following vectors are coplanar.

    $-\hat{i}+\lambda\hat{j}-\lambda\hat{k},2\hat{i}+4\hat{j}+5\hat{k},\;-2\hat{i}+4\hat{j}-4\hat{k}$.

13. Prove that four points with position vectors $\hat{i}+\hat{j}-3\hat{k},2\hat{i}-\hat{j}-\hat{k},-\hat{i}+2\hat{j}+2\hat{k}$ and $2\hat{i}+2\hat{k}$ are coplanar.

14. Using vector method find the area of the trangle with vertices $(1,0,0),(0,1,0)$ and $(0,0,1)$

15. Write the volume of the parallelopiped whose sides are give by $-\hat{j},\hat{k},-\hat{i}$.

16. If the sum of two unit vectors is a unit vector then find the magnitude of their difference.

17. If the magnitude of the difference of two unit vectors is $\sqrt{3}$ then find the magnitude of their sum

18. If $\vec{a}=(1,3,6)$, $\vec{b}=(2,-2,1)$ and $\vec{c}=(-1,0,2)$ find the direction cosine of $\vec{b}-\vec{a}+2\vec{c}$ and unit vector in the direction of $\vec{b}-\vec{a}+2\vec{c}$

Answers

1. $2\hat{i}-4\hat{j}+4\hat{k}$

   Hints: $2\vec{a}-\vec{b}+3\vec{c}=\hat{i}-2\hat{j}+2\hat{k}$

   Unit vectior along $2\vec{a}-\vec{b}+3\vec{c}$

   $$=\frac{1}{3}\left(\hat{i}-2\hat{j}+2\hat{k}\right)$$

   Vector of magnitude 6 units

   $$=6.\frac{1}{3}\left(\hat{i}-\hat{j}+2\hat{k}\right)$$

   $$=2\hat{i}-4\hat{j}+4\hat{k}$$

3. $\vec{b}\times\vec{c}$ is perpendicular to $\vec{b}$ and $\vec{c}$

Also $\vec{a}$ is perpendicular $\vec{b}$ and $\vec{c}$

$\therefore \vec{a}$ and $\vec{b}\times\vec{c}$ are parallel.

$\therefore \vec{a}.\left(\vec{b}\times\vec{c}\right) = \left|\vec{a}\right| \left|\vec{b}\times\vec{c}\right|$

$= \left|\vec{a}\right| \left|\vec{b}\right| \left|\vec{c}\right| \sin 90^0$

$\Rightarrow \left[\vec{a}.\vec{b}\times\vec{c}\right]^2 = |\vec{a}|^2 |\vec{b}|^2 |\vec{c}|^2$

$= a^2 b^2 c^2$

4. $\alpha = 5$

5. Let ApB be a semicircle and 0 be the centre of the circle.

Let $0\vec{A} = \vec{a},\ 0\vec{B} = -\vec{a}$

$\therefore 0\vec{P} = \vec{r}$

$|\overrightarrow{OA}| = |\overrightarrow{OB}| = |\overrightarrow{OP}|$

$\Rightarrow |\vec{a}| = |-\vec{a}| = |\vec{r}|$

$\overrightarrow{AP} = \overrightarrow{OP} - \overrightarrow{OA} = \vec{r} - \vec{a}$

$\overrightarrow{BP} = \overrightarrow{OP} - \overrightarrow{OB} = \vec{r} + \vec{a}$

$\overrightarrow{AP}.\overrightarrow{BP} = \left(\vec{r} - \vec{a}\right).\left(\vec{r} + \vec{a}\right)$

$= \vec{r}^{\,2} - \vec{a}^{\,2}$

$= |\vec{r}|^2 - |\vec{a}|^2 = 0$

$\Rightarrow AP$ is perpendicular to BP

$\Rightarrow \angle APB$ is right angle.

6. 12

7. Given $\vec{a} = \vec{i} + \vec{j} + \vec{k}$

$\hat{i} = 0\hat{i} + \hat{j} - \hat{k}$

Let $\vec{b} = x\hat{i} + y\hat{j} + 2\hat{k}$

Given that $\vec{a}.\vec{b} = 3$

$\Rightarrow \left(\hat{i} + \hat{j} + \hat{k}\right).\left(x\hat{i} + y\hat{j} + 2\hat{k}\right) = 3$

$\Rightarrow x + y + z = 3$ .... (1)

Again $\vec{a}\times\overrightarrow{b} = \vec{c}$

$$\Rightarrow \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 1 \\ x & y & z \end{vmatrix} = 0\hat{i} + \hat{j} - \hat{k}$$

$\Rightarrow \hat{i}\left(z-y\right) - \hat{j}\left(z-x\right) + \hat{k}\left(y-x\right) = 0\hat{i} + \hat{j} - \hat{k}$

$\Rightarrow z - y = 0 \quad \Rightarrow y = z$

$-\left(z - x\right) = 1 \quad \Rightarrow x - z = 1$

$y - x = -1 \quad \Rightarrow x - y = 1$

Solving we get $x = \frac{5}{3}, y = \frac{2}{3}, z = \frac{2}{3}$

$\therefore \vec{b} = \frac{5}{3}\hat{i} + \frac{2}{3}\hat{j} + \frac{2}{3}\hat{k}$

9. 12 sq. unit

11. The scalar projection of $\vec{a}$ on $\vec{b}$

$= \left|\vec{a}\right| \cos\theta$

$= \sqrt{126}.\frac{3}{\sqrt{126}} = 3$

12. Let $\vec{a} = -\hat{i} + \lambda\hat{j} - \lambda\hat{k}$

$\vec{b} = 2\hat{i} + 4\hat{j} + 5\hat{k}$

$\vec{c} = -2\hat{i} + 4\hat{j} - 4\hat{k}$

The vector are coplanar

$\therefore \vec{a}.\left(\vec{b}\times\vec{c}\right) = 0$

$$\Rightarrow \begin{vmatrix} -1 & \lambda & -\lambda \\ 2 & 4 & 5 \\ -2 & 4 & -4 \end{vmatrix} = 0$$

$= \lambda = 2$

13. Let A,B,C,D are four given points whose p.vs are $\hat{i}+\hat{j}-3\hat{k}, 2\hat{i}-\hat{j}-\hat{k}$,

$-\hat{i}+2\hat{j}+2\hat{k}$, and $2\hat{j}+2\hat{k}$

$$\overrightarrow{AB}=\hat{i}-2\hat{j}-2\hat{k}$$

$$\overrightarrow{AC}=-2\hat{i}+\hat{j}-\hat{k}$$

$$\overrightarrow{AD}=-\hat{i}+\hat{j}-\hat{k}$$

$$\overrightarrow{AB}.\left(\overrightarrow{AC}\times\overrightarrow{AD}\right)=0$$

So the points A,B,C and D are coplanar.

14. $\frac{3}{2} sq.unit$

15. 1 cubic unit

16. Let $\overrightarrow{AB}=\hat{a}, \overrightarrow{BC}=\hat{b}$

$$\overrightarrow{AC}=\overrightarrow{AB}+\overrightarrow{BC}$$

$$=\hat{a}+\hat{b}$$

Given that $|\overrightarrow{AB}|=|\vec{a}|=1$

$$|\overrightarrow{BC}|=|\vec{b}|=1$$

$$|\overrightarrow{AC}|=|\hat{a}+\hat{b}|=1.$$

ABC is an equilateral triangle.

Let us produce CB to D such that BC=BD

$$\overrightarrow{BD}=-\overrightarrow{BC}=-\hat{b}$$

$$\overrightarrow{AD}=\overrightarrow{AB}+\overrightarrow{BD}=\hat{a}+\left(-\hat{b}\right)=\hat{a}-\hat{b}$$

$$\angle CAD=60^0+30^0=90^0$$

$$\angle ACD=60^0$$

From the $\Delta\, ACD$,

$$\tan\angle ACD=\frac{|\overrightarrow{AD}|}{|\overrightarrow{AC}|}$$

$$\Rightarrow \tan 60^0=\frac{|\hat{a}-\hat{b}|}{|\hat{a}+\hat{b}|}$$

$$\Rightarrow \sqrt{3}=|\hat{a}-\hat{b}|$$

$$\Rightarrow |\hat{a}-\hat{b}|=\sqrt{3}$$

17. Let $\overrightarrow{OA}=\hat{a}, \overrightarrow{AB}=\hat{b}$

$$\overrightarrow{OB}=\hat{a}+\hat{b}$$

Let us produce BA to C

such that $AC=AB$

$$\therefore \overrightarrow{AC}=-\overrightarrow{AB}=-\hat{b}$$

$$\overrightarrow{OC}=\overrightarrow{OA}+\overrightarrow{AC}=\hat{a}-\hat{b}$$

Given that $|\hat{a}-\hat{b}|=\sqrt{3}$

$$\Rightarrow |\hat{a}-\hat{b}|^2=3$$

$$\Rightarrow \left(\hat{a}-\hat{b}\right)^2=3$$

$$\Rightarrow \hat{a}^2+\hat{b}^2-2\hat{a}.\hat{b}=3$$

$$\Rightarrow 1+1-2\hat{a}.\hat{b}=3$$

$$\Rightarrow 2\hat{a}\,.\,\hat{b}=-1$$

$$\Rightarrow \hat{a}\ .\ \hat{b}=-\frac{1}{2}=\cos 120^0$$

$$\Rightarrow O\,AC=120^0 \Rightarrow \angle AOB=60^0$$

Since we have $\left(\overrightarrow{OA}\right)=|\overrightarrow{AB}|=1$

$$|\hat{a}+\hat{b}|=|\overrightarrow{OB}|=1$$

GROUP-C

Long Questions.

