CLASS-XII (CBSE)

Mathematics

Workbook Cum Question Bank with Answers

SCHEDULED CASTES & SCHEDULED TRIBES
RESEARCH & TRAINING INSTITUTE (SCSTRTI)
ST & SC DEVELOPMENT DEPARTMENT
BHUBANESWAR

MATHEMATICS

Workbook Cum Question Bank with Answers

CLASS-XII (CBSE)

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Unit - I

RELATIONS AND FUNCTIONS

CHAPTER - 1

RELATIONS AND FUNCTIONS

A. Multiple Choice Questions (MCQ)

1. Let R be a relation from a finite set A having m elements to another finite set B having n elements then number of relations from A to B is ____

   (a) 2^{mn} (b) $2^{mn}-1$

   (c) $2m^n$ (d) m^n

2. Let R be a relation defined on a finite set A having n elements. Then no of relations defined on A is ____

   (a) 2^n (b) 2^{n^2}

   (c) n^2 (d) n^n

3. Let R be a relation defined on a set A such that $R = R^{-1}$ then R is ____

   (a) Reflexive (b) Symmetric

   (c) Transitive (d) None of these

4. The relation $R = \{(x, y) \mid x^2 + y^2 = 1, x, y \in R\}$ is ____

   (a) Reflexive (b) Symmetric

   (c) Transitive (d) None of these

5. Give a relation on A = {1, 2, 3} which is both symmetric and antisymmetric.

   (a) R = {(1, 1), (1, 2)}

   (b) R= {(1, 1), (2, 2), (1, 2), (2, 1)}

   (c) R = {(1, 2), (2, 1)}

   (d) None of these

6. If R be the largest equivalence relation on the set A and S is any relation on A then

   (a) $R \subset S$ (b) $S \subset R$

   (c) $R = S$ (d) None of these

7. Let A = {1, 2, 3}. Let a relation R defined on A by R = {(1, 2), (2, 3)}. Then the minimum number of ordered pairs when added to R to make it an equivalence relation is ____

   (a) 10 (b) 8

   (c) 7 (d) 4

8. If R be a relation defined on N by $xRy \Leftrightarrow x + 2y = 8$ then domain of R is ___

   (a) {2, 4, 8} (b) {2, 4, 6, 8}

   (c) {2, 4, 6} (d) {1, 3, 4, 5}

9. If $n(A) = 4$, $n(B) = 6$ then no of one-one functions from A to B is ___

   (a) 24 (b) 4^6

   (c) 6^4 (d) 360

10. Let A, B are two finite sets having m, n elements respectively. Then what is the total no of mappings from A to B ?

    (a) m^n (b) n^m

    (c) mn (d) None of these

11. Let $f: R \to R$ be a function defined as follows. Which of the following is a 1–1 mapping ?

(a) $f(x) = x^2$

(b) $f(x) = \sin x$

(c) $f(x) = \frac{1}{x-3}, \ x \in R\}3\}$

(d) None of these

12. If $f: R \to R$ be a mapping defined by $f(x) = x^2 - 1$ then $f^{-1}(8) =$ _____

(a) {–2, 2} (b) {–3, 3}
(c) {–4, 4} (d) None of these

13. Find the domain of
$f(x) = \sqrt{2x-1} + \sqrt{3-2x}$

(a) (2, 3) (b) (1/2, 3/2)
(c) [1/2, 3/2] (d) None of these

14. If $f(x) = \cos \log_e x$ then
$f(x).f(y) - \frac{1}{2}[f(xy) + f(x/y)] =$ ___

(a) 0 (b) $1/2 f(x) f(y)$

(c) $f(x+y)$ (d) None of these

15. The total number of one-one function from a finite set with m elements to a set with n elements (m < n) is _____

(a) $\frac{n}{(m-n)!}$ (b) $\frac{n!}{(n-m)!}$

(c) n^m (d) m^n

16. If $f(x + 1/x) = x^2 + 1/x^2 \ (x \neq 0)$ then $f(x) =$ _____

(a) $x^2 + 1/x^2$ (b) $x^2 - 2$

(c) $x^2 + 1/x^2 - 2$ (d) None of these

17. If $f: R \to R$ be a mapping defined by $f(x) = x^3 + 5$ then $f'(x) =$ _____

(a) $(x+5)^{1/3}$ (b) $(x-5)^{1/3}$

(c) $(5-x)^{1/3}$ (d) $5-x$

18. $f(x) = (3 - x^7)^{1/7}$ for all $x \in R$ then $fof(x)$ is _____

(a) x (b) 2x
(c) 3x (d) 4x

19. If $f: R \to R$ and $g: R \to R$ are functions defined as $f(x) = x - 3$, $g(x) = x^2 + 1$. Then find the values of x for which $g[f(x)] = 10$.

(a) 0, –6 (b) 2, –2
(c) 1, –1 (d) 0, 6

20. If $f: R \to R$ is given by

$f(x) = \begin{cases} -1, & x \text{ is rational} \\ 1, & x \text{is irrational} \end{cases}$ then

$fof(1 - \sqrt{3}) =$

(a) 1 (b) –1

(c) $\sqrt{3}$ (d) 0

21. The relation defined as "differs by more than 10" in the set Z is ____

(a) Reflexive (b) Symmetric
(c) Transitive (d) None of these

22. The relation of "perpendicular to" over the set of all straight lines in a plane is ____

(a) reflexive

(b) symmetric

(c) transitive

(d) equivalence relation

23. The relation "has the same remainder when divided by 5" over the set of all natural numbers is ______

(a) Reflexive only

(b) Symmetric only

(c) Transitive only

(d) Equivalence relation

24. The relation R defined as mRn if m+n is odd enN is ____

(a) Reflexive

(b) Transitive

(c) Symmetric

(d) None of these

25. The relation R defined as mRn if m/n is odd.

(a) Reflexive

(b) Symmetric

(c) Transitive

(d) Reflexive and Transitive

26. $f: R \to R$ defined as $f(x) = 1 - x$ then the image of 3/2 is ____

(a) 1/2 (b) 1

(c) –1/2 (d) None of these

27. $f: R \to R$ then find the range of $f = \{(x, y) \mid y - 2 = 2\}$

(a) {4} (b) {6}

(c) {–4) (d) None of these

28. Let A = {1, 2, 3}, B = {x, y} . $f: A \to B$ such that f = {(1, x), (2, x), (1, y), (3, y)} then f is ______

(a) one-one (b) onto

(c) constant (d) not a function

29. $f: A \to B$ where A = {1, 2, 3}, B = {x, y) and f = {(1, x), (2, x), (3, y)} is ______

(a) only onto (b) one-one

(c) into (d) bijective

30. $f: R \to R$ defined as $f(x) = x^2$ is ___

(a) one-one

(b) onto

(c) into

(d) neither one-one nor onto

31. $f: R \to R$ defined as $f(x) = 5x + 2$ is ______

(a) one-one (b) onto

(c) into (d) bijective

ANSWER KEYS

1.	(a)	9.	(d)	17.	(b)	25.	(d)
2.	(b)	10.	(b)	18.	(a)	26.	(c)
3.	(b)	11.	(c)	19.	(d)	27.	(a)
4.	(b)	12.	(b)	20.	(b)	28.	(d)
5.	(b)	13.	(c)	21.	(b)	29.	(a)
6.	(b)	14.	(a)	22.	(b)	30.	(d)
7.	(c)	15.	(b)	23.	(d)	31.	(d)
8.	(c)	16.	(b)	24.	(c)		

B. Long Answer Type Questions

1. If (x,y) and $(p,q)\in R^2$ and (x,y) $R(p,q)$ $\Leftrightarrow x^2-y^2=p^2-q^2$. Prove that R is an equivalence relation.

2. A relation R is defined on z as a R b $\Leftrightarrow$ 3a + 4b = 7n, for some integers n. prove that R is an equivalence relation on z.

3. For $a,b\in N$ the relation R is defined as $aRb \Leftrightarrow 4a+b$ is multiple of 5. Prove that R is an equivalence relation on N.

4. For $(x,y),(u,v)\in N\times N$, $(x,y)R(u,v)\Leftrightarrow$ $xv=yu$. Prove that R is an equivalence relation.

5. For $(x,y),(u,v)\in R^2$, $(x,y)R(u,v)\Leftrightarrow$ $x^2+y^2=u^2+v^2$ prove that R is an equivalence relation.

6. A relation R on Z is defined as a R B $\Leftrightarrow$ 4a + b is a multiple of 5. Prove that R is an equivalence relation on z.

7. Prove that congruence modulo n is an equivalence relation over the set of integers.

8. Show that $f:A\rightarrow B$ defined as $f(x)=x^2+4x+7$ is one-one oand onto.

9. Prove that composition of bijective functions is also bijective.

10. If R and S are equivalence relations then $R\cap S$ is also an equivalence relation.

Answer Hints

1. **Reflexive :**

$(x,y)R(x,y)\Rightarrow x^2-y^2=x^2-y^2 \Rightarrow R$ is reflexive.

Symmetric :

$(x,y)R(p,q)\Rightarrow x^2-y^2=p^2-q^2$

$\Rightarrow p^2-q^2=x^2-y^2\Rightarrow (p,q)R(x,y)$

$\Rightarrow R$ is symmetric.

Transitive :

Let $(x,y)R(p,q),(p,q)R(u,v)$

$\Rightarrow x^2-y^2=p^2-q^2, p^2-q^2=u^2-v^2$

$\Rightarrow x^2-y^2=u^2-v^2$

$\Rightarrow (x,y)R(u,v)\Rightarrow R$ is transitive

Hence R is an equivalence relation.

2. **Reflexive :**

For any $a\in z$, $3a+4a=7a$

$\Rightarrow 3a+4a=7n$ for any integer n.

So R is reflexive.

Symmetric :

Let $a\,R\,b\Rightarrow 3a+4b=7n$ for any integer n.

Now $3b+4a=(7a+7b)-(3a+4b) = 7(a+b)-7n$

$=7(a+b-n)=7k$ where $k=a+b-n$.

$\Rightarrow bRa\Rightarrow R$ is symmetric

Transitive :

Let $a\,R\,b, b\,R\,c\Rightarrow 3a+4b$

$=7n,\ 3b+4c=7m$,

for some integer m and n.

Now

(3a+4b) + (3b+4c) = 3a + 7b + 4c = 7 (m+n)

$\Rightarrow$ 3a + 4n = 7(m + n) – 7b = 7 (m + n –b)

= 7p where p = n + m –b $\Rightarrow$ a R c

so R is transitive

Hence R is an equivalence relation

3. $aRb \Leftrightarrow 4a+b$ is a multiple of 5.

Reflexive :

As 4a + a = 5a which is a multiple of 5

$\Rightarrow R$ is reflexive

Symmetric :

Let $aRb \Rightarrow 4a+b$ is a multiple of 5.

Now 4b–b + 5a – 4a = 5(a + b) – (4a + b)

$\Rightarrow bRa$ is true

Transitive :

Let $aRb, bRc \Rightarrow 4a \neq b$ is a multiple of 5 and 4b+c is a multiple of 5.

$\Rightarrow 4a+b+4b+c$ is also a multiple of 5

$\Rightarrow (4a+c)+5b$ is also a multiple of 5

$\Rightarrow 4a+c$ is also a multiple of 5

$\Rightarrow aRc$ is true

Hence R is an equivalence relation

7. $a \equiv b \bmod n \Rightarrow a-b$ is divisible by n

Reflexive :

As a – a = 0 is divisible by n

$\Rightarrow aRa$ is true. So R is reflexive.

Symmetric :

Let $aRb \Rightarrow a-b$ is divisible by n

$\Rightarrow -(a-b)$ is divisible by $n \Rightarrow (b-a)$ is divisible b y n

$\Rightarrow bRa$ is true $\Rightarrow R$ is symmetric.

Transitive :

Let $aRb,\ bRc \Rightarrow a-b$ is divisible by n and b – c is divisible by n.

$\Rightarrow (a-b+b-c)$ is divisible by n.

$\Rightarrow (a-c)$ is divisible by n.

$\Rightarrow R$ is transitive.

Hence congruence modulo n is an equivalence relation.

9. Let $f: A \to B,\ g: B \to C$ are two bijective functions. First - we have to show that $gof: A \to C$ is one-one

Let $gof(a_1) = gof(a_2) = g[f(a_1)] = g[f(a_2)]$

$\Rightarrow f(a_1) = f(a_2)(\because$ g is one-one)

$\Rightarrow a_1 = a_2(\because f$ is one-one

$\Rightarrow$ gof is one-one.

Since g is onto, Let $Z \in C$ then there exists a pre-image y of z under g such that $g(y) = z$.

As f is onto then for $y \in B$ there exists an element $x \in A$ such that $f(x) = y$.

Now $gof(x) = g[f(x)] = g(y) = z$

$\Rightarrow$ Every element $z \in c$ has pre image under gof.

$\Rightarrow gof$ is onto. Hence gof is also a bijective function.

10. Since R and S are reflexive then for $\forall a \in X\ (a,a) \in R$ and $(a,a) \in S$

$\Rightarrow (a,a) \in R \cap S$

$\Rightarrow R \cap S$ is reflexive.

Since R and S are symmetric then $\forall a,b \in x \quad (a,b) \in R \cap S \Rightarrow (a,b) \in R$, $(a,b) \in S$

$\Rightarrow (b,a) \in R,\ (b,a) \in S \Rightarrow (b,a) \in R \cap S$

$\Rightarrow R \cap S$ is symmetric

Since R and S are transitive then for $\forall a,b,c \in x\ (a,b) \in R \cap S,\ (b,c) \in R \cap S$

$\Rightarrow (a,b) \in R,\ (b,c) \in R, (a,b) \in S, (b,c) \in S$

$\Rightarrow (a,c) \in R, (a,c) \in S$

($\because$ R and S are both transitive)

$\Rightarrow (a,c) \in R \cap S \Rightarrow R \cap S$ is transitive.

Hence $R \cap S$ is an equivalence relation.