1. If $\vec{a} = 2\hat{i} + \hat{k}$, $\vec{b} = \hat{i} + \hat{j} + \hat{k}$ and $\vec{c} = 4\hat{i} - 3\hat{j} + 7\hat{k}$ the nfind the vector $\vec{r}$ which satisfres $\vec{r} \times \vec{b} = \vec{c} \times \vec{b}$ and $\vec{r}.\vec{a} = 0$.

2. Prove by vector method that in any trangle ABC, $a = b\cos C + c\cos B$.

3. Prove by vector method that in any triangle $ABC, a^2 = b^2 + c^2 - 2bc\cos A$.

4. If $\vec{a} = 2\hat{i} + \hat{j}$, $\vec{b} = -\hat{i} + 2\vec{k}, \vec{c} = 2\hat{j} + \hat{k}$ find $\vec{a} \times (\vec{b} \times \vec{c})$ and also verify the formula $\vec{a} \times (\vec{b} \times \vec{c}) = (\vec{a}.\vec{c})\vec{b} - (\vec{a}.\vec{b})\vec{c}$

5. By vector method find the area of a triangle whose vertices are $A(2,-3,5)$, $B(30,7)$ and $C(4,0,6)$. Also find $\angle BAC$.

6. Prove by vector method that the diagonals of a rhombus are at right angles

7. It θ be the measure of angle between the vectors $\hat{i} - 3\hat{j} + \hat{k}$ and $\hat{i} + \hat{j} + \hat{k}$ then find the value of $\sin\theta$

8. Obtain the volume of the parallelopiped whose sides are vectors $\vec{a} = 2\hat{i} - 3\hat{j} + 4\hat{k}$, $\vec{b} = \hat{i} + 2\hat{j} - \hat{k}$ and $\vec{c} = 3\hat{i} - \hat{j} + 2\hat{k}$

9. If $\vec{a}, \vec{b}, \vec{c}$ are mutually perpendicular vector of equal magnitude, prove that $\vec{a} - \vec{b} - \vec{c}$ is equally inclined to $\vec{b}$ and $\vec{c}$

10. Prove that the four points with position vectors $4\hat{i} + 5\hat{j} + \hat{k}$, $-\hat{j} - \hat{k}, 3\hat{j} + 9\hat{j} + 4\hat{k}$ and $-4\hat{i} + 4\hat{j} + 4\hat{k}$ are coplanar.

11. Find a unit vector perpendicular to the following two vectors $\vec{a} = 2\hat{j} + 3\hat{j} + 6\hat{k}$ and $\vec{b} = 3\vec{i} - 6\vec{j} + 2\vec{k}$

12. Find the value of λ so that three vector $\vec{a} = 2\vec{j} - \vec{j} + \vec{k}$, $\vec{b} = \hat{i} + 2\hat{j} - 3\hat{k}$ and $\vec{c} = 3\hat{i} + \lambda\hat{j} + 5\hat{k}$ are coplanar.

Answers

1. Given that

$$\vec{a} = 2\hat{i} + \hat{k}$$

$$\vec{b} = \hat{i} + \hat{j} + \hat{k}$$

$$\vec{c} = 4\vec{j} - 3\vec{j} + 7\vec{k}$$

Let $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$

Given that $\vec{r} \times \vec{b} = \vec{c} \times \vec{b}$

$$\Rightarrow \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ x & y & z \\ 1 & 1 & 1 \end{vmatrix} = \begin{vmatrix} \hat{\hat{i}} & \hat{j} & \hat{k} \\ 4 & -3 & 7 \\ 1 & 1 & 1 \end{vmatrix}$$

$$\Rightarrow \hat{i}(y-z) - \hat{j}(x-z) + \hat{k}(x-y)$$

$$= \hat{i}(-3-7) - \hat{j}(4-7) + \hat{k}(4+3)$$

$$\Rightarrow \hat{i}(y-z) + \hat{j}(z-x) + \hat{k}(x-y)$$

$$= -10\hat{i} + 3\hat{j} + 7\hat{k}$$

$\Rightarrow y - z = -10$ .... (1)

$z - x = 3$ ....(2)

$x - y = 7$ ....(3)

Also given that $\vec{r}.\vec{a} = 0$

$\Rightarrow \left(x\hat{i} + yj + z\hat{k}\right).\left(2\hat{i} + \hat{k}\right) = 0$

$\Rightarrow 2x + z = 0$ .... (4)

Solving (1),(2),(3) & (4),

we get $x = -1, y = -8, z = 2$.

$\therefore \vec{r} = -\hat{i} - 8\hat{j} + 2\hat{k}$

8. Given that $\vec{a} = 2\vec{j} + \vec{j} + o\vec{k}$

$\vec{b} = -\hat{j} + o\hat{j} + 2\hat{k}$

$\vec{c} = o\hat{j} + 2\hat{j} + \hat{k}$.

$$\vec{b} \times \vec{c} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 0 & 2 \\ 0 & 2 & 1 \end{vmatrix}$$

$= -4\hat{i} + \hat{j} - 2\hat{k}$

$$\vec{a} \times \left(\vec{b} \times \vec{c}\right) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & 0 \\ -4 & 1 & -2 \end{vmatrix}$$

$= -2\hat{j} + 4\hat{j} + 6\hat{k}$ ....(1)

$\vec{a}.\vec{b} = \left(2\hat{i} + \hat{j} + o\hat{k}\right).\left(-\hat{i} + o\hat{j} + 2\hat{k}\right) = -2$

$\vec{a}.\vec{c} = \left(2\hat{i} + \hat{j} + o\hat{k}\right).\left(-\hat{i} + 2\hat{j} + \hat{k}\right) = 2$

$\left(\vec{a}.\vec{c}\right)\vec{b} - \left(\vec{a}.\vec{b}\right)\vec{c}$

$= 2\left(-\hat{j} + 2\hat{k}\right) - (-2)\left(2\hat{j} + \hat{k}\right)$

$= -2\hat{i} + 4\hat{j} + 6\hat{k}$ .... (2)

From (1) and (2) we see that $\vec{a} \times \left(\vec{b} \times \vec{c}\right) = \left(\vec{a}.\vec{c}\right)\vec{b} - \left(\vec{a}.\vec{b}\right)\vec{c}$.

5. Area of the $\Delta ABC = \dfrac{3\sqrt{3}}{2}$ sq. unit

$m\angle BAC = \cos^{-1}\left(\dfrac{13}{\sqrt{140}}\right)$

6. Let $ABCD$ be a rhombus.

Let $\overrightarrow{AB} = \vec{a}, \overrightarrow{AD} = \vec{b}$

$\overrightarrow{BC} = \overrightarrow{AD} = \vec{b}$

$\overrightarrow{AC} = \overrightarrow{AB} + \overrightarrow{BC} = \vec{a} + \vec{b}$

$\overrightarrow{BD} = \overrightarrow{AD} - \overrightarrow{AB} = \vec{b} - \vec{a}$

Since it is a rhombus, $\left|\vec{a}\right| = \left|\vec{b}\right|$

$\overrightarrow{AC}.\overrightarrow{BD} = \left(\vec{a} + \vec{b}\right).\left(\vec{b} - \vec{a}\right)$

$= \vec{b}^2 - \vec{a}^2$

$= \left|\vec{b}\right|^2 - \left|\vec{a}\right|^2 = 0$

$\Rightarrow AC$ is perpendicular to BD.

$\Rightarrow$ The diagonals of a rbombus are at right angles.

7. $\sin\theta = 4\sqrt{\dfrac{2}{33}}$

8. Volume of the parallelopiped

$\therefore [\vec{a}\ \vec{b}\ \vec{c}]$

$$= \begin{vmatrix} 2 & -3 & 4 \\ 1 & 2 & -1 \\ 3 & -1 & 2 \end{vmatrix} = 7 \text{ cubic unit}$$

$\left(\vec{a} \times \vec{b}\right) \times \vec{c} = 19\hat{i} + 31\hat{j} - 13\hat{k}$

9. $\left|\vec{a}\right| = \left|\vec{b}\right| = \left|\vec{c}\right|$

$\hat{a}.\hat{b} = \hat{b}.\hat{c} = \hat{c}.\hat{a} = 0$

Let $\hat{a} - \hat{b} - \hat{c}$ makes angles

$\alpha \,\&\, \beta$ with two vectors $\vec{b}$ abd $\vec{c}$.

$$\therefore \cos\alpha = \frac{\left(\vec{a} - \vec{b} - \vec{c}\right).\vec{b}}{\left|\vec{a} - \vec{b} - \vec{c}\right|\left|\vec{b}\right|} = \frac{-\left|\vec{b}\right|}{\left|\vec{a} - \vec{b} - \vec{c}\right|}$$

.... (1)

$$\cos\beta = -\frac{\left|\vec{c}\right|}{\left|\vec{a} - \vec{b} - \vec{c}\right|}$$

$$\Rightarrow \frac{-\left|\vec{b}\right|}{\left|\vec{a} - \vec{b} - \vec{c}\right|} = \frac{\left|\vec{c}\right|}{\left|\vec{a} - \vec{b} - \vec{c}\right|}$$

$\Rightarrow \cos\alpha = \cos\beta$

$\Rightarrow \alpha = \beta$

10. $\overrightarrow{OA} = 4\hat{i} + 5\hat{j} + \hat{k}$

$\overrightarrow{OB} = -\hat{j} - \hat{k}$

$\overrightarrow{OC} = 3\hat{i} + a\hat{j} + 4\hat{k}$

$\overrightarrow{OD} = -4\hat{i} + 4\hat{j} + 4\hat{k}$

$\therefore \overrightarrow{AB} = -4\hat{j} - 6\hat{j} - 2\hat{k}$

$\overrightarrow{AC} = -\hat{i} + 4\hat{j} + 3\hat{k}$

$\overrightarrow{AD} = --8\hat{i} - \hat{j} + 3\hat{k}$

$$\overrightarrow{AB}.\left(\overrightarrow{AC} \times \overrightarrow{AD}\right) = \begin{vmatrix} -4 & -6 & -2 \\ -1 & 4 & 3 \\ -8 & -1 & 3 \end{vmatrix} = 0$$

$\Rightarrow \overrightarrow{AB}, \overrightarrow{AC}, \overrightarrow{AD}$ are coplanar

$\Rightarrow$ The points A,B,C,D are coplanar

11. Unit vector perpendicular to $\vec{a}$ and $\vec{b}$

$$= \frac{\vec{a} \times \vec{b}}{\left|\vec{a} \times \vec{b}\right|} = \frac{1}{7}\left(6\hat{i} + 2\hat{j} - 3k\right)$$

12. $\lambda = -4$

CHAPTER - 13

THREE- DIMENSIONAL GEOMETRY

Multiple Choice Questions (MCQ)

Group-A

A. Choose the correct answer from the given choices:

1. What is the image of the point $(6,3,-4)$ with respect ot yz plane?

(a) (6,3,4) (b) (6,-3,-4)

(c) (-6,3,-4) (d) (-6,3,4)

2. If the direction cosines of a straight line are $\left\langle \frac{2}{7},\frac{3}{7},\frac{k}{7}\right\rangle$ then what is the value of k?

(a) ± 4 (b) $\neq 6$

(c) ± 8 (d) ± 10

3. If a line makes an angle 90^0 with $x-axis$ 60^0 with y-axis then what angle it-makes with z-axis?

(a) 30^0 (b) 45^0

(c) 60^0 (d) 90^0

4. What is the value of k for which the line $\frac{x-2}{2}=\frac{1-y}{k}=\frac{z-1}{4}$ is parallel to the plane $2x+6y+3z-4=0$?

(a) 1 (b) 2

(c) 3 (d) 4

5. What is the equation of the plane passing through (1,1,2) and parallel to the plane $x+y+z-1=0$

(a) $x+y+z=0$

(b) $x+y+2z-1=0$

(c) $x+y+z=2$

(d) $x+y+z=4$

6. What is the equation of the plane passing through (1,-2,3) and perpendicular to $y-axis$?

(a) $y=-2$ (b) $y=2$

(c) $y=-4$ (d) $y=4$

7. The equation of the plane perpendicular to z-axis and passing through (1,-2,4) is __________

(a) $x-1=0$

(b) $y+2=0$

(c) $z-4=0$

(d) $x+y+z-3=0$

8. The distance between parallel planes $2x-3y+6z=0$ and $4x-6y+12z-5=0$ is _____

(a) $\frac{1}{2}$ (b) $\frac{1}{7}$

(c) $\frac{4}{7}$ (d) $\frac{6}{7}$

9. A plane whose normal has direction ratios $\langle 3,-2,k\rangle$ is parallel to the line joining $(-1,1,-4)$ and $(5,6,-2)$. Then the value of k is ________

10. The symmetircal form of the line $2x+z-4=0=2y+z$ is ________

(a) $\frac{x-2}{1}=\frac{y-0}{1}=\frac{z-0}{-2}$

(b) $\frac{x-2}{2}=\frac{y-0}{3}=\frac{z-0}{2}$

(c) $\frac{x-2}{-3}=\frac{y-0}{2}=\frac{z-0}{5}$

(d) none of these

11. What is the distance between the parallel planes $2x-y+3z=4$ and $2x-y+3z-18=0$?

(a) $\sqrt{12}$ (b) $\sqrt{14}$

(c) $\sqrt{16}$ (d) $\sqrt{20}$

12. What is the equation of $x-axis$ in symmetric form?

(a) $\frac{x-0}{0}=\frac{y-0}{1}=\frac{z-0}{0}$

(b) $\frac{x-0}{1}=\frac{y-0}{0}=\frac{z-0}{0}$

(c) $\frac{x-0}{0}=\frac{y-0}{0}=\frac{z-0}{1}$

(d) none of these

13. What is the direction cosines of a line passing through $(0,0,0)$ and $(1,2,3)$

(a) $\left\langle \frac{2}{\sqrt{14}},\frac{3}{\sqrt{14}},\frac{1}{\sqrt{14}} \right\rangle$

(b) $\left\langle \frac{3}{\sqrt{14}},\frac{2}{\sqrt{14}},\frac{1}{\sqrt{14}} \right\rangle$

(c) $\left\langle \frac{1}{\sqrt{14}},\frac{2}{\sqrt{14}},\frac{3}{\sqrt{14}} \right\rangle$

(d) none of these

14. What is the equation to the plane perpendicular to $y-axis$, and passing through point $(0,-2,0)$?

(a) $y+1=0$ (b) $y+2=0$

(c) $y+5=0$ (d) $y+6=0$

15. What is the projection of the line segment joining $(1,3,-1)$ and $(3,2,4)$ on $z-axis$?

(a) 4 (b) 5

(c) 6 (d) 7

16. If α,β,γ are the angles which a directed line makes with the positive direction of the coordinate axes, then what is the vlaue of $\sin^2\alpha+\sin^2\beta+\sin^2\gamma$?

(a) 1 (b) 2

(c) 3 (d) 4

17. What is the equation of the line passing through the point $(4,-6,1)$ and parallel to the line $\frac{x-1}{1}=\frac{y+2}{3}=\frac{z-1}{-1}$

(a) $\frac{x-4}{2}=\frac{y+6}{-1}=\frac{z-1}{-1}$

(b) $\frac{x-4}{3}=\frac{y+6}{-1}=\frac{z-1}{2}$

(c) $\frac{x-4}{1}=\frac{y+6}{3}=\frac{z-1}{-1}$

(d) none of these

18. What is the image of the point $(-2,3,-5)$ with respect to the $zx-$plane?

(a) $(2,3,-5)$ (b) $(-2,-3,-5)$

(c) $(3,-2,-5)$ (d) $(2,3,5)$

19. What is the value of k for which the line $\frac{x-2}{3}=\frac{1-y}{k}=\frac{z-1}{4}$ is parallel to the plane $2x+6y+3z-4=0$

(a) 1 (b) 2

(c) 3 (d) 4

20. What are the direction cosines of the line perpendicular to the plane $3x-2y-2z+1=0$

(a) $\left\langle \frac{3}{\sqrt{15}}, \frac{-2}{\sqrt{15}}, \frac{-2}{\sqrt{15}} \right\rangle$

(b) $\left\langle \frac{3}{\sqrt{11}}, \frac{-2}{\sqrt{11}}, \frac{-2}{\sqrt{11}} \right\rangle$

(c) $\left\langle \frac{3}{\sqrt{17}}, \frac{-2}{\sqrt{17}}, \frac{-2}{\sqrt{17}} \right\rangle$

(d) none of these

Answers

1. (c)	2. (b)	3. (a)	4. (c)
5. (d)	6. (a)	7. (c)	8. (a)
9. (b)	10. (a)	11. (b)	12. (b)
13. (c)	14. (b)	15. (b)	16. (b)
17. (c)	18. (b)	19. (c)	20. (c)

B. Fill in the blanks:

1. The equation of the line passing through $(-3,1,2)$ and perpendicular to the plane $2y-z=3$ is _________

2. The direction cosines of $z-axis$ are ______.

3. The distance of the point $(4,5,-3)$ from $y-$axis is _______

4. The equation of the plane that passes through y-axis and z-axis is ______

5. The distance of the point of intersection of the plane $ax+by+cz+d=0$ and $z-axis$ from the origin is _______

6. The projection of the line segment joining $(1,3,-1)$ and $(3,2,4)$ on z-axis is _______

7. The direction cosines of the line through $(1,-1,1)$ and $(2,-5,-3)$ is ______

8. The derection cosines of the line $x=y=z$ are ________

9. The equation ofthe plane passing through the point $(3,1,2)$ and parallel to the plane $2x+2y+2z+1=0$ is ________

10. The equation of the line through the point $(2,3,5)$ and parallel to the line $\frac{x-3}{2}=\frac{y+1}{1}=\frac{z+7}{4}$ is _______