ADDENDUM

1. Let R be a relation from a finite set A having m elements to another finite set B having n elements. Then no. of relations from A to B is

(a) 2^{mn} (b) $2^{mn}-1$

(c) 2^{m^n} (d) m^n

2. Let R be a relation defined on a finite set A having n elements. Then no. of relations defined on A is ______

(a) 2^n (b) 2^{n^2}

(c) n^2 (d) n^n

3. Let R be a relation defined on a set A such that $R = R^{-1}$. Then R is

(a) reflexive (b) symmetric

(c) transitive (d) None of these

4. The relation $R = \{(x, y)/x^2 + y^2 = 1, x, y \in R\}$ is ___

(a) reflexive (b) symmetric

(c) transitive (d) None of these

5. Give a relation on $A = \{1,2,3\}$ which is both symmetric & anti-symmetric.

(a) $R = \{(1,1), (1,2)\}$

(b) $R = \{(1,1), (2,2), (1,2), (2,1)\}$

(c) $R = \{(1,2), (2,1)\}$

(d) None of these

6. If R be the largest equivalence relation on the set A and S is any relation on A, then

(a) $R \subset S$ (b) $S \subset R$

(c) $R = S$ (d) None of these

7. Let $A = \{1,2,3\}$. Let a relation R defined on A by $R = \{(1,2), (2,3)\}$. Then the minimum no. of ordered pairs when added to R to make it an equivalence relation is

(a) 10 (b) 8

(c) 7 (d) 4

8. If R be a relation defined on N by $xRy \Leftrightarrow x + 2y = 8$, then domain of R is

(a) {2, 4, 8} (b) {2, 4, 6, 8}

(c) {2, 4, 6} (d) {1, 3, 4, 5}

9. If $n(A) = 4,\ n(B) = 6$, then no of one-one functions from A to B is ______

(a) 24 (b) 4^6

(c) 6^4 (d) 360

10. Let A, B be two finite sets having m, n elements respectively. What is the total no. of mapping from A to B ?

(a) m^n (b) n^m

(c) mn (d) None of these

11. Let $f : R \to R$ be function defined as follows which of these is a 1-1 mapping?

(a) $f(x) = x^2$

(b) $f(x) = \sin x$

(c) $f(x) = \dfrac{1}{x-3}\ \ x \in R - \{3\}$

(d) None of these

12. If $f : R \to R$ be a mapping defined by $f(x) = x^2 - 1$. Then $f^{-1}(.8) = ...?$

(a) {–2, 2} (b) {–3, 3}

(c) {–4, 4} (d) None of these

13. Find domain of $f(x)=\sqrt{2x-1}+\sqrt{3-2x}$

(a) (2, 3) (b) $\left(-\frac{1}{2},\frac{3}{2}\right)$

(c) $\left[\frac{1}{2},\frac{3}{2}\right]$ (d) None of these

14. If $f(x)=\cos\log_e x$, then

$f(x).f(y)-\frac{1}{2}\left[f(xy)+f\left(\frac{x}{y}\right)\right]=.....?$

(a) 0 (b) $\frac{1}{2}f(x)f(y)$

(c) $f(x+y)$ (d) None of these

15. The total no. of one-one function from a finite set with m elements to a set with n elements $(m<n)$ is

(a) $\frac{m!}{(m-n)!}$ (b) $\frac{n!}{(n-m)!}$

(c) n^m (d) m^n

16. If $f(x)=\left(a-x^n\right)^{\frac{1}{n}}, a>0, m\in z$, then $f[f(x)]=.....?$

(a) x (b) x^n

(c) $a-x^n$ (d) None of these

17. If $f(x)=\log\frac{1+x}{1-x}$, then $f\left(\frac{2x}{1+x^2}\right)=...?$

(a) $f(x)$ (b) $2f(x)$

(c) $\log\frac{1+x}{1-x}$ (d) None of these

18. If $f\left(x+\frac{1}{x}\right)=x^2+\frac{1}{x^2},(x\neq 0)$, then $f(x)=....?$

(a) $x^2+\frac{1}{x^2}$ (b) x^2-2

(c) $x^2+\frac{1}{x^2}-2$ (d) None of these

19. If $f:R\to R$ be a mapping defined by $f(x)=x^3+5$. Then $f^{-1}(x)$ is equal to

(a) $(x+5)^{\frac{1}{3}}$ (b) $(x-5)^{\frac{1}{3}}$

(c) $(5-x)^{\frac{1}{3}}$ (d) $5-x$

20. Find period of the function $f(x)=\cos ec^2 3x+\cot 4x$

(a) 2π (b) π

(c) $\frac{\pi}{2}$ (d) None of these

21. If $f(x)=\left(3-x^7\right)^{\frac{1}{7}}$, for all $x\in R$, then $(fof)(x)$ is

(a) x (b) $2x$

(c) $3x$ (d) $4x$

23. If $f:R\to R$ and $g:R\to R$ be functions defined by $f(x)=x-3$, $g(x)=x^2+1$. Then find values of x, for which $g[f(x)]=10$

(a) 0, –6 (b) 2, –2

(c) 1, –1 (d) 0, 6

24. If $f:R\to R$ is given by

$f(x)=\begin{cases}-1, & x \text{ rational}\\ 1, & x \text{ irrational}\end{cases}$ then

$fof\left(1-\sqrt{3}\right)=.....?$

(a) 1 (b) –1

(c) $\sqrt{3}$ (d) 0

ANSWER KEYS

1. (a) 2^{mn}

Sol. Here $|A| = m; |B| = n$

$\Rightarrow |A \times B| = m^n$

$\therefore$ No. of relations from A to B

= No. of subsets of $A \times B$

$= z^{mn} \because |A| = m \Rightarrow |P(A)| = z^m$

2. (b) 2^{n^2}

Sol. Here $|A| = n$

$\Rightarrow |A \times A| = n.n = n^2$

$\therefore$ No. of relations defined in A

= No. of subsets of $A \times A$

$= 2^{n^2}$

3. (b) symmetric

Sol. R is symmetric

$\because$ Let $(a,b) \in R \Rightarrow (b,a) \in R^{-1}$

$\Rightarrow (b,a) \in R \because R^{-1} = R$

$\therefore (a,b) \in R \Rightarrow (b,a) \in R$

$\Rightarrow R$ is symmetric

4. (b) symmetric

Sol. Let $(x,y) \in R \Rightarrow x^2 + y^2 = 1$

$\Rightarrow y^2 + x^2 = 1$

$\Rightarrow (y,x) \in R$

$\Rightarrow R$ is symmetric

5. (b) $R = \{(1,1), (2,2), (1,2), (2,1)\}$

Sol. $R = \{(1,1),(2,2),(1,2),(2,1)\}$ is both symmetric and antisymmetric on $A = \{1,2,3\}$.

6. (b) $S \subset R$

Sol. The largest equivalence relation on A

$= R = A \times A$

Since S is a relation on A, hence

$S \subseteq A \times A$

Hence appropriate response is $S \subset R$

7. (c) 7

Sol. $A = \{1,2,3\}$. S R to be reflexive, symmetric & transitive, it should be

R = {(1,1), (2,2), (3,3), (1,2), (2,3), (1,3), (2,1), (3,2), (3,11)}

so 7 more ordered pairs to be added to R to make it an equivalence relation.

8. (c) {2, 4, 6}

Sol. $x, y \in N$ such that $x + 2y = 8$

$x \neq 1,\ y = 1$

If $x = 2$, then $y = 3 \Rightarrow (2,3) \in R \because$

$2 + 2 \times 3 = 8 \quad x \neq 3$

If $x = 4$, then $y = 2 \Rightarrow (4,2) \in R \because$

$4 + 2 \times 2 = 8 \quad x \neq 5$

If $x = 6$, then $y = 1 \Rightarrow (6,1) \in R$

$\because 6 + 2 \times 1 = 8$

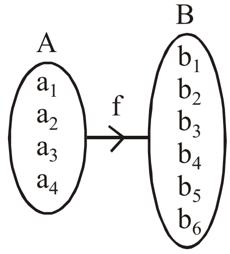
$x \neq 7$ or any number > 7

$\therefore R\{(2,3), (4,2), (6,1)\}$

$\Rightarrow dom\ R = \{2,4,6\}$

9. (d) 360

Sol. a_1 is related to elements of B in 6 diff. ways.



a_2 is related to elements of B in 5 diff. ways.

similarly, a_3 is related to elements of B in 4 diff ms

and a_4 is related to elements of B in 3 diff. ms

$\therefore$ total no. of one-one functions from A to B $= 6 \times 5 \times 4 \times 3 = 360$

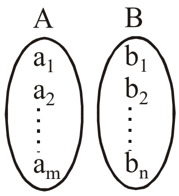
10. (b) n^m

Sol. a_1 is related to elements of B in n different ways

a_2 is related to elements of B in n different ways

$\vdots$

a_m is related to elements of B in n different ways



$\therefore$ total no. of functions for A to B

$= n.n.n....n(m \text{ times})$

$= n^m$

11. (c) $f(x) = \dfrac{1}{x-3} \quad x \in R - \{3\}$

Sol. $f(x) = x^2$

Let $f(x) = f(y)$

$\Rightarrow x^2 = y^2$

$\Rightarrow x = y, -y$

$\Rightarrow f$ is not 1-1

$f(x) = \sin x$. Let $f(x) = f(y)$

$\Rightarrow \sin x = \sin y$

$\Rightarrow x = 2n\pi + y$

$\Rightarrow f$ is not 1 – 1

$f(x) = \dfrac{1}{x-3}$ .Let $f(x) = f(y)$

$\Rightarrow \dfrac{1}{x-3} = \dfrac{1}{y-3}$

$\Rightarrow x - 3 = y - 3$

$\Rightarrow x = y$

$\Rightarrow f$ is 1 – 1

12. (b) {–3, 3}

Sol. Let $f^{-1}(.8) = x$

$\Rightarrow f(x) = .8$

$\Rightarrow x^2 - 1 = .8 \because f(x) = x^2 - 1$

$\Rightarrow x^2 = 9 \Rightarrow x = -3, 3$

$\Rightarrow f^{-1}(8) = \{-3, 3\}$

13. (c) $\left[\dfrac{1}{2}, \dfrac{3}{2}\right]$

Sol. For domain of $f(x), 2x - 1 \geq 0$

$\Rightarrow x \geq \dfrac{1}{2}$

Similarly $3 - 2x \geq 0$

$\Rightarrow 3 \geq 2x$

$\Rightarrow \dfrac{3}{2} \geq x \Rightarrow x \leq \dfrac{3}{2}$

$\therefore$ Domain of $f(x) = x \geq \dfrac{1}{2} \wedge x \leq \dfrac{3}{2}$

$= \left[\dfrac{1}{2}, \dfrac{3}{2}\right]$

14. (a) 0

Sol. Here $f(x) = \cos \log_e x$

$f(y) = \cos \log_e y$

$\therefore f(x).f(y) - \dfrac{1}{2}\left[f(xy) = f\left(\dfrac{x}{y}\right)\right]$

$= \cos \log_e x . \cos \log_e y - \dfrac{1}{2}$

$\left[\cos\log_e(xy)+\cos\log_e\left(\frac{x}{y}\right)\right]$

$=\cos\log_e x.\cos\log_e y-\frac{1}{2}$

$\left[\cos(\log_e x+\log_e y)+\cos(\log_e x-\log_e y)\right]$

$=\cos\log_e x.\cos\log_e y-\frac{1}{2}\times 2\cos$

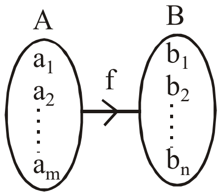
$\frac{\log_e x+\log_e y+\log_e x-\log_e y}{2}\times$

$\frac{\cos\log_e x+\log_e x-\log_e x+\log_e y}{2}$

$=\cos\log_e x.\cos\log_e y-\cos\log_e x.\cos\log_e y$

$=0$

15. (b) $\frac{n!}{(n-m)!}$

Sol. 

a_1 is related to elements of D in n diff. m,s

a_2 is related to elements of D in (n–1) diff. n,s

a_3 is related to elements of D in (n–2) diff. n,s

⋮

a_m is related to elements of D in $\left[n-(m-1)\right]$ diff n,s

∴ Total no. of 1 – 1 functions for A to B

$=n.(n-1).(n-2)....(n-m+1)$

$=\frac{n(n-1)(n-2)....(n-m+1)(n-m).(n-m-1).....3.2.1}{1.2.3.....(n-m)}$

$=\frac{n!}{(n-m)!}$

16. (a) x

Sol. Here $f\left[f(x)\right]$

$f[y]$ where

$y=f(x)=\left(a-x^n\right)^{\frac{1}{n}}\Rightarrow y^n=a-x^n$

$=\left(a-y^n\right)^{\frac{1}{n}}=\left[a-\left(a-x^n\right)\right]^{\frac{1}{n}}$

$=\left(x^n\right)^{\frac{1}{n}}=x$

17. (b) $2f(x)$

Sol. Here $f\left(\frac{2x}{1+x^2}\right)=f(y)$ where $y=\frac{2x}{1+x^2}$

$=\log\frac{1+y}{1-y}\because f(x)=\log\frac{1+x}{1-x}$

$=\log(1+y)-\log(1-y)$

$=\log\left[1+\frac{2x}{1+x^2}\right]-\log\left[1-\frac{2n}{1+x^2}\right]$

$=\log\left[\frac{1+x^2+2x}{1+x^2}\right]-\log\left[\frac{1+x^2-2x}{1+x^2}\right]$

$=\log\left[1+x^2+2x\right]-\log\left(1+x^2\right)$

$=\log\left[1+x^2-2x\right]+\log\left(1+x^2\right)$

$=\log\left[(1+x)^2\right]-\log\left[(1-x)^2\right]$

$=2\log(1+x)-2\log(1-x)$

$=2\left[\log(1+x)-\log(1-x)\right]=2.\log\frac{1+x}{1-x}$

$=2f(x)$

18. (b) x^2-2

Sol. Here $f\left(x+\frac{1}{x}\right)=x^2+\frac{1}{x^2}$

$=\left(x+\frac{1}{x}\right)^2-2$

$\Rightarrow f(y)=y^2-2$ where $y=x+\frac{1}{x}$

$\Rightarrow f(x)=x^2-2$

19. (b) $(x-5)^{\frac{1}{3}}$

20. $f:R\to R$ defined by $f(x)=x^3+5$ is bijective.

$\Rightarrow f^{-1}$ exists.

Let $f(x)=y\Rightarrow x=f^{-1}(y)$

$\Rightarrow x^3+5=y$

$\Rightarrow x^3=y-5\Rightarrow x=(y-5)^{\frac{1}{3}}$

$\therefore f^{-1}(y)=x=(y-5)^{\frac{1}{3}}$

$\Rightarrow f^{-1}(x)=(x-5)^{\frac{1}{3}}$

21. (b) π

Sol. $f(x)=\cos ec^2 3x+\cot 4x$

$=\frac{1}{\sin^2 3x}+\frac{1}{\tan 4x}=\frac{1}{1-\cos 6x}+\frac{1}{\tan 4x}$

$\therefore$ period of $f(x)=$ LCM of [period of cos 6x, period of tan 4x]

$=$ LCM of $\left[\frac{2\pi}{6},\frac{\pi}{4}\right]$

($\therefore$ period of $\cos mx=\frac{2\pi}{m}$ & $\tan mx=\frac{\pi}{m}$

$=$ LCM of $=\left[\frac{\pi}{3}\ \&\ \frac{\pi}{4}\right]=\pi$

22. (a) x

Sol. Here $(fof)(x)=f[f(x)]$

$=f\left[(3-x^2)^{\frac{1}{7}}\right]\because f(x)=(3-x^7)^{\frac{1}{7}}$

$=f(y)$; say where $y=(3-x^7)^{\frac{1}{7}}$

$\Rightarrow y^7=3-x^7$

$=(3-y^7)^{\frac{1}{7}}$

$=(3-3+x^7)^{\frac{1}{7}}=x$

23. (d) 0, 6

Sol. Here $f(x)=x-3,\ g(x)=x^2+1$

$\therefore g[f(x)]=g(x-3)\because f(x)=x-3$

$=(x-3)^2+1\ \because g(x)=x^2+1$

$=x^2+9-6x+1$

$=x^2-6x+10$

since $g[f(x)]=10\Rightarrow x^2-6x+10=10$

$\Rightarrow x^2-6x=0\Rightarrow x(x-6)=0\Rightarrow x=0,6$

24 (b) -1

Sol. Here $fof\left(1-\sqrt{3}\right)=f\left[f\left(1-\sqrt{3}\right)\right]$

$=f(1)\because 1-\sqrt{3}$ irrational

$=-1\because 1$ is rational.