11. The distance between the parallel planes $x-y-z+1=0$ and $y+z-x+1=0$ is ________

12. The lines $x=ay+b, z=cy+d$ and $x=a^1y+b^1, z=c^1y+d^1$ are perpendicular of $a\,a^1+c\,c^1$ equal to ______.

13. The direction ratios of the line $6x-2=3y+1=2y-2$ is _____

14. The equation of the lines passing through $(2,-1,3)$ and $(4,2,1)$ is _____

15. The measure of the angle between two main diagonals of a cube is ______

16. If the line $\frac{x-4}{1}=\frac{y-2}{1}=\frac{z-k}{2}$ line on the plane $2x-4y+z$ then the value of k is _________

17. The image of the point $(1,3,4)$ in the plane $2x-y+z+3=0$ is ________

18. The equation of $yz-$plane is ________

19. The equation of the line $2x+z-4=0=2y+z$ in symmetrical form is ______

20. The distance between the parallel planes $2x-y+3z=4$ and $2x-y+3z-18=0$ is __________

21. The perpenduclar distance of the point (2,1,0) from the plane 2x+y+2z+5=0 is _________.

Answers

1. $\frac{x+3}{0}=\frac{y-1}{2}=\frac{z-2}{-1}$
2. $<0,0.1>$
3. 5
4. $x=0$
5. $-\frac{d}{c}$
6. 5
7. $\left\langle\frac{1}{\sqrt{33}},\frac{-4}{\sqrt{33}},\frac{-4}{\sqrt{33}}\right\rangle$
8. $\left\langle\frac{1}{\sqrt{3}},\frac{1}{\sqrt{3}},\frac{1}{\sqrt{3}}\right\rangle$
9. $x+y+z-6=0$
10. $\frac{x-2}{2}=\frac{y-3}{1}=\frac{z-5}{4}$
11. $\frac{2}{\sqrt{3}}$
12. -1
13. $\langle 1,2,3\rangle$
14. $\frac{x-2}{2}=\frac{y+1}{3}=\frac{z-3}{-2}$
15. $\cos^{-1}\frac{1}{3}$
16. 7
17. $(-3,5,2)$
18. $x=0$
19. $\frac{x-2}{1}=\frac{y-0}{1}=\frac{z-0}{-2}$
20. $\frac{4}{\sqrt{14}}$

C. Answer in one word

1. What is the equation of x-axis is symmetric form?
2. What is the equation of the plane through (0,0,0) perpendicular to the line joining (0,0,1) and (0,0,-1)?
3. What is the distance between the parallel planes $x-y-z+1=0$ and $y+z-x+1=0$?
4. If the planes $2x+y+kz-1=0$ and $kx-y+z+2=0$ are pependicular then k=?
5. What is the angle between the planes $x+y=0$ and $y+z=1$?
6. What is the distance between the planes $2x-3y+6z+1=0$ and $4x-6y+12z-5=0$?
7. The angle between the planes $x+y+1=0$ and $y+z+1=0$ is ?
8. What is the equation of the plane passing through (1,1,2) and parallel to $x+y+z=1$?
9. What is the distance between parallel planes $2x-3y+6z+1=0$ and $4x-6y+12z-5=0$?
10. What is the equation of the plane passing through (1,0,0), (0,1,0) and (0,0,1)?
11. If the lines $x=ay+b, z=cy+d$ and are perpendicular then what is the value of aa^1+cc^1?
12. What is the symmetricel form of $y-axis$.
13. What is the equation of the line passing through (-1,0,1) and perpendicular to the plane $x+2y+1=0$?
14. What is the vector equation of thel ine which is passing through the point (5,-2,4) and parallel to the vector $2\hat{i}-\hat{j}+3\hat{k}$.

15. What is the vector equation of al ine passing through the point (1,2,3) and parall to the line whose equation is $\frac{x-3}{2}=\frac{y+1}{7}=\frac{z-2}{-3}$

16. If l,m,n be the direction cosines of a line which is perpendicular to the plane $x-3y+2z+1=0$ then what relation we get between l,m,n and $1,-3,2$?

17. If the line $\frac{x-3}{2}=\frac{y+k}{-1}=\frac{z+1}{-5}$ line on the plane $2x-y+z-7=0$ then $k=$?

18. What is the image of the point (3,-2,1) in the plane $3x-y+4z=2$?

Answers

1. $\frac{x-0}{1}=\frac{y-0}{0}=\frac{z-0}{0}$
2. $z=0$
3. $\frac{2}{\sqrt{3}}$
4. $k=\frac{1}{3}$
5. $\frac{\pi}{3}$
6. $\frac{1}{2}$
7. $\frac{\pi}{3}$
8. $x+y+z=4$
9. $\frac{1}{2}$
10. $x+y+z=1$
11. -1
12. $\frac{x}{0}=\frac{y}{1}=\frac{z}{0}$
13. $\frac{x+1}{1}=\frac{y-0}{2}=\frac{z-1}{0}$
14. $\vec{r}=4\hat{i}-2\hat{j}+4\hat{k}+t\left(2\hat{i}-\hat{j}+3\hat{k}\right)$
15. $\vec{r}=2\hat{i}-\hat{j}+\hat{k}+t\left(2\hat{i}-\hat{j}+3\hat{k}\right)$
16. $\frac{l}{1}=\frac{m}{-2}=\frac{n}{2}$
17. $k=2$
18. $(0,-1,-3)$

D. Answer in one sentence:

1. What are the skew lines.

2. Difine the shortest distance between two lines in the space.

3. Find the equation of the line through the point (2,3,5) and parallel to hte line $\frac{x-3}{2}=\frac{y+1}{1}=\frac{z+7}{4}$.

4. Write the equation of the plane passing through the point (3,1,2) and parallel to the plane $2x+2y+2z+1=0$.

5. Write down the direction ratios of the line $2x=3y=4z$.

6. Write the direction cosines of the line $x=y=z$.

7. What are the direction cosenes of the line through (1,-1,1) and (2,-5,-3)?

8. What are the projection of the line segment joing (1,3,-1) and (3,2,4) on $z-$ axis?

9. Write the distance of the point of intersection of the plane $ax+by+cz+d=0$ and z-axis from the origin.

10. Write the equaion to the plane that passes through y-axis and z-axis.

11. What is the distance of the point (4,5,-3) from y-axis?

12. Write the direction cosines of z-axis.

13. Write the equation of the line passing through (-3,1,2) and perpendicular to the plane $2y-z=3$

14. Write the angle between the planes $3z-5y+2z-8=0$ and $2x+4y+7z+16=0$

15. What are the ratio in which the line segment joining the points (1,2,-2) and (4,3,4) is divided by the $xy-$ plane.

16. Write are the direction cosines ofthe line perpendicular to the plane $3x-2y-2z+1=0$

17. Write the equation of the plane with intercept on axes 1,-1,3.

18. Find the value of k for which the line $\frac{x-2}{3}=\frac{1-y}{k}=\frac{z-1}{4}$ is parallel to the plane $2x+6y+3z-4=0$

Answers

1. Two lines in space are said to be skew lines if they are neither parallel nor intersect.

2. The shortest distance between two lines l_1 and l_2 is the distance PQ between two points $P\,\&\,Q$ where PQ is perpendicular to both l_1 and l_2.

3. The equation of the line passing through the point (2,3,5) and parallel to the line $\frac{x-3}{2}=\frac{y-1}{1}=\frac{z+7}{4}$ is

$$\frac{x-2}{2}=\frac{y-3}{1}=\frac{z-5}{4}$$

4. The equation of the plane passing through the point (3,1,2) and parallel to the plane $2x+2y+2z+1=0$ is $2x+2y+2z-12=0$

5. The given line is $2x=3y=4z$

$$\Rightarrow \frac{x}{1/2}=\frac{y}{1/3}=\frac{2}{1/4}$$

The direction ratios of the line are $\left\langle \frac{1}{2},\frac{1}{3},\frac{1}{4} \right\rangle$

6. The direction cosines of the line $x=y=z$ are $\left\langle \frac{1}{\sqrt{3}},\frac{1}{\sqrt{3}},\frac{1}{\sqrt{3}} \right\rangle$.

7. The direction cosines of the line through (1,-1,1) and (2,-5,-3) are $\left\langle \frac{1}{\sqrt{33}},\frac{-4}{\sqrt{33}},\frac{-4}{\sqrt{33}} \right\rangle$.