CHAPTER - 2

INVERSE TRIGONOMETRIC FUNCTIONS

A. Multiple Choice Questions (MCQ)

1. Principal value of $\sin^{-1}(\sin 2\pi/3) =$ ______
   (a) $\pi/3$ (b) $2\pi/3$
   (c) $\pi+\pi/3$ (d) None of these

2. The value of $\sin\left[\cot^{-1}\left\{\tan\left(\cos^{-1}x\right)\right\}\right] =$ ______
   (a) sin x (b) x
   (c) $\sin^{-1}x$ (d) None of these

3. Value of $2\sin^{-1}\frac{1}{2}+\cos^{-1}\frac{1}{2} =$ ______
   (a) $\frac{\pi}{3}$ (b) $\frac{2\pi}{3}$
   (c) $\frac{5\pi}{3}$ (d) None of these

4. Value of $\cos\left[2\cos^{-1}(0.8)\right] =$ ___
   (a) 2.8 (b) 0.28
   (c) 0.24 (d) None of these

5. If $\cot^{-1}x+\cot^{-1}y+\cot^{-1}z=\frac{\pi}{2}$ then $x+y+z=$ ______
   (a) xyz (b) $\frac{1}{x}+\frac{1}{y}+\frac{1}{z}$
   (c) $\frac{1}{xyz}$ (d) None of these

6. $\sin^{-1}\frac{3}{5}+\tan^{-1}\frac{1}{7} =$ ______
   (a) π (b) $\frac{\pi}{2}$
   (c) $\frac{\pi}{3}$ (d) $\frac{\pi}{4}$

7. If $x+y+z=xyz$ then $\tan^{-1}x+\tan^{-1}y+\tan^{-1}z =$ ______
   (a) $\frac{\pi}{2}$ (b) 0
   (c) $\frac{\pi}{4}$ (d) $\frac{\pi}{6}$

8. If $\sin^{-1}x+\sin^{-1}y=\frac{2\pi}{3}$ then $\cos^{-1}x+\cos^{-1}y =$ ______
   (a) $\frac{\pi}{6}$ (b) $\frac{\pi}{4}$
   (c) $\frac{\pi}{3}$ (d) $\frac{\pi}{2}$

9. If $\sin^{-1}\frac{x}{5}+\operatorname{cosec}^{-1}\frac{5}{9}=\frac{\pi}{2}$ then $x =$ ______
   (a) 1 (b) 2
   (c) 3 (d) 4

10. Solution of $\sin^{-1}\left(\frac{2a}{1+a^2}\right)-\cos^{-1}\left(\frac{1-b^2}{1+b^2}\right)=\tan^{-1}\frac{2x}{1-x^2}$ is ______
    (a) $\frac{a-b}{1-ab}$ (b) $\frac{a-b}{1+ab}$
    (c) $\frac{1+ab}{a-b}$ (d) $\frac{1-ab}{a-b}$

11. If $A=\tan^{-1}x$ then value of $\sin 2A =$ ______
    (a) $\frac{2x}{1-x^2}$ (b) $\frac{2x}{\sqrt{1-x^2}}$
    (c) $\frac{2x}{1+x^2}$ (d) None of these

12. Value of $\sec^2\left(\tan^{-1} 2\right)+\operatorname{cosec}^2\left(\cot^{-1} 3\right)=$ ______

(a) 16 (b) 14

(c) 15 (d) None of these

13. If value of $\sin^{-1} x=\frac{\pi}{5}$ for some $x\in(-1,1)$ then value of $\cos^{-1} x=$ ______

(a) $\frac{3\pi}{10}$ (b) $\frac{5\pi}{10}$

(c) $\frac{7\pi}{10}$ (d) None of these

14. Value of $\tan\left(\frac{\pi}{4}+2\cot^{-1} 3\right)=$ ______

(a) 5 (b) 2

(c) 7 (d) None of these

15. $\tan^{-1}\left(\frac{x}{y}\right)-\tan^{-1}\left(\frac{x-y}{x+y}\right)=$ ______

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{3}$

(c) $\frac{\pi}{2}$ (d) None of these

16. $\tan\left[\frac{1}{2}\sin^{-1}\frac{2x}{1+x^2}+\frac{1}{2}\cos^{-1}\frac{1-y^2}{1+y^2}\right]=$ ___

(a) $\frac{x+y}{1-xy}$ (b) $\frac{x-y}{1+xy}$

(c) $x+y$ (d) None of these

17. Value of $\sin\cos^{-1}\tan\sec^{-1}\sqrt{2}=$ ____

(a) 0 (b) $\frac{\pi}{2}$

(c) $\frac{\pi}{3}$ (d) None of these

18. Principal value of $\sin^{-1}\frac{1}{2}$ is ______

(a) $\frac{\pi}{3}$ (b) $\frac{2\pi}{3}$

(c) $\frac{\pi}{6}$ (d) None of these

19. The curve $y^2=\sin\left(\sin^{-1} x\right)$ is a ______

(a) line (b) circle

(c) parabola (d) ellipse

20. If $x+y=\frac{1}{2}=xy$ then $\tan^{-1} x+\tan^{-1} y=$ ______

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{2}$

(c) π (d) None of these

ANSWER KEYS

1.	(a)	6.	(d)	11.	(c)	16.	(a)
2.	(b)	7.	(b)	12.	(c)	17.	(a)
3.	(b)	8.	(c)	13.	(a)	18.	(c)
4.	(b)	9.	(c)	14.	(c)	19.	(c)
5.	(a)	10.	(b)	15.	(a)	20.	(a)

B. Long Answer Type Questions

1. Prove that

$$\sin^{-1}\frac{1}{3}+\sin^{-1}\frac{1}{3\sqrt{11}}+\sin^{-1}\frac{3}{\sqrt{11}}=\frac{\pi}{2}$$

2. Prove that

$$\cot^{-1}\left(\frac{ab+1}{a-b}\right)+\cot^{-1}\left(\frac{bc+1}{b-c}\right)$$

$$+\cot^{-1}\left(\frac{ca+1}{c-a}\right)=0$$

3. Prove that

$$\cos^{-1}\left(\frac{-1}{\sqrt{1+x}}\right)-\cos^{-1}$$

$$\sqrt{\frac{1}{1+x}}=\frac{\pi}{2},x>0$$

4. If $\tan^{-1}x+\tan^{-1}y+\tan^{-1}z=\pi$ then prove that $x+y+z=xyz$.

5. If $\sin^{-1}x+\sin^{-1}y+\sin^{-1}z=\pi$ then prove that $x\sqrt{1-x^2}+y\sqrt{1-y^2}+z\sqrt{1-z^2}=2xyz$

6. If $\cos^{-1}x+\cos^{-1}y+\cos^{-1}z=\pi$ then prove that $x^2+y^2+z^2+2xyz=1$

7. If $r^2=x^2+y^2+z^2$ then prove that

$$Tan^{-1}\frac{yz}{xr}+Tan^{-1}\frac{xz}{yr}+Tan^{-1}\frac{xy}{zr}=\frac{\pi}{2}$$

8. $Tan^{-1}\sqrt{\frac{xr}{yz}}+Tan^{-1}\sqrt{\frac{yr}{xz}}+Tan^{-1}\sqrt{\frac{zr}{xy}}=\pi$

where $r=x+y+z$

9. Solve

$$Tan^{-1}(x+1)+Tan^{-1}(x-1)=\tan^{-1}\frac{6}{17}\quad x>0.$$

10. Solve

$$Tan^{-1}\frac{2x}{1-x^2}=\sin^{-1}\frac{2a}{1+a^2}-\cos^{-1}\frac{1-b^2}{1+b^2}$$

11. Solve

$$Tan^{-1}\left(\frac{x-1}{x-2}\right)+Tan^{-1}\left(\frac{x+1}{x+2}\right)=\frac{\pi}{4}$$

12. Prove that

$$Tan^{-1}+Tan^{-1}2+Tan^{-1}3=\pi$$

$$=2\left(Tan^{-1}1+Tan^{-1}\frac{1}{2}+Tan^{-1}\frac{1}{3}\right)$$

13. $Tan\left[Tan^{-1}a+Tan^{-1}b+Tan^{-1}c\right]$

$$=\cot\left[\cot^{-1}a+\cot^{-1}b+\cot^{-1}c\right]$$

ANSWER HINTS

1. $\sin^{-1}\frac{1}{3}+\sin^{-1}\frac{1}{3\sqrt{11}}+\sin^{-1}\frac{3}{\sqrt{11}}$

$=\sin^{-1}\frac{1}{3}+\sin^{-1}\left[\frac{1}{3\sqrt{11}}\sqrt{1-\frac{9}{11}}+\frac{3}{\sqrt{11}}\right.$

$\left.\sqrt{1-\left(\frac{1}{3\sqrt{11}}\right)^2}\right]$

$=\sin^{-1}\frac{1}{3}+\sin^{-1}\left[\frac{1}{3\sqrt{11}}.\frac{\sqrt{2}}{\sqrt{11}}+\frac{3}{\sqrt{11}}\sqrt{\frac{98}{99}}\right]$

$=\sin^{-1}\frac{1}{3}+\sin^{-1}\left[\frac{\sqrt{2}}{33}+\frac{3}{\sqrt{11}}\frac{7\sqrt{2}}{3\sqrt{11}}\right]$

$=\sin^{-1}\frac{1}{3}+\sin^{-1}\left[\frac{\sqrt{2}}{33}+\frac{7\sqrt{2}}{11}\right]$

$=\sin^{-1}\frac{1}{3}+\sin^{-1}\left[\frac{\sqrt{2}+21\sqrt{2}}{33}\right]$

$=\sin^{-1}\frac{1}{3}+\sin^{-1}\frac{22\sqrt{2}}{33}=\sin^{-1}\frac{1}{3}+\sin^{-1}\frac{2\sqrt{2}}{3}$

Then again apply $\sin^{-1}x+\sin^{-1}y$.

2. $\cot^{-1}\left(\frac{ab+1}{a-b}\right)+\cot^{-1}\left(\frac{bc+1}{b-c}\right)+\cot^{-1}\left(\frac{ca+1}{c-a}\right)$

$=\tan^{-1}\left(\frac{a-b}{1+ab}\right)+\tan^{-1}\left(\frac{b-c}{bc+1}\right)+\tan^{-1}\left(\frac{c-a}{1+ca}\right)$

$=\tan^{-1}a-\tan^{-1}b+\tan^{-1}b-\tan^{-1}c+\tan^{-1}c$
$-\tan^{-1}a=0$

4. Let $\tan^{-1}x=\alpha,\ \tan^{-1}y=\beta,\ \tan^{-1}z=\gamma$

$\tan^{-1}x+\tan^{-1}y+\tan^{-1}z=\pi$

$\Rightarrow\alpha+\beta+\gamma=\pi$

$\alpha+\beta=\pi-\gamma\Rightarrow\tan(\alpha+\beta)=\tan(\pi-\gamma)$

$\Rightarrow\frac{\tan\alpha+\tan\theta}{1-\tan\alpha\tan\beta}=-\tan\gamma$

Then replace the values of $\tan\alpha,\tan\beta,\tan\gamma$.

5. Let $\sin^{-1}x=\alpha,\sin^{-1}y=\beta,\sin^{-1}z=\gamma$

$\Rightarrow x=\sin\alpha, y=\sin\beta, z=\sin\gamma$

$\sin^{-1}x+\sin^{-1}y+\sin^{-1}z=\pi\Rightarrow\alpha+\beta+\gamma=\pi$

LHS $=x\sqrt{1-x^2}+y\sqrt{1-y^2}+z\sqrt{1-z^2}$

$=\sin\alpha\sqrt{1-\sin^2\alpha}+\sin\beta\sqrt{1-\sin^2\beta}$

$+\sin\gamma\sqrt{1-\sin^2\gamma}$

$=\sin\alpha\cos\alpha+\sin\beta\cos\beta+\sin\gamma\cos\gamma$

$=\frac{1}{2}[2\sin\alpha\cos\alpha+2\sin\beta\cos\beta+2\sin\gamma\cos\gamma]$

$=\frac{1}{2}[\sin 2\alpha+\sin 2\beta+\sin 2\gamma]$

Let $\cos^{-1}x=\alpha,\ \cos^{-1}y=\beta,\ \cos^{-1}z=\gamma$

$\cos^{-1}x+\cos^{-1}y+\cos^{-1}z=\pi\Rightarrow\alpha+\beta+\gamma=\pi$

Here $x=\cos\alpha, y=\cos\beta, z=\cos\gamma$

LHS $x^2+y^2+z^2=2xyz=\cos^2\alpha+\cos^2\beta$

$+\cos^2\gamma+2\cos\alpha\cos\beta\cos\gamma$

$$=\frac{1+\cos 2\alpha}{2}+\frac{1+\cos 2\beta}{2}+\cos^2\gamma$$

$$+2\cos\alpha\cos\beta\cos\gamma$$

$$=\left(\frac{1}{2}+\frac{1}{2}\right)+\frac{1}{2}\left[\cos 2\alpha+\cos 2\beta\right]$$

$$+\cos^2\gamma+2\cos\alpha\cos\beta\cos\gamma$$

$$=1+\frac{1}{\not 2}\not 2\cos\frac{2\alpha+2\beta}{2}\cos\frac{2\alpha-2\beta}{2}$$

$$+\cos^2\gamma+2\cos\alpha\cos\beta\cos\gamma$$

$$=1+\cos(\alpha+\beta)\cos(\alpha-\beta)+\cos^2\gamma$$

$$+2\cos\alpha\cos\beta\cos\gamma$$

$$=1-\cos\gamma\cos(\alpha-\beta)+\cos^2\gamma+2\cos\alpha\cos\beta\cos\gamma$$

Then proceed

$$\left|\begin{array}{l}\alpha+\beta+\gamma=\pi\\ \alpha+\beta=\pi-\gamma\\ \cos(\alpha+\beta)=\cos(\pi-\gamma)=-\cos\gamma\end{array}\right.$$

7. $Tan^{-1}\frac{yz}{xr}+Tan^{-1}\frac{xz}{yr}+Tan^{-1}\frac{xy}{zr}$

$$=Tan^{-1}\left(\frac{\frac{yz}{xr}+\frac{dz}{yr}}{1-\frac{bz}{xr}.\frac{xz}{yr}}\right)+Tan^{-1}\frac{xy}{zr}$$

$$=T\,an^{-1}\left(\frac{\frac{y^2z+x^2z}{xyz}}{1-\frac{z^2}{r^2}}\right)+Tan^{-1}\frac{xy}{zr}$$

$$=Tan^{-1}\left(\frac{z\frac{(x^2+y^2)}{xyr}}{\frac{r^2-z^2}{r^2}}\right)+Tan^{-1}\frac{xy}{zr}\Big|r^2-z^2=x^2+y^2$$

$$=Tan^{-1}\left(\frac{zr}{xy}\right)+Tan^{-1}\frac{xy}{zr}$$

Then apply $\tan^{-1}x+\tan^{-1}y$

10. Put $x=\tan\alpha, a=\tan\beta,\ b=\tan\gamma$

Then use $\tan\alpha=\frac{2\tan\alpha}{1-\tan^2\alpha},\sin 2\beta$

$$=\frac{2\tan\beta}{1+\tan^2\beta},\cos 2\beta=\frac{1-\tan^2\gamma}{1+\tan^2\gamma}$$