8. The direction cosenes of z-axis are (0,0,1). The projection of the line segment joinig (1,3,-1) and (3,2,4) on z-axis is

$$0(3-1)+0(2-3)+1(4+1)=5$$

9. Required distance from the orign $=-\frac{d}{c}$

10. Equation of the plane that passes through y-axis and z-axis is $x=0$

11. The distance of the point (4,5,-3) from $y-axis\ is\ \sqrt{4^2+(3)^2}=\sqrt{16+9}=\sqrt{25}=5$

12. The direction cosines of z-axis are $(0,0,1)$.

13. The equation of the line passing through (-3,1,2) and perpendicular to the point $2y-z=3$ is $\frac{x+3}{0}=\frac{y-1}{2}=\frac{z-2}{-1}$.

14. The angle between the planes $3x-5y+2z-8=0$ and $2x+4y+7z+16=0$.

15. The line divides $xy-$plane in the ratio $1:2$.

16. The direction cosines of the normal are $\left\langle \frac{3}{\sqrt{17}},\frac{-2}{\sqrt{17}},\frac{-2}{\sqrt{17}} \right\rangle$.

17. The equation of the plane is $x-y+\frac{z}{3}=1$.

18. The line is $\frac{x-2}{3}=\frac{y-1}{-k}=\frac{z-1}{4}$

the line in parallel to the plans $\therefore 2.3+6(-k)+3.4=0$

$$\Rightarrow k=3$$

GROUP-B

Short Type (Questions & Answers)

1. The projection of a line segment $\overline{OP}$ through the origin O on the coordinatcs axes are 6,2,3. Find the length of the line segment $\overline{OP}$ and its direction cosines.

2. Find the equation of the plane passing through the point (-1,3,2) and perpendicular to the planes $x+2y+2z=5$ and $3x+3y+2z=8$.

3. Find the locus of the point which are equidistant from the points (1,2,3) and (3,2,-1).

4. Find the equation of the plane through the points (1,2,-3), (2,3,-4) and perpendicular to the plane $x+y+z+1=0$

5. Find the perpendicular distance of the point (-1,3,9) from the line $\frac{x-13}{5}=\frac{y+8}{-8}=\frac{z-31}{1}$.

6. Prove the measure of the angle between two main diagonals of a cube is $\cos^{-1}\frac{1}{3}$

7. Find the equation of the plane passing through the intersection of the planes $x+2y+3y-4=0$ and $2x+y-z+3=0$. and also perpendicular to the plane $2x-y+2z+3=0$

8. If the point $(1,y,z)$ lines on the straight line through (3,2,-1) and(-4,6,3) then find y and z.

9. Find the coordinates of the point of intersection of the line $\frac{x-1}{1}=\frac{y+2}{3}=\frac{z-1}{-1}$ and the plane $2x+y+z=9$.

10. Find the coordinatis of lthe point at which the perpendicular from the origin meet the line joining the points (-9,4,5) and (11,0,-1)

11. Find the image of the point (-2,0,3) with respect to the plane $y=3$

12. Find the point where the line $\frac{x-2}{1}=\frac{y}{-1}=\frac{z-1}{2}$ intersect the plane $2x+y+z=2$.

Hints & Solutions

1. The projection of $\overline{OP}$ on the coordinate axes are 6,2,3.

The coordinats of p are (6,2,3)

Length of $OP=\sqrt{6^2+2^2+3^2}$

$=\sqrt{36+4+9}=\sqrt{49}=7$.

The direction cosisnes of OP are $\left\langle\frac{6}{7},\frac{2}{7},\frac{3}{7}\right\rangle$.

2. The given point is (-1,3,2).

The equation of the plane passing through (-1,3,2) is

$a(x+1)+b(y-3)+c(z-2)=0)$ ......(1)

The given planes are

$x+2y+2z=5$ ....(2)

$3x+3y\times 2z=8$ ....(3)

Since the plane (1) is perpendicular of (2) and (3)

$$a.1+b.2+c.2=0$$

$$a.3+b.3+c.2=0$$

$$\Rightarrow a+2b+2c=0$$

$$3a+3b+2c=0$$

By cross multiplicaiton, we get

$$\frac{a}{2.2-3.2}=\frac{b}{2.3-1.2}=\frac{c}{1.3-3.2}$$

$$\Rightarrow \frac{a}{-2}=\frac{b}{4}=\frac{c}{-3}=k(say)$$

$$\Rightarrow a=-2k, b=4k\ c=-3k$$

From (1) the required plane is $-2k(x+1)+4k(y-3)-3k(2-2)=0$

$$\Rightarrow 2x-4y+32+8=0$$

3. $x-2z=0$

4. The equation of the plane passing throw (1,2,-3) is

$$a(x-1)+b(y-2)+c(z+3)=0 \text{ ....(1)}$$

Since it is passing throug (2,3,-4) is $a(2-1)+b(3-2)+C(-4+3)=0$

$$\Rightarrow a+b-c=0 \qquad \text{..........(2)}$$

Since the plane (1) is perpendicular to $x+y+z+1=0$, we have $a.1+b.1+c.1=0$

$$\Rightarrow a+b+c=0 \qquad \text{...........(3)}$$

From (2) and (3)

$$\frac{a}{-1}=\frac{b}{0}=\frac{c}{1}$$

The required plane is

$$-(x-1)+0(y-2)+1(2+3)=0$$

$$\Rightarrow x-z-4=0$$

5. The given line is

$$\frac{x-13}{5}=\frac{y+8}{-8}=\frac{z-31}{1}=r(say) \text{ ....(1)}$$

Let P be the point (-1,3,9)

let PM be the perpen-

dicular from P to the line (1)

Any point on the line (1) is $(5r+13,-8r-8,r+31)$.

The coordinates of M are $(5r+13,-8r-8,r+31)$ for some value of r.

The d.r.s of PM are

$$\langle 5r\_13-(-1),-8r-8-3,r+31-9\rangle$$

$$=\langle 5r+14,-8r-11,r+22\rangle$$

Since PM is perpendicular to the line (1) we have

$$5(5r+14)+(-8)(-8r-11)+1(r+22)=0$$

$$\Rightarrow r=-2$$

The coordinates of M are $(-10+13,16-8,-2+31)=(3,8,29)$

$$PM=\sqrt{3-(-1)^2+(8-3)^2(29-9)^2}$$

$$=\sqrt{16+25+400}=\sqrt{441}=21$$

7. $11x+4y-9z+3y=0$

8. Let A and B be two given points whose coordinates are (3,2,-1) and (-4,6,3) respectively.

Let C be the point $(1,y,z)$.

The d.rs of AB are

$< -4-3, 6-2, 3-(-1) >$

$= < -7, 4, 4 >$

The d.rs of AC are $< 1-3, y-2, z-(-1) >$

$= < -2, y-2, z+1 >$

Since A,B,C are collinear, we have

$$\frac{-2}{-7} = \frac{y-2}{4} = \frac{z+1}{4}$$

$$\Rightarrow \frac{y-2}{4} = \frac{z+1}{4} = \frac{2}{7}$$

$$\therefore \frac{y-2}{4} = \frac{2}{7} \text{ and } \frac{z+1}{4} = \frac{2}{7}$$

$$\Rightarrow y = \frac{22}{7}, z = \frac{1}{7}$$

9. $(3, 4, -1)$

10. Let A and B be the given points whose coordinates are (-9,4,5) and (11,0,-1) respectively.

Let OC be the perpendicular from the origin O to AB.

Let C divides AB in the ratio $k:1$

The coordinates of C are $\left(\frac{11k-9}{k+1}, \frac{4}{k+1}, \frac{-k+5}{k+1}\right)$

The direction ratios of OC are

$$\left\langle \frac{11k-9}{k+1}, \frac{4}{k+1}, \frac{-k+5}{k+1} \right\rangle$$

The direction ratios of AB are

$< 11-(-a), 0-4, -1-5 >$

$= < 20, -4, -6 >$

Since OC is perpendicular to AB, we have

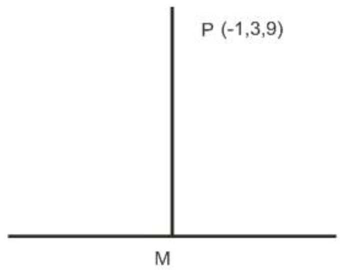
$$20.\left(\frac{11k-9}{k+1}\right) + (-4)\left(\frac{4}{k+1}\right) + (-6)$$

$$\left(\frac{-k+5}{k+1}\right) = 0$$

$$\Rightarrow (k = 1)$$

Then the coordinates of C are (1,2,2).

11. The coordinates of image are (-2,6,3)



12. The coordinates of the point of intersection is (1,1,-1)

GROUP-C

Long Type questions.

1. Find the shortest distance between the following two lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{x-2}{3} = \frac{y-4}{4} = \frac{z-5}{5}$. Find the also the quations of the line of shortest distance.

2. Find the perpendicular distance from the point $(7, 2, 4)$ to the plane passing through three

points A (2,5,-3), B(-2,-3,5) and C(5,3,-3).

3. Prove that the lines $\frac{x+3}{2}=\frac{y+5}{2}=\frac{z-7}{-3}$ and $\frac{x+1}{4}=\frac{y+1}{5}=\frac{z+1}{-1}$ are coplanar. Find the equation of the plane conaining them

4. Prove that the four point $(0,4,3)(-1,-5,-3)$, $(-2,-2,1)$ and $(1,1,-1)$ one plane. Find the equation of the plane

5. Show that the lines $\frac{x-4}{1}=\frac{y+3}{-4}=\frac{z+1}{7}$ and $\frac{x-1}{2}=\frac{y+1}{-3}=\frac{z+10}{8}$ are coplanar Find their point of intersection and equation of the plane in which they lie.

6. Find the distance from the line $\frac{x}{2}=\frac{y}{3}=\frac{z}{1}$ to the point $(4,5,2)$.

7. A variable plane is at a constant distance 3r from the origin and meets the axes in A,B and C. Show that the loucs of the centrod at the triangle ABC is $x^{-2}+y^{-2}+z^{-2}=r^{-2}$

8. If the edge of a rectangular parllelopiped are of length a,b,c then the angle between four diagonals are.

$$\cos^{-1}\left(\frac{\pm a^2 \pm b^2 \pm c^2}{a^2+b^2+c^2}\right)$$

9. If l_1, m_1, n_1 and l_2, m_2, n_2 be the direction cosines of two mutualy perpendicular lines, show that the direction cosines of the line perpendicular to both of them are $m_1, n_2 - m_2 n_1, n_1, n_1 l_2 - n_2 l_1, l_1 m_2 - l_2 m_2$

10. Find the image of the point (2,3,4) with respect to the plane $x-y+2z=4$ obtain the foot of the perpendicular from P on the plane and the corresponding perpendicular distance.

11. Prove that the straight lines whose direction cosines are connected by the relations

$l+2m+3m=0$

$3lm-4\ln+mn=0$ are perpendicular to each other.

12. Find the equation of the plane through the points (2,2,1) and (9,3,6) and perpendicular to the plane $2x+6y+6z-1=0$.

13. A plane through the point (-1,3,0) is perpendicular to both the planes $x+2y+2z-5=0$ and $3x+3y+2z-8=0$.

14. Find the equation of the plane passing through the point (1,-1,0) and the line of intersection of the planes $2x-y+2z+3=0$ $3x-2y+6z+8=0$

15. Find the point of intersection of the line $\frac{x-2}{3}=\frac{y+1}{4}=\frac{z-2}{12}$ and the planes $x-y+z=5$.

Hints & Solutions

1. Two given lines are

$$\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4} \text{ ....(1)}$$

$$\frac{x-2}{3}=\frac{y-4}{4}=\frac{z-5}{5} \text{ ....(2)}$$

The shortest distance between the above two lines is

$$S.D=\frac{\begin{vmatrix} x_2-x_1 & y_2-y & z_2-z_1 \\ l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \end{vmatrix}}{\sqrt{(m_1n_2-m_2l_1)^2+(m_2n_1-m_1n_2)^2+(l_1m_2-l_2m_1)^2}}$$

Here $\begin{vmatrix} x_2-x_1 & y_2-y_2 & z_2-z_1 \\ l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \end{vmatrix}$

$$=\begin{vmatrix} 2-1 & 4-2 & 5-3 \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{vmatrix}$$

$$=\begin{vmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{vmatrix}$$

$$=1(15-16)-2(10-12)+2(8-9)$$

$$=-1-2(-2)+2(-1)$$

$$=-1+4-2=1$$

Again

$$\sqrt{(m_1n_2-m_2n_2)^2+(m_2n_1-m_1n_2)^2+(l_1m_2-l_2m_1)^2}$$

$$=\sqrt{(3.5-4.4)^2+(4.3-2.5)^2+(2.4-3.3)^2}$$

$$=\sqrt{1+4+1}=\sqrt{6}$$

Required shortest distance $=\frac{1}{\sqrt{6}}=\frac{1}{6}\sqrt{6}$ unit

Two given lines are

$$\frac{x-1}{2}=\frac{y-2}{3}-\frac{z-3}{4}-r_1\,(say)$$

$$\frac{x-2}{3}=\frac{y-4}{4}=\frac{z-5}{5}=r_2$$

Let PQ be the shortest distance

Any point on the line (1) is $(2r_1+1,3r_1+2,4r_1+3)$ for some value any point on the line (2) is $(3r_2+2,4r_2+4,5r_2+5)$.

The coordinates of Q are also $(3r_2+2,4r_2+4,5r_2+5)$ for same value r_2.

The direction ratios of PQ are

$$<3r_2+2-2r_1-1,4r_2+4-3r_1-2,5r_2+5$$
$$-4r_1-3>$$

$$=<3r_2-2r_1+1,4r_2-3r_1+2,5r_2-4r_1+2>$$

The direction ratios of the lines (1) and (2) are $<2,34>$ and $<3,4,5>$ respectively.

Since PQ is perpendicular to the gives(1) and (2).

$$(3r_2-2r+1)+3(4r_2-3r_1+2)+4$$
$$(5r_2-4r_1+2)=0$$

and $3(3r_2-2r_1+1)+4(4r_2-3r_1+2)+5$
$(5r_2-4r_1+2)=0.$

$$\Rightarrow 38r_2-29r_1+16=0 \qquad ....(4)$$

$$50\ r_2-38r_1+21=0 \qquad ....(5)$$

solving (4) and (5), we get

$$\frac{r_2}{(-29)21+38\times16}=\frac{r_1}{16.50-38.21}$$

$$=\frac{1}{-38.38+29\times50}$$

$$\Rightarrow\frac{r_2}{-1}=\frac{r_1}{2}=\frac{1}{36}$$

$$\Rightarrow r_1=-\frac{1}{36},r_2=\frac{2}{36}=\frac{1}{18}$$

The direction ratio of PQ are

$$<3\left(-\frac{1}{36}\right)-2.\frac{1}{18}+1,4\left(-\frac{1}{36}\right)-3\frac{1}{18}+2,\ 5\left(-\frac{1}{36}\right)-4.\frac{1}{18}+2>$$

$$=<-\frac{1}{12}-\frac{1}{9}+1,\frac{-1}{9}-\frac{1}{6}+2,\frac{-5}{36}+\frac{2}{9}-\frac{2}{9}+2>$$

$$=<\frac{29}{36},\frac{31}{18},\frac{59}{36}>$$

Also the coordinates of pare

$$\left(2.\frac{1}{18}+1,3.\frac{1}{18}+2,4.\frac{1}{18}+3\right)$$

$$=\left(\frac{1}{9}+1,\frac{1}{6}+2,\frac{2}{9}+3\right)=\left(\frac{10}{9},\frac{13}{6},\frac{29}{9}\right)$$

The shortest distance PQ between two lines (1) and (2) passes through the point $P\left(\frac{10}{9},\frac{13}{6},\frac{29}{9}\right)$ where direction retios are $\left(\frac{29}{36},\frac{31}{18},\frac{59}{36}\right)$

The equation of the shortest distance PQ is

$$\frac{x-\frac{10}{9}}{\frac{29}{36}}=\frac{y-\frac{13}{6}}{\frac{31}{18}}=\frac{z-\frac{29}{9}}{\frac{59}{36}}$$

2. Three given points are $A(2,5,-3),B(-2,-3,5)$ and $C(5,3,-3)$

Equation of the plane passing throgh the above three points is

$$\begin{vmatrix} x-2 & y-5 & z+3 \\ -2-2 & -3-5 & 5+3 \\ 5-2 & 3-5 & -3+3 \end{vmatrix}=0$$

$$\Rightarrow \begin{vmatrix} x-2 & y-5 & z+3 \\ -4 & -8 & 8 \\ 3 & -2 & 0 \end{vmatrix}=0$$

$$\Rightarrow 2x+3y+4z-7=0 \quad ....(1)$$

Let P be the point $(7,2,4)$

The perpendicular distance from the point $P(7,2,4)$ to the plane (1)

$$=\frac{2.7+3.3+4.4-7}{\sqrt{2^2+3^2+4^2}}$$

$$=\frac{14+6+16-7}{\sqrt{4+9+16}}=\frac{29}{\sqrt{29}}=\sqrt{29} \text{ unit}$$

3. The given lines are

$$\frac{x+3}{2}=\frac{y+5}{3}=\frac{z-7}{-3}=r_1(say) \quad .... (1)$$

$$\frac{x+1}{4}=\frac{y+1}{5}=\frac{z+1}{-1}=r_2(say) \quad ....(2)$$

Any point on the line (1) is

$$(2r_1-3,3r_1-5,-3r_1+7)$$

Any point on the line (2) is

$$(4r_2-1,5r_2-1,-r_2-1)$$

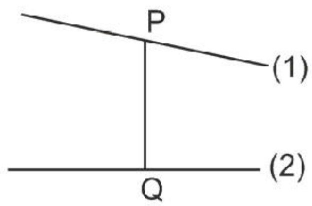
If the lines are coplanar, then they must intersect.

$\therefore$ At the point of intersection

$$2r_1-3=4r_2-1$$

$$3r_1-5=5r_2-1$$

$$-3r_1+7=-r_2-1$$



$\Rightarrow r_1 - 4r_2 - 2 = 0$ .... (3)

$3r_1 - 5r_2 - 4 = 0$ ....(4)

$3r_1 - r_2 - 8 = 0$ ....(5)

Solving (3) and (4) , we get

$r_1 = 3$ and $r_2 = 1$

These values of r_1 and r_2 satifies the equation (5) .