13. $Tan\left[Tan^{-1}a+Tan^{-1}b+Tan^{-1}c\right]$

$$=Tan\left[\frac{\pi}{2}-\cot^{-1}a+\frac{\pi}{2}-\cot^{-1}b+\frac{\pi}{2}-\cot^{-1}c\right]$$

$$=Tan\left[3\frac{\pi}{2}-\left\{\cot^{-1}a+\cot^{-1}b+\cot^{-1}c\right\}\right]$$

$$=\cot\left[\cot^{-1}a+\cot^{-1}b+\cot^{-1}c\right]$$

ADDENDUM

1. Principal value of $\sin^{-1}\left(\sin\frac{2\pi}{3}\right) = ...?$

(a) $\frac{\pi}{3}$ (b) $\frac{2\pi}{3}$

(c) $\pi + \frac{\pi}{3}$ (d) None of these

2. Find value of $\sin\left[\cot^{-1}\left[\tan\left(\cos^{-1} x\right)\right]\right]$

(a) $\sin x$ (b) x

(c) $\sin^{-1} x$ (d) None of these

3. Evaluate $2\sin^{-1}\frac{1}{2} + \cos^{-1}\frac{1}{2}$

(a) $\frac{\pi}{3}$ (b) $\frac{2\pi}{3}$

(c) $\frac{5\pi}{3}$ (d) None of these

4. Evaluate $\cos\left(2\cos^{-1}(0.8)\right)$

(a) 2.8 (b) 0.28

(c) 0.028 (d) None of these

5. Find value of $\sin\left[\frac{1}{4}\sin^{-1}\frac{\sqrt{63}}{8}\right]$

(a) $\frac{1}{2}$ (b) $\frac{1}{\sqrt{2}}$

(c) $\frac{1}{2\sqrt{2}}$ (d) None of these

6. If $\cot^{-1} x + \cot^{-1} y + \cot^{-1} z = \frac{\pi}{2}$, then $x + y + z = ?$

(a) $x y z$ (b) $\frac{1}{x} + \frac{1}{y} + \frac{1}{z}$

(c) $\frac{1}{xyz}$ (d) None of these

7. Evaluate $\sin^{-1}\frac{3}{5} + \tan^{-1}\frac{1}{7}$

(a) π (b) $\frac{\pi}{2}$

(c) $\frac{\pi}{3}$ (d) $\frac{\pi}{4}$

8. If $x + y + z = xyz$, then $\tan^{-1} x + \tan^{-1} y + \tan^{-1} z = ...$

(a) $\frac{\pi}{2}$ (b) 0

(c) $\frac{\pi}{4}$ (d) $\frac{\pi}{6}$

9. If $\sin^{-1} x + \sin^{-1} y = \frac{2\pi}{3}$, then $\cos^{-1} x + \cos^{-1} y = ?$

(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{4}$

(c) $\frac{\pi}{3}$ (d) $\frac{\pi}{2}$

10. If $\sin^{-1}\frac{x}{5} + \operatorname{cosec}^{-1}\frac{5}{9} = \frac{\pi}{2}$, then x = ...?

(a) 1 (b) 2

(c) 3 (d) 4

11. The solution of

$\sin^{-1}\left(\frac{2a}{1+a^2}\right) - \cos^{-1}\left(\frac{1-b^2}{1+b^2}\right) = \tan^{-1}\frac{2x}{1-x^2}$ is

(a) $\frac{a-b}{1-ab}$ (b) $\frac{a-b}{1+ab}$

(c) $\frac{1+ab}{a-b}$ (d) $\frac{1-ab}{a-b}$

12. If $A = \tan^{-1} x$, then value of $\sin 2A = ...$

(a) $\frac{2x}{1-x^2}$ (b) $\frac{2x}{\sqrt{1-x^2}}$

(c) $\frac{2x}{1+x^2}$ (d) None of these

13. Evaluate :

$\sec^2(\tan^{-1}2)+\operatorname{cosec}^2(\cot^{-1}3)= .....?$

(a) 16 (b) 14

(c) 15 (d) None of these

14. $\Delta ABC, \angle A=\frac{\pi}{2}$. Then

$\tan^{-1}\frac{b}{a+c}+\tan^{-1}\frac{c}{a+b}= ....?$

where a, b, c are sides of ΔABC.

(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{4}$

(c) $\frac{\pi}{3}$ (d) None of above

15. If value of $\sin^{-1}x=\frac{\pi}{5}$ for some $x\in(-1,1)$, then value of $\cos^{-1}x= ...?$

(a) $\frac{3\pi}{10}$ (b) $\frac{5\pi}{10}$

(c) $\frac{7\pi}{10}$ (d) None of these

16. Evaluate $\tan\left(\frac{\pi}{4}+2\cot^{-1}3\right)$

(a) 5 (b) 2

(c) 9 (d) None of these

17. Evaluate $\tan^{-1}\left(\frac{x}{y}\right)-\tan^{-1}\frac{x-y}{x+y}$

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{3}$

(c) $\frac{\pi}{2}$ (d) None of these

18. Evaluate : $\sin^{-1}\frac{1}{\sqrt{5}}+\cos^{-1}\frac{3}{\sqrt{10}}$

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{3}$

(c) $\frac{\pi}{2}$ (d) None of these

19. Evaluate :

$\tan\left[\frac{1}{2}\sin^{-1}\frac{2x}{1+x^2}+\frac{1}{2}\cos^{-1}\frac{1-y^2}{1+y^2}\right]$

(a) $\frac{x+y}{1-xy}$ (b) $\frac{x-y}{1+xy}$

(c) $x+y$ (d) None of these

20. Evaluate : $\sin\cos^{-1}\tan\sec^{-1}\sqrt{2}$

(a) 0 (b) $\frac{\pi}{2}$

(c) $\frac{\pi}{3}$ (d) None

ANSWER KEYS

1. (a) $\frac{\pi}{3}$

Sol. $\sin^{-1}\left[\sin\frac{2\pi}{3}\right]=\sin^{-1}\left[\sin\left(\pi-\frac{\pi}{3}\right)\right]$

$$=\sin^{-1}\sin\frac{\pi}{3}=\frac{\pi}{3}$$

2. (b) x

Sol. Here $\sin\left[\cot^{-1}\left\{\tan\left(\cos^{-1}x\right)\right\}\right]$

$$=\sin\left[\cot^{-1}\left\{\tan\left(\tan^{-1}\frac{\sqrt{1-x^2}}{x}\right)\right\}\right]$$

$\because \cos^{-1}x=y\Rightarrow x=\cos y$

$\Rightarrow \tan y=\frac{\sin y}{\cos y}=\frac{1-x^2}{x}$

$\Rightarrow y=\tan^{-1}\frac{\sqrt{1-x^2}}{x}$

$$=\sin\left[\cot^{-1}\frac{\sqrt{1-x^2}}{x}\right]$$

$$=\sin\ \sin^{-1}x=x$$

$$\because \cot^{-1}\frac{\sqrt{1-x^2}}{x}=z$$

$$\Rightarrow \cot z=\frac{\sqrt{1-x^2}}{x}$$

$$\Rightarrow \cos ecz=\sqrt{1+\cot^2 z}$$

$$=\sqrt{1+\frac{1-x^2}{x^2}}=\sqrt{\frac{x^2+1-x^2}{x^2}}=\sqrt{\frac{1}{x^2}}=\frac{1}{x}$$

$$\Rightarrow \sin z=x\Rightarrow z=\sin^{-1}x$$

3. (b) $\frac{2\pi}{3}$

Sol. $2\sin^{-1}\frac{1}{2}+\cos^{-1}\frac{1}{2}$

$$=2\times\frac{\pi}{6}+\frac{\pi}{3}=\frac{\pi}{3}+\frac{\pi}{3}=\frac{2\pi}{3}$$

4. (b) 0.28

Sol. Here $\cos\left(2\cos^{-1}(0.8)\right)=.....?$

since $2\cos^{-1}x=\cos^{-1}\left(2x^2-1\right)$

$$\Rightarrow 2\cos^{-1}(0.8)=\cos^{-1}\left[2(0.8)^2-1\right]$$

$$=\cos^{-1}\left[2\times 0.64-1\right]$$

$$=\cos^{-1}\left[1.28-1\right]$$

$$=\cos^{-1}\left[0.28\right]$$

$$\therefore \cos\left(2\cos^{-1}(0.8)\right)$$

$$=\cos\ \cos^{-1}(0.28)=0.28$$

5. (b) $\frac{1}{\sqrt{2}}$

Sol. Here $\sin\left[\frac{1}{4}\sin^{-1}\frac{\sqrt{63}}{8}\right]$

$=\sin\theta$, where $\theta=\frac{1}{4}\sin^{-1}\frac{\sqrt{63}}{8}$

$$\Rightarrow \sin^{-1}\frac{\sqrt{63}}{8}=4\theta$$

$$\Rightarrow \sin 4\theta=\frac{\sqrt{63}}{8}$$

$$\Rightarrow \cos 4\theta=\sqrt{1-\sin^2 4\theta}$$

$$=\sqrt{1-\frac{63}{64}}=\sqrt{\frac{1}{64}}=\frac{1}{8}$$

$$\Rightarrow \cos 2\theta=\sqrt{\frac{1+\cos 4\theta}{2}}=\sqrt{\frac{1+\frac{1}{8}}{2}}=\frac{3}{4}$$

$$\Rightarrow \sin\theta=\sqrt{\frac{1-\cos 2\theta}{2}}=\sqrt{\frac{1-\frac{3}{4}}{2}}=\sqrt{\frac{1}{8}}$$

$$=\frac{1}{2\sqrt{2}}$$

6. (a) $x\,y\,z$

Sol. Here $\cot^{-1}x+\cot^{-1}y+\cot^{-1}z=\frac{\pi}{2}$

$$\Rightarrow \cot^{-1}\frac{xyz-x-y-z}{xy+yz+xz-1}=\frac{\pi}{2}$$

$$\Rightarrow \frac{xyz-x-y-z}{xy+yz+xz-1}=\cot\frac{\pi}{2}=0$$

$$\Rightarrow x+y+z=xyz$$

7. (d) $\frac{\pi}{4}$

Sol. Here $\sin^{-1}\frac{3}{5}+\tan^{-1}\frac{1}{7}$

$$\frac{3/5}{\sqrt{1-\frac{9}{25}}}+\tan^{-1}\frac{1}{7}$$

$$\because \sin^{-1}x=\tan^{-1}\frac{x}{\sqrt{1-x^2}}$$

$= \tan^{-1}\frac{3}{4} + \tan^{-1}\frac{1}{7} = \tan^{-1}\frac{\frac{3}{4}+\frac{1}{7}}{1-\frac{3}{4}\times\frac{1}{7}}$

$= \tan^{-1}\left(\frac{21+4}{28-3}\right) = \tan^{-1}\left(\frac{25}{25}\right) = \tan^{-1}1 = \frac{\pi}{4}$

8. (b) 0

Sol. Here $\tan^{-1}x + \tan^{-1}y + \tan^{-1}z$

$= \tan^{-1}\left(\frac{x+y+z-xyz}{1-xy-yz-xz}\right)$

$= \tan^{-1}\frac{0}{1-xy-yz-xz} \because x+y+z = xyz$

$= \tan^{-1}(0) = 0$

9. (c) $\frac{\pi}{3}$

Sol. Here $\sin^{-1}x + \sin^{-1}y = \frac{2\pi}{3}$

$\Rightarrow \frac{\pi}{2} - \cos^{-1}x + \frac{\pi}{2} - \cos^{-1}y = \frac{2\pi}{3}$

$\left(\because \sin^{-1}x + \cos^{-1}x = \frac{\pi}{2}\right)$

$\Rightarrow \pi - \left(\cos^{-1}x + \cos^{-1}y\right) = \frac{2\pi}{3}$

$\Rightarrow \cos^{-1}x + \cos^{-1}y = \pi - \frac{2\pi}{3} = \frac{\pi}{3}$

10. (c) 3

Sol. Here $\sin^{-1}\frac{x}{5} + \operatorname{cosec}^{-1}\frac{5}{9} = \frac{\pi}{2}$

$\Rightarrow \sin^{-1}\frac{x}{5} = \frac{\pi}{2} - \operatorname{cosec}^{-1}\frac{5}{4}$

$\Rightarrow \sin^{-1}\frac{x}{5} = \sec^{-1}\frac{5}{4}$

$\left(\because \sec^{-1}x + \operatorname{cosec}^{-1}x = \frac{\pi}{2}\right)$

$= \cos^{-1}\frac{4}{5}\left(\because \cos^{-1}x = \sec^{-1}\frac{1}{x}\right)$

$= \sin^{-1}\sqrt{1-\frac{16}{25}} \because \cos^{-1}x = \sin^{-1}\sqrt{1-x^2}$

$= \sin^{-1}\frac{3}{5}$

$\Rightarrow \frac{x}{5} = \frac{3}{5} \Rightarrow x = 3$

11. (b) $\frac{a-b}{1+ab}$

Sol. Here $\sin^{-1}\left(\frac{2a}{1+a^2}\right) - \cos^{-1}\left(\frac{1-b^2}{1+b^2}\right) = \tan^{-1}\frac{2n}{1-n^2}$

$\Rightarrow 2\tan^{-1}a - 2\tan^{-1}b = 2\tan^{-1}x$

$\because 2\tan^{-1}x = \tan^{-1}\frac{2x}{1-x^2}$

$= \sin^{-1}\frac{2x}{1+x^2}$

$= \cos^{-1}\frac{1-x^2}{1+x^2}$

$\Rightarrow \tan^{-1}x = \tan^{-1}a - \tan^{-1}b$

$= \tan^{-1}\frac{a-b}{1+ab}$

$\Rightarrow x = \frac{a-b}{1+ab}$

12. (c) $\frac{2x}{1+x^2}$

Sol. $A = \tan^{-1}x \Rightarrow x = \tan A$

$\therefore \sin 2A = \frac{2\tan A}{1+\tan^2 A} = \frac{2x}{1+x^2}$

13. (c) 15

Sol. Here $\sec^2\left(\tan^{-1}2\right) + \operatorname{cosec}^2\left(\cot^{-1}3\right)$

$= \sec^2\alpha + \operatorname{cosec}^2\beta$

where

$\alpha = \tan^{-1}2$ & $\beta = \operatorname{Cot}^{-1}3$

$\Rightarrow \cot\beta = 3$

$\Rightarrow \tan\alpha = 2$

$\Rightarrow \sec^2\alpha = 1 + \tan^2\alpha = 1 + 4 = 5$

$\text{Sec}^2\alpha + \text{Co}\sec^2\beta$

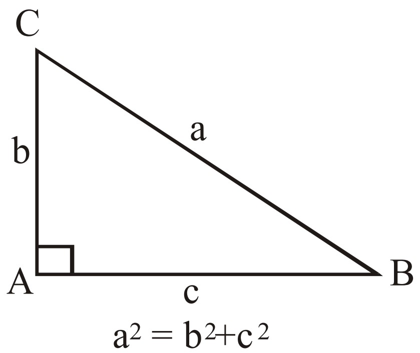
$\text{Sec}^2\alpha + \text{Co}\sec^2\beta = 5 + 10 \quad B = \cot^{-1}3 = 15$

$\Rightarrow \cot\beta = 3$

$\Rightarrow (\cos ec^2\beta = 1 + \cot^2\beta = 1 + 9 = 10)$

14. (b) $\frac{\pi}{4}$

Sol. $\tan^{-1}\frac{b}{a+c} + \tan^{-1}\frac{c}{a+b}$



$$\tan^{-1}\frac{\frac{b}{a+c}+\frac{c}{a+b}}{1-\frac{b}{a+c}\times\frac{c}{a+b}}$$

$$= \tan^{-1}\frac{ab+b^2+ac+c^2}{(a+c)(a+b)}\times\frac{(a+c)(a+b)}{(a+c)(a+b)-bc}$$

$$= \tan^{-1}\frac{ab+ac+a^2}{a^2+ac+ab+bc-bc}$$

$$= \tan^{-1}\frac{a(a+b+c)}{a(a+b+c)} \because b^2+c^2=a^2$$

$$= \tan^{-1}1 = \frac{\pi}{4}$$

15. (a) $\frac{3\pi}{10}$

Sol. $\sin^{-1}x = \frac{\pi}{5}$

$$\Rightarrow \cos^{-1}x = \frac{\pi}{2} - \frac{\pi}{5}$$

$$\left(\because \sin^{-1}x + \cos^{-1}x = \frac{\pi}{2}\right)$$

$$= \frac{3\pi}{10}$$

16. (b) 2

Sol. $\tan\left(\frac{\pi}{4} + 2\cot^{-1}3\right)$

$$= \tan\left(\frac{\pi}{4} + 2\tan^{-1}\frac{1}{3}\right) = \tan\left[\frac{\pi}{4} + \tan^{-1}\frac{2\times\frac{1}{3}}{1-\frac{1}{9}}\right]$$

$$= \tan\left[\frac{\pi}{4} + \tan^{-1}\frac{2}{3}\times\frac{9}{8}\right] = \tan\left[\frac{\pi}{4} + \tan^{-1}\frac{3}{4}\right]$$

$$= \frac{\tan\frac{\pi}{4} + \tan\tan^{-1}\frac{3}{4}}{1 - \tan\frac{\pi}{4}.\tan\tan^{-1}\frac{3}{4}} = \frac{1+\frac{3}{4}}{1-\frac{3}{4}}$$

$$= \frac{7}{4}\times\frac{4}{1} = 7$$

17. (a) $\frac{\pi}{4}$

Sol. $\tan^{-1}\left(\frac{x}{y}\right) - \tan^{-1}\frac{x-y}{x+y}$

$$= \tan^{-1}\frac{\frac{x}{y} - \frac{x-y}{x+y}}{1+\frac{x}{y}\left(\frac{x-y}{x+y}\right)}$$

$$= \tan^{-1}\frac{x(x+y)-y(x-y)}{y(x+y)}$$

$$\times\frac{y(x+y)}{y(x+y)+x(x-y)}$$

$$= \tan^{-1}\frac{x^2+xy-xy+y^2}{xy+y^2+x^2-xy} = \tan^{-1}\frac{x^2+y^2}{x^2+y^2} = \tan^{-1}1 = \frac{\pi}{4}$$

18.. (a) $\frac{\pi}{4}$

Sol. $\sin^{-1}\frac{1}{\sqrt{5}}+\cos^{-1}\frac{1}{\sqrt{10}}$

$\sin^{-1}\frac{1}{\sqrt{5}}+\frac{\pi}{2}-\sin^{-1}\frac{3}{\sqrt{10}}$

$=\frac{\pi}{2}+\left(\sin^{-1}\frac{1}{\sqrt{5}}-\sin\frac{3}{\sqrt{10}}\right)$

$=\frac{\pi}{2}+\sin^{-1}\left[\frac{1}{\sqrt{5}}\sqrt{1-\frac{9}{10}}-\frac{3}{\sqrt{10}}\sqrt{1-\frac{1}{5}}\right]$

$\because \sin^{-1}x-\sin^{-1}y=\sin^{-1}\left\{x\sqrt{1-y^2}-y\sqrt{1-x^2}\right\}$

$=\frac{\pi}{2}+\sin^{-1}\left(\frac{1}{\sqrt{5}\sqrt{10}}-\frac{6}{\sqrt{5}\sqrt{10}}\right)$

$=\frac{\pi}{2}+\sin^{-1}\left(\frac{-5}{\sqrt{5}\sqrt{10}}\right)=\frac{\pi}{2}+\sin^{-1}\left[-\frac{1}{\sqrt{2}}\right]$

$=\frac{\pi}{2}-\frac{\pi}{4}=\frac{\pi}{4}$

19.. (a) $\frac{x+y}{1-xy}$

Sol. $\tan\left[\frac{1}{2}\sin^{-1}\frac{2x}{1+x^2}+\frac{1}{2}\cos^{-1}\frac{1-y^2}{1+y^2}\right]$

$=\tan\left[\frac{1}{2}\times 2\tan^{-1}x+\frac{1}{2}\times 2\tan^{-1}y\right]$

$=\tan\left[\tan^{-1}x+\tan^{-1}y\right]$

$=\tan\left[\tan^{-1}\frac{x+y}{1-xy}\right]=\frac{x+y}{1-xy}$

20.. (a) 0

Sol. $\sin\cos^{-1}\tan\sec^{-1}\sqrt{2}$

$=\sin\cos^{-1}\tan\frac{\pi}{4}\ \because \sec\frac{\pi}{4}=\sqrt{2}$

$=\sin\cos^{-1}1$

$=\sin 0\ (\because \cos 0=1)$

$=0$

Unit - II

ALGEBRA

CHAPTER - 1

MATRICES

A. Multiple Choice Questions (MCQ)

1. If A is a non singular matrix then $\frac{1}{|A|} A \times AdjotA =$

   (a) zero matrix (b) unit matrix

   (c) A^T (d) None of these

2. If the coordinates (x, y) of the part P satisfy the equation $\begin{bmatrix} \frac{x}{3} & \frac{y}{2} \end{bmatrix} \begin{bmatrix} \frac{x}{3} & \frac{y}{3} \end{bmatrix} = 1$, the point describes a ______

   (a) circle (b) parabola

   (c) ellipse (d) hyperbola

3. If A is any $m \times n$ matrix such that AB and BA are both defined then B is an

   (a) $m \times n$ matrix (b) $n \times n$ matrix

   (c) $n \times m$ matrix (d) $m \times m$ matrix

4. A matrix $A = [a_{ij}]$ is an upper triangular matrix if

   (a) It is a square matrix and $a_{ij} = 0$ if $i < j$

   (b) It is a square matrix and $a_{ij} = 0$ if $i > j$

   (c) It is not a square matrix and $a_{ij} = 0$ if $i < j$

   (d) It is not a square matrix and $a_{ij} = 0$ if $i > j$

5. If $E(\theta) = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$ then $E(\alpha)E(\beta)$ equals to

   (a) E(0) (b) $E(\alpha\beta)$

   (c) $E(\alpha + \beta)$ (d) $E(\alpha - \beta)$

6. The matrix A satisfying the equation $\begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix} A = \begin{bmatrix} 1 & 1 \\ 0 & -1 \end{bmatrix}$ is ______

   (a) $\begin{bmatrix} 1 & 4 \\ -1 & 0 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & -4 \\ 1 & 0 \end{bmatrix}$

   (c) $\begin{bmatrix} 1 & 4 \\ 0 & -1 \end{bmatrix}$ (d) None of these

7. If $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} J = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$ then B = ______

   (a) $I\cos\theta + J\sin\theta$

   (b) $I\sin\theta + J\cos\theta$

   (c) $I\cos\theta - J\sin\theta$

   (d) $-I\cos\theta + J\sin\theta$

8. If A is a square matrix such that $AA^T = I = A^T A$ then A is ______

(a) symmetric matrix

(b) skew symmetric matrix

(c) diagonal matrix

(d) orthogonal matrix

9. If A and B are two square matrices such that AB = A and BA = B then

(a) A, B are idempotent

(b) only A is idempotent

(c) only B is idempotent

(d) None of these

10. The inverse of a symmetric matrix is _____

(a) symmetric

(b) skew symmetric

(c) diagonal

(d) None of these

11. If A and B are matrices of same order then $(A+B)^2 = A^2 + B^2 + 2AB$ is possible if and only if ______

(a) AB = I (b) BA = I

(c) AB = BA (d) None of these

12. If A and B are square matrices of same order then adj(AB) = ______

(a) (adj A) (adj B)

(b) (adj B) (adj A)

(c) adj A + adj B

(d) adj A – adj B

13. If A is a square matrix of order $n \times n$ and k is a scalar then adj (KA) =______

(a) $k\ adj\ A$ (b) $k^n\ adj\ A$

(c) $k^{n-1}\ adj\ A$ (d) $k^{n+1}\ adj\ A$

14. If A is a square matrix of order $n \times n$ then adj (adj A) = ______

(a) $|A^n| A$ (b) $|A|^{n-1} A$

(c) $|A|^{n-2} A$ (d) $|A|^{n-3} A$

15. If A is a signular matrix then A. adj A is ______

(a) Identity matrix

(b) null matrix

(c) scalar matrix

(d) None of these

16. If $A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ and $n \in N$ then A^n =_____

(a) $2^n A$ (b) $2^{n-1} A$

(c) nA (d) None of these

17. If $A = \begin{bmatrix} 1 & 3 \\ 3 & 4 \end{bmatrix}$ and $A^2 - kA - 5I_2 = 0$ then value of k is ____

(a) 3 (b) 5

(c) 7 (d) –7

18. If $A = [a_{ij}]$ is a square matrix of order $n \times n$ and k is a scalar then |kA| =______

(a) $k^n |A|$ (b) $k|A|$

(c) $k^{n-1} |A|$ (d) None of these

19. If A and B are matrices such that AB and A+B both are defined then

(a) A and B are any two matrices

(b) A and B are square matrices not necessarily of same order

(c) A and B are square matrices of same order

(d) number of columns of A = number of columns of B.

20. If $\begin{bmatrix} 1 & -\tan\theta \\ \tan\theta & 1 \end{bmatrix}\begin{bmatrix} 1 & \tan\theta \\ -\tan\theta & 1 \end{bmatrix}$

$=\begin{bmatrix} a & -b \\ b & a \end{bmatrix}$ then

(a) a = 1, b = 1

(b) $\cos 2\theta, b = \sin 2\theta$

(c) $a = \sin 2\theta, b = \cos 2\theta$

(d) None of these

21. Which of the following is incorrect

(a) $A^2 - B^2 = (A+B)(A-B)$

(b) $(A^T)^T = A$

(c) $(AB)^n = A^n B^n$

(d) $(A-I)(I+A) = 0 \Leftrightarrow A^2 = I$

22. If A is invertible matrix then which of the following is correct

(a) A^{-1} is multivalued

(b) A^{-1} is singular

(c) $(A^{-1})^T = (A^T)^{-1}$

(d) $|A| \neq 0$

23. For a matrix $A = A^2 + I = 0$ where I is the identity matrix then A = ____

(a) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ (b) $\begin{bmatrix} -i & 0 \\ 0 & -i \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 2 \\ -1 & 1 \end{bmatrix}$ (d) $\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$

24. If A is an orthogonal matrix then

(a) |A| = 0 (b) $|A| = \pm 1$

(c) $|A| = \pm 2$ (d) None of these

25. Order of $[x\, y\, z]\begin{bmatrix} a & h & g \\ h & b & f \\ g & f & c \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix}$ is ___

(a) 3×1 (b) 1×1

(c) 1×3 (d) 3×3

26. If A and B are two matrices such that AB = B and BA = A then $A^2 + B^2 =$ ______

(a) 2 AB (b) 2 BA

(c) A + B (d) AB

27. $\begin{bmatrix} 1 & 3 \\ 3 & 10 \end{bmatrix}^{-1} =$

(a) $\begin{bmatrix} 10 & 3 \\ 3 & 1 \end{bmatrix}$ (b) $\begin{bmatrix} 10 & -3 \\ -3 & 1 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 3 \\ 3 & 10 \end{bmatrix}$ (d) $\begin{bmatrix} -1 & -3 \\ -3 & -10 \end{bmatrix}$

28. If $A + B = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$ and $A + 2B = \begin{bmatrix} -1 & 1 \\ 0 & -1 \end{bmatrix}$ then A = ______

(a) $\begin{bmatrix} 1 & 1 \\ 2 & 1 \end{bmatrix}$ (b) $\begin{bmatrix} 2/3 & 1/3 \\ 1/3 & 2/3 \end{bmatrix}$

(c) $\begin{bmatrix} 1/3 & 1/3 \\ 2/3 & 1/3 \end{bmatrix}$ (d) None of these

29. From the matrix equation AB = AC we can conclude B = C provided

(a) A is singular

(b) A is non singular

(c) A is symmetric

(d) A is square

30. If I_3 is identity matrix of order 3 then $(I_3)^{-1} =$ ______

(a) 0

(b) $3I_3$

(c) I_3

(d) not necessary exists

31. If A and B are two square matrices of same order then

(a) $(AB)^T = B^T A^T$

(b) $(AB)^T = A^T B^T$

(c) $AB = 0 \Rightarrow |A| = 0, |B| = 0$

(d) $AB = 0 \Rightarrow A = 0,\ B = 0$

ANSWER KEYS

1.	(b)	6.	(c)	11.	(c)	16.	(b)	21.	(a)	26.	(c)
2.	(c)	7.	(a)	12.	(b)	17.	(b)	22.	(d)	27.	(b)
										28.	(c)
3.	(c)	8.	(d)	13.	(c)	18.	(a)	23.	(b)	29.	(b)
4.	(b)	9.	(a)	14.	(c)	19.	(b)	24.	(b)	30.	(c)
5.	(c)	10.	(a)	15.	(b)	20.	(b)	25.	(b)	31.	(a)

B. Long Answer Type Questions

1. Find the inverse of $\begin{bmatrix} 1 & 1 & 2 \\ 0 & 1 & 2 \\ 1 & 2 & 1 \end{bmatrix}$

2. Matrices X and Y are such that $3x + 4y = I$ and $x - 2y = 2I$ where I denoted the identity matrix of order 3.

3. Find the adjoint of the matrix $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 4 \\ 1 & 0 & 2 \end{bmatrix}$

4. Solve by matrix method $3x - 2y + z = 1$, $2x + y - 5z = 2$, $x - y - 2z = 3$

5. Solve by matrix method $x + 2y + 3z = 8$, $2x + y + z = 8,\ x + y + 2z = 6$.

6. Solve by matrix inversion method

$x + y + z = 4,\ 2x - y + 3z = 1, 3x + xy - z = 1$

7. $A = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$ then show that

$A^k = \begin{bmatrix} 1+2k & -4k \\ k & 1-2k \end{bmatrix}$

(use method of induction)

8. If $A = \begin{bmatrix} 0 & -\tan\alpha/2 \\ \tan\alpha/2 & 0 \end{bmatrix}$ then show that $(I + A)(I - A)^{-1} = \begin{bmatrix} \cos\alpha & \sin\alpha \\ \sin\alpha & -\cos\alpha \end{bmatrix}$

where $I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

9. Find x, y, u and v is $\begin{bmatrix} x+1 & y \\ -u & v \end{bmatrix}$

$= \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} y & u \\ x & v \end{bmatrix}$

10. If $\begin{bmatrix} x & y \\ x & x/2+l \end{bmatrix} + \begin{bmatrix} y & x+l \\ x+z & x/2 \end{bmatrix} = \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$ then find x, y, z, l.