So the lines are coplanar.

Equation of the plane containing these two lines is

$$\begin{vmatrix} x+3 & y+5 & z-7 \\ 2 & 3 & -3 \\ 4 & 5 & -1 \end{vmatrix} = 0$$

$\Rightarrow 6x - 5y - 3 = 0$

4. Four given points are $(0,4,3),(-1,-5,-3),(-2,-2,1)$ and $(1,1,-1)$

$$\begin{vmatrix} x_4 - x_1 & y_4 + y_1 & z_4 - z_1 \\ x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ x_3 - x_1 & y_3 - y_1 & z_3 - z_1 \end{vmatrix}$$

$$= \begin{vmatrix} 1-0 & 1-4 & -1-3 \\ -1-0 & -5-4 & -1-3 \\ -2-0 & -2-4 & 1-3 \end{vmatrix}$$

$$= \begin{vmatrix} 1 & -3 & -4 \\ -1 & -9 & -6 \\ -2 & -6 & -2 \end{vmatrix}$$

$= (18-36) - (-3) - 4(-12)$

$= 18 - 30 + 48 = 0$

So the four points are coplanar.

Equation of the plane containing the first three points is

$$\begin{vmatrix} x - x_1 & y - y_1 & z - z_1 \\ x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ x_3 - x_1 & y_3 - y_1 & z_3 - z_1 \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} x-0 & y-4 & z-3 \\ -1-0 & -5-4 & -3-3 \\ -2-0 & -2-4 & 1-3 \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} x & y-4 & z-3 \\ -1 & -9 & -6 \\ -2 & -6 & -2 \end{vmatrix} = 0$$

$\Rightarrow x(18-36) - (y-4)(2-12) + (2-3)$
$(6-18) = 0$

$\Rightarrow 9x - 10y + 12z + 4 = 0$

5. Two given lines are

$\frac{x-4}{1}, \frac{y+3}{-4} = \frac{z+1}{7}$ .... (1)

$\frac{x-1}{2} = \frac{y+1}{-3} = \frac{z+10}{8}$ ....(2)

The points on the line (1) and (2) are $(4,-3,-1)$ and $(1,-1,-10)$ respecitively

$\therefore$ Two lines (1) and (2) are coplanar

$$\text{if} \begin{vmatrix} x_2 - x_1 & 4_2 - y_1 & z_2 - z_1 \\ l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \end{vmatrix} = 0$$

$$\text{Here} \begin{vmatrix} 1-4 & -1+3 & -10+1 \\ 1 & -4 & 7 \\ 2 & -3 & 8 \end{vmatrix}$$

$$= \begin{vmatrix} -3 & 2 & -9 \\ 1 & -4 & 7 \\ 2 & -3 & 8 \end{vmatrix}$$

$= -3(-32+21) - 2(8-14) - 9(-3+8)$

$= 33 + 12 - 45 = 0$

So two given lines are colanar.

Equation (1) and (2) can be written as

$$\frac{x-y}{1} = \frac{y+3}{-4} = \frac{z+1}{7} = r_1 (say)$$

$$\frac{x-1}{2} = \frac{y+1}{-3} = \frac{z+10}{8} = r_2 (say)$$

Any point on the line (1) is $(r_1 + 4, -4r_1 - 3, 7r_1 - 1)$

Any point on the line is $(2r_2 + 1, -3, r_2 - 1, 8r_2 - 10)$

At the point of intersection,

$$r_1 + y = 2r_2 + 1$$

$$-4r_1 - 3 = -r_2 - 1$$

$$r_1 - 1 = 8r_2 - 10$$

$$r_1 - 2r_2 + 3 = 0 \qquad \text{.......... (3)}$$

$$4r_1 - 3r_2 + 2 = 0 \qquad \text{.......... (4)}$$

$$7r_1 - 8r_2 + 9 = 0 \qquad \text{.......... (5)}$$

Solving (3) & (4), we get

$$\frac{r_1}{-4+9} = \frac{r_2}{12-2} = \frac{1}{-3+8}$$

$$\Rightarrow \frac{r_1}{5} = \frac{r_2}{10} = \frac{1}{5}$$

The value of $r_1 \& r_2$ satisfy the equation (5).

The point of intersection is given

$$(1+4, -4-3, 7-1) = (5, -7, 6)$$

Equation of the plane is

$$\begin{vmatrix} x-4 & y-3 & z+1 \\ 1 & -4 & 7 \\ 2 & -3 & 8 \end{vmatrix} = 0$$

$$\Rightarrow 11x - 6y - 5z - 67 = 0$$

6. The given line is

$$\frac{x}{2} = \frac{y}{3} = \frac{z}{1} = r(say) \text{ .... (1)}$$

Let P be the given point whose coordinates are (4,5,2). From P, let as draw a perpendicular PM to the line (1)

Any point on the is $(2r, 3r, r)$ coordinates of M are also $(2r, 3r, r)$ for some value of r.

Direction ratio of PM are $< 2r-4, 3r-5, r-2 >$ d.rs of the given line are $< 2,3,1 >$

Since PM is perpendcular to the given line, we have $2(2r-4) + 3(3r-5) + (r-2) = 0$

$$\Rightarrow 4r - 8 + ar - 15 + r - 2 = 0$$

$$\Rightarrow 14r = 25$$

$$\Rightarrow r = \frac{25}{14}$$

The coordinates M are

$$\left(2.\frac{25}{14}, 3.\frac{25}{14}, \frac{25}{14}\right)$$

$$= \left(\frac{50}{14}, \frac{75}{14}, \frac{25}{14}\right)$$

$$PM = \sqrt{\left(\frac{50}{14} - 4\right)^2 + \left(\frac{75}{14} - 5\right)^2 + \left(\frac{25}{14} - 2\right)^2}$$

$$= \sqrt{\left(\frac{6}{14}\right)^2 + \left(\frac{5}{14}\right)^2 + \left(\frac{3}{14}\right)^2}$$

$$= \sqrt{\frac{5}{14}}$$

7. Let the equation of the plane be $lx + my + nz = 3r$ .... (1)

Where $l^2 + m^2 + n^2 = 1$ .... (2)

At $A, y + 0, z = 0$

let $OA = x$,

From (1),

$l\,x_1 + mo + n.o = 3r$

$\Rightarrow lx_1 = 3r$

$\Rightarrow x_1 = \frac{3r}{e}$

Semilarly $y_1 = \frac{3r}{m}, z_1 = \frac{3r}{n}$

The coordinaty of $A, B \,\&\, C$ are $\left(\frac{3r}{l}, 0, 0\right)$, $\left(0, \frac{3r}{m}, 0\right)$, $\left(0, 0, \frac{3r}{n}\right)$

Let (x, y, z) be the coordinaty of the centoid of the tringle ABC.

$$\therefore x = \frac{\frac{3r}{l} + 0 + 0}{3} = \frac{r}{l}$$

Semilaly $y = \frac{r}{m}, z = \frac{r}{n}$

$\therefore l = \frac{r}{x}, m = \frac{r}{y}, n = \frac{r}{z}$

From (2), we get

$$\frac{r^2}{x^2} + \frac{r^2}{y^2} + \frac{r^2}{z^2} = 1$$

$$\Rightarrow r^2\left(\frac{1}{x^2} + \frac{1}{y^2} + \frac{1}{z^2}\right) = 1$$

$$\Rightarrow \frac{1}{x^2} + \frac{1}{y^2} + \frac{1}{z^2} = \frac{1}{r^2}$$

$$\Rightarrow x^{-2} + y^{-2} + z^{-2} = r^{-2}$$

8. Let O A B C D E F G be a rectangular parallelopiped $OA = a, OC = b, OE = C$.

Let us take O as origin, OA along $x - axis$, OC along $y - axis$ and OE along $z - axis$.

The coordinates of the corner points are $O(0,0,0)$, $A(a,0,0)$, $B(a,b,0)$, $C(0,b,0)$, $D(0,b,\mathrm{c})$, $\mathrm{E}(0,0,c)$, $F(a,0,\mathrm{c})$, $G(a,b,c)$

The d.rs of OG are $< a-0, b-0, 0-c >$ $=< a, b, c >$

The d.rs of EB are $< a-0, b-0, 0-\mathrm{c} >$ $=< a, b, -c >$

Let O be the angle between OG and EB

$$\cos\theta = \frac{a.a + b.b + c(-c)}{\sqrt{a^2+b^2+c^2}\sqrt{a^2+b^2(-c)^2}}$$

$$= \frac{a^2+b^2-c^2}{a^2+b^2+c^2}$$

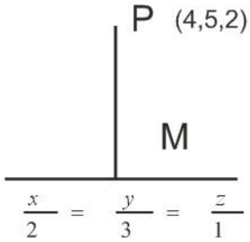
$$\theta = \cos^{-1}\left(\frac{a^2+b^2-c^2}{a^2+b^2+c^2}\right)$$

Similarly we can find the angle between other diagonals.