ADDENDUM

1. If A is any $m \times n$ matrix such that AB and BA are both defined then B is an

   (a) $m \times n$ matrix

   (b) $n \times n$ matrix

   (c) $n \times m$ matrix

   (d) $m \times m$ matrix

2. A matrix $A = \left[a_{ij}\right]$ is an upper triangular matrix if

   (a) it is a square matrix and $a_{ij} = 0, i < j$

   (b) it is a square matrix and $a_{ij} = 0, i > j$

   (c) it is not a square matrix and $a_{ij} = 0, i > j$

   (d) it is not a square matrix and $a_{ij} = 0, i < j$

3. If $E(\theta) = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$, then $E(\alpha)E(\beta)$ is equal to

   (a) $E(0)$ (b) $E(\alpha\beta)$

   (c) $E(\alpha+\beta)$ (d) $E(\alpha-\beta)$

4. The matrix A satisfying the equation

   $\begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix} A = \begin{bmatrix} 1 & 1 \\ 0 & -1 \end{bmatrix}$ is

   (a) $\begin{bmatrix} 1 & 4 \\ -1 & 0 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & -4 \\ 1 & 0 \end{bmatrix}$

   (c) $\begin{bmatrix} 1 & 4 \\ 0 & -1 \end{bmatrix}$ (d) None of these

5. If $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, j = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$, then B equals

   (a) $I\cos\theta + J\sin\theta$

   (b) $I\sin\theta + J\cos\theta$

   (c) $I\cos\theta - J\sin\theta$

   (d) $-I\cos\theta + J\sin\theta$

6. If A is a square matrix such that $AA^T = I = A^T A$, then A is

   (a) a symmetric matrix

   (b) a skew symmetric matrix

   (c) a diagonal matrix

   (d) an orthogonal matrix

7. If A is an orthogonal matrix, then A^{-1} equals

   (a) A (b) A^T

   (c) A^2 (d) None of these

8. If A and B are two invertible matrices, then the inverse of AB equals to

   (a) AB (b) BA

   (c) $A^{-1}B^{-1}$ (d) $B^{-1}A^{-1}$

9. If A, B are two square matrices such that AB = A and BA = B then

   (a) A, B are idempotent

   (b) only A is idempotent

   (c) only B is idempotent

   (d) None of these

10. The inverse of a symmetric matrix is

(a) symmetric

(b) skew symmetric

(c) diagonal

(d) None of these

11. If A is a skew symmetric matrix and n is a positive integer, then A^n is a

(a) symmetric matrix

(b) skew symmetric matrix

(c) diagonal matrix

(d) None of these

12. If A and B are matrices of same order, then $(A+B)^2 = A^2 + 2AB + B^2$ is possible, if and only if

(a) AB = I (b) BA = I

(c) AB = BA (d) None of these

13. If A and B are square matrices of same order, then $adj(AB)$ is equals is

(a) (adj A) (adj B)

(b) (adj B) (adj A)

(c) adj A + adj B

(d) adj A + adj B

14. If A is a square matrix of order $n \times n$ and k is a scalar then $adj(kA)$ is equal to

(a) $k\ adj\ A$ (b) $k^n\ adj\ A$

(c) $k^{-1}\ adj\ A$ (d) $k^{n+1}\ adj\ A$

15. If A is square matrix of order $n \times n$ then $adj(adjA)$ is equal to

(a) $|A|^n A$ (b) $|A|^{n-1} A$

(c) $|A|^{n-2} A$ (d) $|A|^{n-3} A$

16. If A is a singular matrix then $A\ adjA$ is

(a) identity matrix

(b) null matrix

(c) scalar matrix

(d) None of these

17. If $A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ and $n \in N$, then A^n is equal to

(a) $2^n A$ (b) $2^{n-1} A$

(c) nA (d) None of these

18. If $A = \begin{bmatrix} 1 & 3 \\ 3 & 4 \end{bmatrix}$ and $A^2 - kA - 5I_2 = 0$, then value of k is

(a) 3 (b) 5

(c) 7 (d) –7

19. If $A = [a_{ij}]$ is a scalar matrix of order $n \times n$ such that $a_{ij} = k$ for all i then $|A|$ is

(a) nk (b) $n+k$

(c) n^k (d) k^n

20. If $A = [a_{ij}]$ is a square matrix of order $n \times n$ and k is a scalar, then $|kA| =$

(a) $k^n|A|$ (b) $k|A|$

(c) $k^{n-1}|A|$ (d) None of these

21. If A and B are matrices such that AB and A + B both are defined then

(a) A and B can be any two matrices

(b) A and B are square matrices not necessarily of same order

(c) A and B are square matrices of same order

(d) number of columns of A is same as number of rows of B

22. If $\begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix} A \begin{bmatrix} -3 & 2 \\ 5 & -3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$,

then matrix A equals

(a) $\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$ (d) $\begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}$

23. If $\begin{bmatrix} 1 & -\tan\theta \\ \tan\theta & 1 \end{bmatrix} \begin{bmatrix} 1 & \tan\theta \\ -\tan\theta & 1 \end{bmatrix} = \begin{bmatrix} a & -b \\ b & a \end{bmatrix}$ then

(a) $a = 1, b = 1$

(b) $a = \cos 2\theta, b \sin 2\theta$

(c) $a = \sin 2\theta,\ b = \cos 2\theta$

(d) None of these

24. If matrix A is such that $3A^3 + 2A^2 + 5A + I = 0$, then A^{-1} is equal to

(a) $-\left(3A^2 + 2A + 5\right)$

(b) $3A^2 + 2A + 5$

(c) $3A^2 - 2A - 5$

(d) None of these

25. Which of the following is incorrect

(a) $A^2 - B^2 = (A+B)(A-B)$

(b) $\left(A^T\right)^T = A$

(c) $(AB)^n = A^n B^n$ when A, B commute

(d) $(A-I)(I+A) = 0 \Leftrightarrow A^2 = I$

26. If A is an invertible matrix, then which of the following is correct.

(a) A^{-1} is multivalued

(b) A^{-1} is singular

(c) $\left(A^{-1}\right)^T = \left(A^T\right)^{-1}$

(d) $|A| \neq 0$

27. For a matrix $A^2 + I = 0$, where I is identify matrix then A equals

(a) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ (b) $\begin{bmatrix} -i & 0 \\ 0 & -i \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 2 \\ -1 & 1 \end{bmatrix}$ (d) $\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$

28. If A is a skew symmetric matrix of odd order then |A| is equal to

(a) 0 (b) 1

(c) −1 (d) None of these

29. If A is an orthogonal matrix then

(a) $|A| = 0$ (b) $|A| = \pm 1$

(c) $|A| = \pm 2$ (d) None of these

30. If A is non-singular square matrix of order 'n' then $|adj\ A|$ is equal to

(a) $|A|^n$ (b) $|A|^{n-1}$

(c) $|A|^{n-2}$ (d) None of these

31. Let $A = \left[a_{ij}\right]_{n\times n}$ be a square matrix. let c_{ij} be co-factor of a_{ij}. If $C = \left[C_{ij}\right]$, then

(a) $|C| = |A|$

(b) $|C| = |A|^{n-1}$

(c) $|C| = |A|^{n-2}$

(d) None of these

32. The order of $\begin{bmatrix} x & y & z \end{bmatrix}\begin{bmatrix} a & h & g \\ h & b & f \\ g & f & c \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix}$ is

(a) 3×1 (b) 1×1

(c) 1×3 (d) 3×3

33. If * and B are two matrices such that $AB = B$ and $BA = A$, then $A^2 + B^2 =$

(a) $2AB$ (b) $2BA$

(c) $A+B$ (d) AB

34. $\begin{bmatrix} 1 & 3 \\ 3 & 10 \end{bmatrix}^{-1} =$

(a) $\begin{bmatrix} 10 & 3 \\ 3 & 1 \end{bmatrix}$ (b) $\begin{bmatrix} 10 & -3 \\ -3 & 1 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 3 \\ 3 & 10 \end{bmatrix}$ (d) $\begin{bmatrix} -1 & -3 \\ -3 & -10 \end{bmatrix}$

35. If $A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ then $A^4 =$

(a) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$

(c) $\begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix}$ (d) $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

36. If $A = \begin{bmatrix} 5 & 2 \\ 3 & 1 \end{bmatrix}$ then $A^{-1} =$

(a) $\begin{bmatrix} 1 & -2 \\ -3 & 5 \end{bmatrix}$ (b) $\begin{bmatrix} -1 & 2 \\ 3 & -5 \end{bmatrix}$

(c) $\begin{bmatrix} -1 & -2 \\ -3 & -5 \end{bmatrix}$ (d) $\begin{bmatrix} 1 & 2 \\ 3 & 5 \end{bmatrix}$

37. For the equations

$x + 2y + 3z = 1$,

$2x + y + 3z = 2$,

$5x + 5y + 9z = 4$

(a) there is one solution

(b) there exists infinitely many solution

(c) there is no solution

(d) None of these

38. If $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$, then $A^2 =$

(a) $\begin{bmatrix} 8 & -5 \\ -5 & 3 \end{bmatrix}$ (b) $\begin{bmatrix} 8 & -5 \\ 5 & 3 \end{bmatrix}$

(c) $\begin{bmatrix} 8 & -5 \\ -5 & -3 \end{bmatrix}$ (d) $\begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix}$

39. If $A = \begin{bmatrix} 4 & x+2 \\ 2x-3 & x+1 \end{bmatrix}$ is symmetric matrix, then $x =$

(a) 3 (b) 5

(c) 2 (d) 4

40. If $A + B = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$ and $A = 2B = \begin{bmatrix} -1 & 1 \\ 0 & -1 \end{bmatrix}$. Then $A =$

(a) $\begin{bmatrix} 1 & 1 \\ 2 & 1 \end{bmatrix}$ (b) $\begin{bmatrix} 2/3 & 1/3 \\ 1/3 & 2/3 \end{bmatrix}$

(c) $\begin{bmatrix} 1/3 & 1/3 \\ 2/3 & 1/3 \end{bmatrix}$ (d) None of these

41. $\begin{bmatrix} -6 & 5 \\ -7 & 6 \end{bmatrix}^{-1} =$

(a) $\begin{bmatrix} -6 & 5 \\ -7 & 6 \end{bmatrix}$ (b) $\begin{bmatrix} 6 & -5 \\ -7 & 6 \end{bmatrix}$

(c) $\begin{bmatrix} 6 & 5 \\ 7 & 6 \end{bmatrix}$ (d) $\begin{bmatrix} 6 & -5 \\ 7 & -6 \end{bmatrix}$

42. From the matrix equation $AB = AC$ we can conclude $B = C$ provided

(a) A is singular

(b) A is non singular

(c) A is symmetric

(d) A is square

43. If I_3 is identify matrix of order 3, then $(I_3)^{-1} =$

(a) 0

(b) $3I_3$

(c) I_3

(d) Not necessarily exists

44. If A & B are two square matrices of same order and A' & B' are transpose of A & B then

(a) $(AB)' = B'A'$

(b) $(AB)' = A'B'$

(c) $AB = 0 \Rightarrow |A| = 0$ or $|B| = 0$

(d) $AB = 0 \Rightarrow A = 0$ or $B = 0$

45. If A and B are square matrices of order 3 such that $|A| = -1$, $|B| = 3$ then $|3\ AB| =$

(a) –9 (b) –81

(c) 27 (d) 81

ANSWER KEYS

1. (b) $n \times n$ matrix

Sol. Since AB exists therefore no of col of A = no of rows of B as B has n rows

As BA exists $\Rightarrow$ no of col. of B = no of rows of A.

$\Rightarrow B$ has m columns

Order of B is $n \times m$.

2. (b) it is a square matrix and $a_{ij} = 0, i > j$

3. (c) $E(\alpha + \beta)$

Sol. $E(\alpha)E(\beta) = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\theta \end{bmatrix}\begin{bmatrix} \cos\beta & \sin\beta \\ -\sin\beta & \cos\beta \end{bmatrix}$

$\begin{bmatrix} \cos(\alpha+\beta) & \sin(\alpha+\beta) \\ -\sin(\alpha+\beta) & \cos(\alpha+\beta) \end{bmatrix} = E(\alpha+\beta)$

4. (a) $\begin{bmatrix} 1 & 4 \\ -1 & 0 \end{bmatrix}$

Sol. $\begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix} A = \begin{bmatrix} 1 & 1 \\ 0 & -1 \end{bmatrix}$

$\Rightarrow A = \begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix}^{-1} \begin{bmatrix} 1 & 1 \\ 0 & -1 \end{bmatrix}$

$= \begin{bmatrix} 1 & -3 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 4 \\ 0 & -1 \end{bmatrix}$

5. (a) $I\cos\theta + J\sin\theta$

Sol. $I\cos\theta + J\sin\theta = \begin{bmatrix} \cos\theta & 0 \\ 0 & \cos\theta \end{bmatrix}$

$+ \begin{bmatrix} 0 & \sin\theta \\ -\sin\theta & 0 \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$

6. (d) an orthogonal matrix

Sol. By defn. $AA' = I = A'A$ (If A is orthogonal matrix)

7. (b) A^T

Sol. A is orthogonal matrix

$\Rightarrow AA^{-1} = I = A^T A$

$\Rightarrow A^{-1} = A^T$

8. (d) $B^{-1}A^{-1}$

Sol. By the defn $(AB)^{-1} = B^{-1}A^{-1}$

9. (a) A, B are idempotent

Sol. $AB = A$ and $BA = B$

As $AB = A$

$\Rightarrow (AB)A = AA$

$\Rightarrow A(BA) = A^2$

$\Rightarrow AB = A^2 \, (As \;\; BA = B)$

$\Rightarrow A = A^2 \, (As \;\; AB = A)$

Again $BA = B$

$\Rightarrow (BA)B = BB$

$\Rightarrow B(AB) = B^2$

$\Rightarrow BA = B^2 \, (As \;\; AB = A)$

$\Rightarrow B = B^2 \, (As \, BA = B)$

10. (a) symmetric

Sol. A is symmetric matrix

$\Rightarrow AA^{-1} = I$

$\Rightarrow \left(AA^{-1}\right)^T = I$

$\Rightarrow \left(A^{-1}\right)^T A^T = I$

$\Rightarrow \left(A^{-1}\right)^T = \left(A^T\right)^{-1} = A^{-1} \left(As \;\; A = A^T\right)$

$\Rightarrow A^{-1}$ is symmetric matrix

11. (d) None of these

Sol. A is skew symm matrix

$\Rightarrow A^T = -A$

$\Rightarrow \left(A^T\right)^n = (-A)^n = (-1)^n A^n$

$$\Rightarrow \left(A^n\right)^T = \begin{cases} A^n \text{ if } n \text{ is even} \\ -A^n \text{ if } n \text{ is odd} \end{cases}$$

12. (c) AB = BA

Sol. $(A+B)^2 = (A+B)(A+B)$

$(A+B)^2 = A^2 + AB + BA + B^2$

$= A^2 + 2AB + B^2 \Leftrightarrow AB = BA$

13. (b) (adj B) (adj A)

Sol. $(AB)(adj \;\; AB) = |AB| I = (adj \; AB)(AB)$ (1)

Now $(AB)(adj \;\; B \;\; adjA) = A(B.adj \, B) adjA$

$= A(|B| I) adj \;\; A$

$= |B| \, (A.adj \;\; A)$

$= |B| \, |A| I$

$= |B| \, |A|$

$= |A \, B| I$ (2)

From (1) & (2)

$(AB)(adj \;\; AB) = (AB)(adj \, B \;\; adjA)$

$\Rightarrow adj(AB) = adj \, B.adjA$

14. (c) $k^{-1} \;\; adj \;\; A$

Sol. $(KA)(adj \, KA) = |KA| I_n$

$\Rightarrow K(A \;\; adj \;\; KA) = K^n |A| I_n$

$\Rightarrow A \;\; adj \;\; (KA) = K^{n-1} |A| I_n$

$$\Rightarrow A\ adj\ (KA) = K^{n-1}A(adj\ A)$$

$$\Rightarrow A\ adj\ (KA) = A(K^{n-1}adj\ A)$$

$$\Rightarrow adj\ KA = K^{n-1}adj\ A$$

15. (c) $|A|^{n-2} A$

Sol. For any square matrix X

$$X(adj\ X) = |X| I_n$$

$$\Rightarrow (adj\ A)(adj\ (adj\ A)) = |adj\ A| I_n$$

$$\Rightarrow (adj\ A)(adj\ (adj\ A)) = |A|^{n-1}$$

$$\Rightarrow (A\ adj\ A)(adj\ (adj\ A)) = |A|^{n-1} A$$

$$\Rightarrow |A| I_n (adj\ (adj\ A)) = |A|^{n-1} A$$

$$\Rightarrow adj(adj\ A) = |A|^{n-2} A$$

16. (b) null matrix

Sol. we know $A(adj\ A) = |A| I_n$

As A is singular

$$\Rightarrow |A| = 0$$

$$\Rightarrow A(adj\ A) = 0$$

$\Rightarrow A(adj\ A)$ is singular matrix

17. (b) $2^{n-1} A$

Sol. $A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

$$A^2 = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix} = 2A$$

$$A^3 = 2(AA) = 2A^2 = 2(2A) = 2^2 A$$

$$A^n = 2^{n-1} A$$

18. (b) 5

Sol. $A^2 - 5I_2 = \begin{bmatrix} 10 & 15 \\ 15 & 25 \end{bmatrix} - \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix} = \begin{bmatrix} 5 & 15 \\ 15 & 20 \end{bmatrix} = 5A$

$$\Rightarrow K = 5$$

19. (d) k^n

Sol. By the defn $|A| = nk$

20. (a) $k^n |A|$

Sol. $A = [a_{ij}]$

$$\Rightarrow |KA| = K^n |A|$$

21. (b) A and B are square matrices not necessarily of same order

Sol. By the defn of matrix

22. (a) $\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$

Sol. $\begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix} A \begin{bmatrix} -3 & 2 \\ 5 & -3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$$\Rightarrow A = \begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix}^{-1}\left(\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} -3 & 2 \\ 5 & -3 \end{bmatrix}\right)$$

$$= \begin{bmatrix} 2 & -1 \\ -3 & 2 \end{bmatrix}\left(\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 3 & 2 \\ 5 & 3 \end{bmatrix}\right)$$

$$\Rightarrow \begin{bmatrix} 2 & -1 \\ -3 & 2 \end{bmatrix}\begin{bmatrix} 3 & 2 \\ 5 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$$

23. (b) $a = \cos 2\theta, b \sin 2\theta$

Sol. $\begin{bmatrix} 1 & -\tan\theta \\ \tan\theta & 1 \end{bmatrix}\begin{bmatrix} 1 & \tan\theta \\ -\tan\theta & 1 \end{bmatrix} = \begin{bmatrix} a & -b \\ b & a \end{bmatrix}$

$$\frac{1}{1+\tan^2\theta}\begin{bmatrix} 1-\tan^2\theta & -2\tan\theta \\ 2\tan\theta & 1-\tan^2\theta \end{bmatrix} = \begin{bmatrix} a & -b \\ b & a \end{bmatrix}$$

$$\begin{bmatrix} \cos 2\theta & -\sin\theta \\ \sin 2\theta & \cos 2\theta \end{bmatrix} = \begin{bmatrix} a & -b \\ b & a \end{bmatrix}$$

$a = \cos 2\theta, b = \sin 2\theta$

24. (d) None of these

Sol. $3A^3 + 2A^2 + 5A + I = 0$

$I = -3A^3 - 2A^2 - 5A$

$IA^{-1} = \left(-3A^3 - 2A^2 - 5A\right) A^{-1}$

$\Rightarrow A^{-1} = -3A^2 - 2A - 5I$

25. (a) $A^2 - B^2 = (A+B)(A-B)$

Sol. As $AB \neq BA$

26. (d) $|A| \neq 0$

Sol. By defn of matrix

if $|A| \neq 0$ then A is an invertible matrix

27. (b) $\begin{bmatrix} -i & 0 \\ 0 & -i \end{bmatrix}$

Sol. For $A = \begin{bmatrix} -i & 0 \\ 0 & -i \end{bmatrix}$

$A^2 = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} = -I$

$\Rightarrow A^2 + I = 0$

28. (a) 0

Sol. A is sqew symmetric matrix odd order say (2n + 1)

$\Rightarrow A^T = -A$

$\Rightarrow |A^T| = |-A|$

$\Rightarrow |A^T| = -|-A|^{2n+1} |A|$

$\Rightarrow |A^T| = -|A|$

$\Rightarrow |-A| = -|A|$

$\Rightarrow |A| = -|A|$

$\Rightarrow 2|A| = 0$

$\Rightarrow |A| = 0$

29. (b) $|A| = \pm 1$

Sol. A is orthogonal matrix

$\Rightarrow AA^T = I = A^T A$

$\Rightarrow |AA^T| = |A^T A| = |I|$

$\Rightarrow |A||A| = |A^T||A| = |I|$

$\Rightarrow |A|^2 = 1$

$\Rightarrow |A| = \pm 1$

30. (b) $|A|^{n-1}$

Sol. As $A(adj\ A) = |A| I = (adjA) A$

$\Rightarrow |A|\ |adjA| = |A|^n = |adj\ A||A|$

$\Rightarrow |adjA| = |A|^{n-1}\ \left(As\ |A| \neq 0\right)$

31. (b) $|C| = |A|^{n-1}$

Sol.. Same as Q 30

32. (b) 1×1

Sol. $[x\,y\,z]_{1\times3} \begin{bmatrix} a & h & g \\ h & b & f \\ g & f & c \end{bmatrix}_{3\times3} \begin{bmatrix} x \\ y \\ z \end{bmatrix}_{3\times1} = [\ \ ]_{1\times1}$

33. (c) $A + B$

Sol. $A^2 + B^2 = AA + BB = A(BA) + B(AB)$

$\begin{pmatrix} As\ AB = B \\ BA = A \end{pmatrix}$

$= (AB)A + (BA)B$

$= BA + AB$

$= A + B$

34. (b) $\begin{bmatrix} 10 & -3 \\ -3 & 1 \end{bmatrix}$

Sol. $\begin{bmatrix} 1 & 3 \\ 2 & 10 \end{bmatrix} = \frac{1}{1}\begin{bmatrix} 10 & -3 \\ -3 & 1 \end{bmatrix} = \begin{bmatrix} 10 & -3 \\ -3 & 1 \end{bmatrix}$

35. (a) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Sol. $A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

$A^2 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I_2$

$\Rightarrow A^4 = I_4 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

36. (b) $\begin{bmatrix} -1 & 2 \\ 3 & -5 \end{bmatrix}$

Sol. $A = \begin{bmatrix} 5 & 2 \\ 3 & 1 \end{bmatrix}$

$A^{-1} = \frac{1}{-1}\begin{bmatrix} 1 & -2 \\ -3 & 5 \end{bmatrix} = \begin{bmatrix} -1 & 2 \\ 3 & -5 \end{bmatrix}$

37. (a) there is one solution

Sol. As $\begin{vmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \\ 5 & 5 & 9 \end{vmatrix} = 1(-6) - 2(3) + 3(5) = 3 \neq 0$

so there is only one solution

38. (d) $\begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix}$

Sol. $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$

$A^2 = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix}$

39. (b) 5

Sol. symmetric matrix

$A_{12} = A_{21}$

$\Rightarrow x + 2 = 2x - 3$

$\Rightarrow x = 5$

40. (c) $\begin{bmatrix} 1/3 & 1/3 \\ 2/3 & 1/3 \end{bmatrix}$

Sol. $3A = \begin{bmatrix} 1 & 1 \\ 2 & 1 \end{bmatrix}$

$A = \begin{bmatrix} 1/3 & 1/3 \\ 2/3 & 1/3 \end{bmatrix}$

41. (a) $\begin{bmatrix} -6 & 5 \\ -7 & 6 \end{bmatrix}$

Sol. $\begin{bmatrix} -6 & 5 \\ -7 & 6 \end{bmatrix}^{-1} = \frac{1}{-1}\begin{bmatrix} 6 & -5 \\ 7 & -6 \end{bmatrix} = \begin{bmatrix} -6 & 5 \\ -7 & 6 \end{bmatrix}$

42. (b) A is non singular

Sol. AB = AC

If A is non sigular matrix

then B = C

43. (c) I_3

Sol. $(I_3)^{-1} = I_3$

44. (a) $(AB)' = B'A'$ (b) $(AB)' = A'B'$

Sol. $(AB)' = B'A'$

45. (a) –9

Sol. $|3AB| = 3|A|\,|B| = -9$

CHAPTER - 2

DETERMINANTS

A. Multiple Choice Questions (MCQ)

1. The value of $\begin{vmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \\ 3 & 4 & 5 \end{vmatrix} =$ ____

(a) 4 (b) 11
(c) 0 (d) None of these

2. The value of $\begin{vmatrix} 0.2 & 0.1 & 3 \\ 0.4 & 0.2 & 7 \\ 0.6 & 0.3 & 2 \end{vmatrix} =$ ____

(a) 5.2 (b) 7.2
(c) 9.4 (d) 0

3. $\begin{vmatrix} \sin^2\theta & \cos^2\theta & 1 \\ \cos^2\theta & \sin^2\theta & 1 \\ -10 & 12 & 2 \end{vmatrix} =$ _____

(a) $6\sin^2\theta$ (b) $7\cos^2\theta$
(c) 0 (d) None of these

4. If $1, \omega, \omega^2$ are cube roots of unity then

$\begin{vmatrix} 1 & \omega & \omega^2 \\ \omega & \omega^2 & 1 \\ \omega^2 & 1 & \omega \end{vmatrix} =$ ____

(a) 2 (b) 3
(c) 4 (d) 0

5. If a, b, c are in AP then value of

$\begin{vmatrix} x+1 & x+2 & x+a \\ x+2 & x+3 & x+b \\ x+3 & x+4 & x+c \end{vmatrix}$ is ____

(a) 3 (b) –3
(c) 0 (d) 1

6. If $p+q+r=0=a+b+c$ then the value of $\begin{vmatrix} pa & qb & rc \\ qc & ra & pb \\ rb & pc & qa \end{vmatrix} =$ ____

(a) 0 (b) $pa+qb+rc$
(c) 1 (d) None of these

7. If $a>0, b>0, c>0$ are pth, qth and rth term of a GP then the value of

$\begin{vmatrix} \log a & p & 1 \\ \log b & q & 1 \\ \log c & r & 1 \end{vmatrix} =$ ____

(a) 1 (b) 0
(c) –1 (d) None of these

8. If A is an invertible matrix then $\det\left(A^{-1}\right) =$ ___

(a) det A (b) $\dfrac{1}{\det A}$
(c) 1 (d) None of these

9. The determinant $\begin{vmatrix} a & b & ax+b \\ b & c & bx+c \\ ax+b & bx+c & 0 \end{vmatrix} = 0$ if a, b, c are in ____

(a) AP
(b) GP
(c) HP
(d) x is a root of $ax^2+bx+c=0$

10. If $a_1, a_2, ..., a_n$ are in GP then the value of

$$\begin{vmatrix} \log a_n & \log a_{n+1} & \log a_{n+2} \\ \log a_{n+3} & \log a_{n+4} & \log a_{n+4} \\ \log a_{n+5} & \log a_{n+6} & \log a_{n+7} \end{vmatrix} = \_\_\_$$

(a) 0 (b) 1
(c) 2 (d) None of these

11. If x, y, z are all distinct and

$$\begin{vmatrix} x & x^2 & 1+x^3 \\ y & y^2 & 1+y^3 \\ z & z^2 & 1+z^3 \end{vmatrix} = 0$$

then the value of $xyz = \_\_\_\_$
(a) –2 (b) –1
(c) –3 (d) None of these

12. If $\begin{vmatrix} b+c & c+a & a+b \\ a+b & b+c & c+a \\ c+a & a+b & b+c \end{vmatrix} = k \begin{vmatrix} a & b & c \\ c & a & b \\ b & c & a \end{vmatrix}$

then $k = \_\_\_\_$
(a) 1 (b) 2
(c) 3 (d) 4

13. If $a^2 + b^2 + c^2 = 0$ and

$$\begin{vmatrix} b^2+c^2 & ab & ac \\ ab & c^2+a^2 & bc \\ ac & bc & a^2+b^2 \end{vmatrix} = ka^2b^2c^2$$

then k = ___
(a) 2 (b) 1
(c) 4 (d) 3

14. If $a \neq b \neq c$ then the value of x satisfying

$$\begin{vmatrix} 0 & x-a & x-b \\ x+a & 0 & x-c \\ x+b & x+c & 0 \end{vmatrix} = 0$$ is ___

(a) a (b) c
(c) b (d) 0

15. The system of equations $x + y + z = 1$, $x + ky + z = k$, $x + y + k = k^2$ have no solution of k = ____

(a) 0 (b) 1
(c) –1 (d) –2

16. If B is a non singular matrix and A is a square matrix then $\det\left(B^{-1}AB\right) = \_\_\_$

(a) det (B) (b) det (A)
(c) det (B^{-1}) (d) det (A^{-})

17. If a, b, c are non zero real numbers then

$$\begin{vmatrix} bc & ca & ab \\ ca & ab & bc \\ ab & bc & ca \end{vmatrix} = 0 \text{ if } \_\_\_\_$$

(a) $\frac{1}{a} + \frac{1}{b} + \frac{1}{c} = 0$ (b) $\frac{1}{a} - \frac{1}{b} + \frac{1}{c} = 0$