So the angle between the diagonals

$$\cos^{-1}\left(\frac{\pm a^2 \pm b^2 \pm c^2}{a^2+b^2+c^2}\right)$$

9. Let OA and OB are two mutually perpendicular lines, whose d.cs are



$< l_1, m_1, n_1 >$ and $< l_2, m_2, n_2 >$

Let OC be the line which is perpenidcular to OA and OB.

Let d.rs of OC be $< l, m, n >$,

Since OC is perpendicular to both OB and OB,

$$l\,l_1 + mm_1 + nn_1 = 0$$

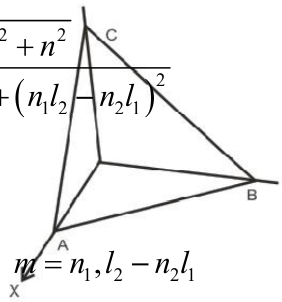
$$l\,l_2 + mm_2 + nn_2 = 0$$

By cross multipliction, we get

$$\frac{l}{m_1n_2 - m_2n_1} = \frac{m}{n_1l_2 = n_2l_1} = \frac{n}{l_1m_2 - l_2m_1}$$

$$= \frac{\sqrt{l^2+m^2+n^2}}{(m_1n_2-m_2n_1)^2 + (n_1l_2 - n_2l_1)^2}$$

$$= \frac{1}{\sin 90} = 1$$



$\therefore l = m_1n_2 - m_2n_1,\ m = n_1, l_2 - n_2l_1$

$n = l_1m_2 - l_2m_1$

Then d.rs of the line OC are

$$\langle m_1n_2 - m_2n_1, n_1l_2 - n_2l_1, l_1m_2 - n_2 \rangle$$

10. The equation of the plane is

$x - y + 2z = 4$ ....(1)

Let P be a point whose

coordinaty are (2,3,4)

From P, let as a perpendicular PM to the plane

Let as produce PM to O such that $PM = MQ$.

The point Q is called the image of P the d.rs of PM are $\langle 1,-1,2 \rangle$

Equations of the line PM are

$$\frac{x-2}{1} = \frac{y-3}{-1} = \frac{z-4}{2} \text{ (say)}$$

Any point on the line are $(r+2-r+3, 2r+4)$

The coordinaty of Q are $(r+2, -r+3, 2r+4)$ for some value of r.

Since M is the middlepoint of PQ, the coordinaty of M are

$$\left(\frac{r+2+2}{2}, \frac{-r+3+3}{2}, \frac{2r+4+4}{2}\right)$$

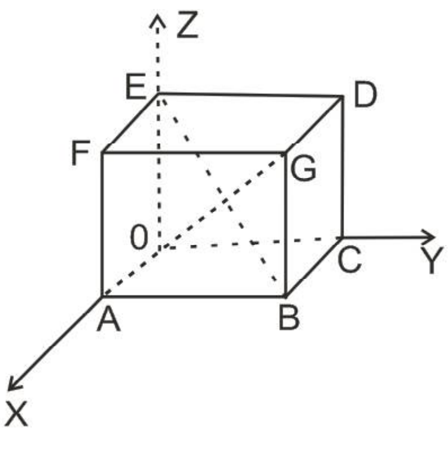
$$= \left(\frac{r+4}{2}, \frac{-r+6}{2}, \frac{2r+8}{2}\right)$$

Since it is a point on the plane (1), we have

$$\frac{r+4}{2} - \frac{-r+6}{2} + 2\left(\frac{2r+8}{2}\right) = 4$$

$$\Rightarrow r + 4 + r - 6 + 4r + 16 = 8$$

$$\Rightarrow r = -1$$



The coordinaty of the foot of the perpendicular

$$=\left(\frac{-1+4}{2},\frac{1+6}{2},\frac{-2+8}{2}\right)$$

$$=\left(\frac{3}{2},\frac{7}{2},\ -1+4\right)$$

$$=\left(\frac{3}{2},\frac{7}{2}+3\right)$$

$$PM=\sqrt{\left(2-\frac{3}{2}\right)^2+\left(3-\frac{7}{2}\right)^2+\left(4-\frac{6}{3}\right)^2}$$

$$=\frac{\sqrt{6}}{2}\text{ unit.}$$

11. The given relations are

$l+2m+3n=0$ .... (1)

$3lm-4\ln+nm=0$ .... (2)

From (1), we get $l=-(2m+3n)$

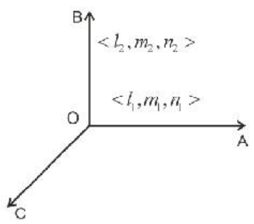
From (2), we get

$$-3(2m+3n)m+4(2m+3n)n+nm=0$$

$$\Rightarrow m^2=2n^2$$

$$\Rightarrow m=\pm\sqrt{2}n$$

$$l=-(2m+3n)$$



$$=-\left(\pm2\sqrt{2}n+3n\right)$$

$$=-\left(3\pm2\sqrt{2}\right)n$$

The d.rs of two lines are $-\left(3\pm2\sqrt{2},\sqrt{2},1\right)$

and $\left(-3,2\sqrt{2},-\sqrt{2},1\right)$

Let these d.rs be $\langle a_1,b_1,c_1\rangle$ & $\langle a_2,b_2,c_2\rangle$

$$\therefore a_1a_2+b_2+c_1c_2$$

$$=-\left(3+2\sqrt{2}\right).\left(2\sqrt{2}-3\right)+\sqrt{2}.\left(\sqrt{2}\right)+1$$

$$=-(8-9)-2+1$$

$$=1-2+1=0$$

So two lines are perpendicular to each other.

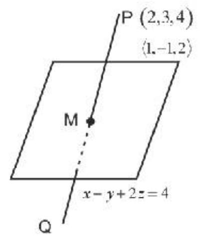
12. Given plane is

$2x+6y+6z-1=0$ .... (1)

Let A and B be two given points whose coordinates are (2,2,1) and (a,3,6).

Equation of the pane passing through (2,2,1) is

$a(x-2)+b(y-2)+c(z-1)=0$ .... (2)



Since it is passing through $(a,3,6)$, we get

$$a(9-2)+b(3-2)+c(6-1)=0$$

$\Rightarrow 7a+b+5c=0$ ....(3)

From (3) & (4) by cross multiplication,

$$\frac{a}{6-30}=\frac{b}{10-42}=\frac{c}{42-2}$$

$$\Rightarrow\frac{a}{-24}=\frac{b}{-32}=\frac{c}{40}$$

$$\Rightarrow\frac{a}{3}=\frac{b}{4}=\frac{c}{-5}=k(say)$$

$$\therefore a=3k,b=4k,c=-5k$$

The required plane is

$$3k(x-2)+4k(y-2)-5k(2-1)=0$$

$\Rightarrow 3(x-2)+4(y-2)-5(z-1)=0$

13. Two given planes are

$x+2y+2z-5=0$ ....(1)

$3x+3y+2z-8=0$ .... (2)

Equation of the plane passing through $(-1,3,0)$ is where $a(x+1)+b(y-3)+c(z-0)=0$ a,b,c are the d.rs of its normal. Since this plane is perpendicular to

the planes (1) & (2), we have

$a+2b+2c=0$ .... (4)

$3a+3b+2c=0$ .... (5)

By cross multiplication, we have

$$\frac{a}{4-6}=\frac{b}{6-2}=\frac{c}{3-6}$$

$$\Rightarrow \frac{a}{-2}=\frac{b}{4}=\frac{c}{-3}=k(say)$$

$$\Rightarrow a=-2k, b=4k, c=-3k$$

The required plane is

$$-2x(x+1)+4x(y-3)-3x(z-0)=0$$

$$\Rightarrow 2(x+1)-4(y-3)+3z=0$$

14. The given planes are

$2x-y+2z+3=0$ .... (1)

$3x-2y+6z+8=0$ .... (2)

Equation of the plane passing through the intersection of the planes (1) & (2) is

$$2x-y+2z+3+k\ (3x-2y+6z+8)=0$$

Since it is passing through $(1,-1,0)$ we have

$$2+1+0+3+k(3+2+0+8)=0$$

$$\Rightarrow 13k=-6$$

$$\Rightarrow k=\frac{-6}{13}$$

Required plane is

$$(2x-y+2z+3)-\frac{6}{13}\ (3x-2y+6z+8)$$

$=0$

$\Rightarrow 8x-y-10z-9=0$

15. The equation of the plane is

$x-y+z=5$ .... (1)

Given line is

$\frac{x-2}{3}=\frac{y+1}{4}=\frac{z-2}{12}=r(say)$ .... (2)

Let the line (2) intersect the plane (1) at the point P.

Any point on the line (2) is

$$\langle 3r+2, 4r-1, 12r+2\rangle$$

The coordinaty of P are

$(3r+2, 4r-1, 12r+2)$ for some value of r.

This point wil satisfy the equation (1)

$3r+2-(4r-1)+12r+2=5$

$\Rightarrow 3r+2-4r+1+12r+2=5$

$\Rightarrow 11r=0$ $\Rightarrow r=0$

The coordinaty of P are $(3.0+2, 4.0-1, 12.0+2)$

$=(2,-1,2)$

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