(c) $\frac{1}{b} + \frac{1}{c} - \frac{1}{a} = 0$ (d) $\frac{1}{b} - \frac{1}{c} - \frac{1}{a} = 0$

18. $\begin{vmatrix} ka & k^2+a^2 & 1 \\ kb & k^2+b^2 & 1 \\ kc & k^2+c^2 & 1 \end{vmatrix} = \_\_\_\_$

(a) $k(a+b)(b+c)(c+a)$

(b) $kabc\left(a^2+b^2+c^2\right)$

(c) $k(a-b)(b-c)(c-a)$

(d) $k(a+b-c)(b+c-a)(c+a-b)$

19. Value of $\begin{vmatrix} 1 & \omega^3 & \omega^5 \\ \omega^3 & 1 & \omega^4 \\ \omega^5 & \omega^4 & 1 \end{vmatrix} = \_\_\_\_\_$

(a) $(1-\omega)^2$ (b) 3
(c) –3 (d) None of these

20. Value of $\begin{vmatrix} x+1 & \omega & \omega^2 \\ \omega & x+\omega^2 & 1 \\ \omega^2 & 1 & x+\omega \end{vmatrix} =$ ____

(a) 1 (b) ω
(c) ω^2 (d) 0

21. $\begin{vmatrix} 1 & 1 & 1 \\ 1 & 1+x & 1 \\ 1 & 1 & 1+y \end{vmatrix} =$ ____

(a) $x+y$ (b) xy
(c) $x-y$ (d) $1+x+y$

22. If $\begin{vmatrix} x+a & b & c \\ a & x+b & c \\ a & b & x+c \end{vmatrix} = 0$ then $x =$ ____

(a) $a+b+c$
(b) $-(a+b+c)$
(c) $0, a+b+c$
(d) $0, -(a+b+c)$

23. If A and B are square matrices of order 3 and |A| = –1, |B| = 3 then |3AB| = ___

(a) –9 (b) –81
(c) –27 (d) 81

24. $\begin{vmatrix} x & 4 & y+z \\ y & 4 & z+x \\ z & 4 & x+y \end{vmatrix} =$ ____

(a) 4 (b) $x+y+z$
(c) xyz (d) 0

25. $\begin{vmatrix} 11 & 12 & 13 \\ 12 & 13 & 14 \\ 13 & 14 & 15 \end{vmatrix} =$ ____

(a) 1 (b) 0
(c) –1 (d) 67

26. A root of the equation

$\begin{vmatrix} 3-x & -6 & 3 \\ -6 & 3-x & 3 \\ 3 & 3 & -6-x \end{vmatrix} = 0$ is ____

(a) 6 (b) 3
(c) 0 (d) None of these

27. The value of $\begin{vmatrix} -1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{vmatrix} =$ ____

(a) –4 (b) 0
(c) 1 (d) 4

28. If $1, \omega, \omega^2$ are cube roots of unity then

$\begin{vmatrix} 1 & \omega^n & \omega^{2n} \\ \omega^{2n} & 1 & \omega^n \\ \omega^n & \omega^{2n} & 1 \end{vmatrix} =$ _____

(a) 0 (b) ω
(c) ω^2 (d) 1

ANSWER HINTS

1.	(c)	8.	(b)	15.	(d)	22.	(d)
2.	(d)	9.	(b)	16.	(b)	23.	(a)
3.	(c)	10.	(a)	17.	(a)	24.	(d)
4.	(d)	11.	(b)	18.	(c)	25.	(b)
5.	(c)	12.	(b)	19.	(b)	26.	(c)
6.	(a)	13.	(c)	20.	(d)	27.	(d)
7.	(b)	14.	(d)	21.	(b)	28.	(a)

B. Long Answer Type Questions

1. Show that $\begin{vmatrix} x & y & z \\ x^2 & y^2 & z^2 \\ yz & zx & xy \end{vmatrix}$

$= (x-y)(y-z)(z-x)(xy+yz+zx)$

2. Prove that $\begin{vmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ bc & ca & ab \end{vmatrix}$

$= (b-c)(c-a)(a-b)(bc+ca+ab)$

3. $\begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix}$

$= abc\left(\dfrac{1}{a}+\dfrac{1}{b}+\dfrac{1}{c}+1\right)$, prove it

4. Solve $\begin{vmatrix} x-a & 0 & 0 \\ a & x-b & 0 \\ a & b & x-c \end{vmatrix} = 0$

5. Prove that $\begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix}$

$= (a+b+c)^3$

6. Prove that $\begin{vmatrix} (b+c)^2 & a^2 & bc \\ (c+a)^2 & b^2 & ca \\ (a+b)^2 & c^2 & ab \end{vmatrix}$

$= (a^2+b^2+c^2)(a+b+c)(b-c)(c-a)(a-b)$

7. Prove that $\begin{vmatrix} a^3-x^3 & a^2 & a \\ b^3-x^3 & b^2 & b \\ c^3-x^3 & c^2 & c \end{vmatrix}$

$= (a-b)(a-c)(b-c)(abc-x^3)$

8. Prove that $\dfrac{1}{bc+ca+ab}$

$\begin{vmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ bc & ca & ab \end{vmatrix} = (b-c)(c-a)(a-b)$

9. Prove that $\begin{vmatrix} y+z & x & x \\ y & z+x & y \\ z & z & x+y \end{vmatrix} = 4xyz$

10. Prove that $\begin{vmatrix} a+3b & a+5b & a+7b \\ a+4b & a+6b & a+8b \\ a+5b & a+7b & a+9b \end{vmatrix} = 0$

11. Prove that $\begin{vmatrix} a^2+1 & ab & ac \\ ab & b^2+1 & bc \\ ac & bc & c^2+1 \end{vmatrix}$

$= 1+a^2+b^2+c^2$ and write its minimum value.

12. Prove that $\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^3 & b^3 & c^3 \end{vmatrix}$

$= (b-c)(c-a)(a-b)(a+b+c)$

13. Show that $\begin{vmatrix} a+b+c & -c & -b \\ -c & a+b+c & -a \\ -b & -a & a+b+c \end{vmatrix}$

$= 2(b+c)(c+a)(a+b)$

14. Show that $\begin{vmatrix} b+c & a+b & a \\ c+a & b+c & b \\ a+b & c+a & c \end{vmatrix}$

$= a^3+b^3+c^3-3abc$

15. Solve by Cramer's rule
$2x-y+2z=5,\ 3x-2y-z=-1,$
$x+3y+2z=8$

16. Solve by determinant method
$2x-y+z-3=0,\ x+2y-z-1=0,$
$2x+y+z-6=0$

ANSWER KEYS

1. Replacing c_1 by $c_1 - c_2$ and c_2 by $c_2 - c_3$

$$\begin{vmatrix} x-y & y-z & z \\ x^2-y^2 & y^2-z^2 & z^2 \\ yz-zx & zx-xy & xy \end{vmatrix} = (x-y)(y-z)$$

$$\begin{vmatrix} 1 & 1 & z \\ x+y & y+z & z^2 \\ -z & -x & xy \end{vmatrix}$$

Replacing c_1 by $c_1 - c_2$

$$(x-y)(y-z)\begin{vmatrix} 1-1 & 1 & z \\ x+y-y-z & y+z & z^2 \\ -z+x & -x & xy \end{vmatrix}$$

$$(x-y)(y-z)\begin{vmatrix} 0 & 1 & z \\ x-z & y+z & z^2 \\ x-z & -x & xy \end{vmatrix}$$

$$=(x-y)(y-z)(x-z)\begin{vmatrix} 0 & 1 & z \\ 1 & y+z & z^2 \\ 1 & -x & xy \end{vmatrix}$$

Replacing R_2 by $R_2 - R_3$

$$(x-y)(y-z)(x-z)\begin{vmatrix} 0 & 1 & z \\ 0 & y+z+x & z^2-xy \\ 1 & -x & xy \end{vmatrix}$$

Then expand directly

2. Replacing c_1 by $c_1 - c_2$ and c_2 by $c_2 - c_3$

$$c_1 - \begin{vmatrix} a-b & b-c & c \\ a^2-b^2 & b^2-c^2 & c^2 \\ bc-ca & ca-ab & ab \end{vmatrix}$$

$$=(a-b)(b-c)\begin{vmatrix} 1 & 1 & c \\ a+b & b+c & c^2 \\ -c & -a & ab \end{vmatrix}$$

Again replacing c_1 by $c_1 - c_2$

$$(a-b)(b-c)\begin{vmatrix} 1-1 & 1 & c \\ a+b-b-c & b+c & c^2 \\ -c+a & -a & ab \end{vmatrix}$$

$$=(a-b)(b-c)\begin{vmatrix} 0 & 1 & c \\ a-c & b+c & c^2 \\ a-c & -a & ab \end{vmatrix}$$

$$=(a-b)(b-c)(a-c)\begin{vmatrix} 0 & 1 & c \\ 1 & b+c & c^2 \\ 1 & -a & ab \end{vmatrix}$$

Expand directly

3. Dividing a, b, c in 1st, 2nd, 3rd columns directly

$$abc\begin{vmatrix} \frac{1}{a}+1 & \frac{1}{b} & \frac{1}{c} \\ \frac{1}{a} & 1+\frac{1}{b} & \frac{1}{c} \\ \frac{1}{a} & \frac{1}{b} & 1+\frac{1}{c} \end{vmatrix}$$

Replacing c_1 by $c_1 + c_2 + c_3$

$$abc\begin{vmatrix} 1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c} & \frac{1}{b} & \frac{1}{c} \\ 1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c} & 1+\frac{1}{b} & \frac{1}{c} \\ 1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c} & \frac{1}{b} & 1+\frac{1}{c} \end{vmatrix}$$

$$abc\left(1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)\begin{vmatrix} 1 & \frac{1}{b} & \frac{1}{c} \\ 1 & 1+\frac{1}{b} & \frac{1}{c} \\ 1 & \frac{1}{b} & 1+\frac{1}{c} \end{vmatrix}$$

Replacing R_1 and $R_1 - R_2$ and R_2 by $R_2 - R_3$

$$abc\left(1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)\begin{vmatrix}0 & -1 & 0\\ 0 & 1 & -1\\ 1 & 1/b & 1+1/c\end{vmatrix}$$

Then expand directly.

4. Expand directly.

5. Replacing R_1 by $R_1+R_2+R_3$

$$\begin{vmatrix}a+b+c & a+b+c & a+b+c\\ 2b & b-c-a & 2b\\ 2c & 2c & c-a-b\end{vmatrix}$$

$$=(a+b+c)\begin{vmatrix}1 & 1 & 1\\ 2b & b-c-a & 2b\\ 2c & 2c & c-a-b\end{vmatrix}$$

Replacing c_2 by c_2-c_1 and c_3 by c_3-c_1

$$(a+b+c)\begin{vmatrix}1 & 0 & 0\\ 2b & -c-a-b & 0\\ 2c & 0 & -c-a-b\end{vmatrix}$$

Then expand directly.

6. Replacing c_1 by c_1-2c_3

$$\begin{vmatrix}b^2+c^2 & a^2 & bc\\ c^2+a^2 & b^2 & ca\\ a^2+b^2 & c^2 & ab\end{vmatrix}$$

Replacing c_1 by c_1+c_2

$$\begin{vmatrix}a^2+b^2+c^2 & a^2 & bc\\ a^2+b^2+c^2 & b^2 & ca\\ a^2+b^2+c^2 & c^2 & ab\end{vmatrix}=\left(a^2+b^2+c^2\right)$$

$$\begin{vmatrix}1 & a^2 & bc\\ 1 & b^2 & ca\\ 1 & c^2 & ab\end{vmatrix}$$

Replacing R_2 by R_2-R_1 and R_3 by R_3-R_1

$$\left(a^2+b^2+c^2\right)\begin{vmatrix}1 & a^2 & bc\\ 0 & b^2-a^2 & ca-bc\\ 0 & c^2-a^2 & ab-bc\end{vmatrix}$$

$$=\left(a^2+b^2+c^2\right)\begin{vmatrix}1 & a^2 & bc\\ 0 & (a+b)(a-b) & c(a-b)\\ 0 & (c+a)(c-a) & b(a-c)\end{vmatrix}$$

$$=\left(a^2+b^2+c^2\right)(a-b)(c-a)$$

$$\begin{vmatrix}1 & a^2 & bc\\ 0 & a+b & c\\ 0 & c+a & -b\end{vmatrix}$$

Then expand directly.

7. Splitting the determinant as

$$\begin{vmatrix}a^3 & a^2 & a\\ b^3 & b^2 & b\\ c^3 & c^2 & c\end{vmatrix}-\begin{vmatrix}x^3 & a^2 & a\\ x^3 & b^2 & b\\ x^3 & c^2 & c\end{vmatrix}$$

$$=abc\begin{vmatrix}a^2 & a & 1\\ b^2 & b & 1\\ c^2 & c & 1\end{vmatrix}-x^3\begin{vmatrix}1 & a^2 & a\\ 1 & b^2 & b\\ 1 & c^2 & c\end{vmatrix}$$

Then proceed.

Replacing c_1 by c_1-c_2 and c_2 by c_2-c_3

9. Expand it directly

10. Replacing R_2 by R_2-R_1 and R_3 by R_3-R_2

$$\begin{vmatrix}a+3b & a+5b & a+7b\\ b & b & b\\ b & b & b\end{vmatrix}=b^2$$

$$\begin{vmatrix}a+3b & a+5b & a+7b\\ 1 & 1 & 1\\ 1 & 1 & 1\end{vmatrix}=0$$

Value is 0 since $R_2=R_3$

11. $$a^2b^2c^2\begin{vmatrix} 1+\frac{1}{a^2} & 1 & 1 \\ 1 & 1+\frac{1}{b^2} & 1 \\ 1 & 1 & 1+\frac{1}{c^2} \end{vmatrix}$$

$$=a^2b^2c^2\begin{vmatrix} 1+\frac{1}{a^2} & 1 & 0 \\ 1 & 1+\frac{1}{b^2} & -\frac{1}{b^2} \\ 1 & 1 & \frac{1}{c^2} \end{vmatrix}$$

(Replacing $c_3 \to c_3 - c_2$)

Then expand it directly.

12. Replacing c_1 by $c_1 - c_2$ and c_2 by $c_2 - c_3$

$$\begin{vmatrix} 1-1 & 1-1 & 1 \\ a-b & b-c & c \\ a^3-b^3 & b^3-c^3 & c^3 \end{vmatrix}$$

$$\begin{vmatrix} 0 & 0 & 1 \\ a-b & b-c & c \\ (a-b)(a^2+ab+b^2) & (b-c)(b^2+bc+c^2) & c^3 \end{vmatrix}$$

$$(a-b)(b-c)\begin{vmatrix} 0 & 0 & 1 \\ 1 & 1 & c \\ a^2+ab+b^2 & b^2+bc+c^2 & c^3 \end{vmatrix}$$

Then expand it directly.

13. Replacing c_1 by $c_1 + c_2$ and c_2 by $c_2 + c_3$

$$\begin{vmatrix} a+b & -(b+c) & -b \\ a+b & b+c & -a \\ -(a+b) & b+c & a+b+c \end{vmatrix}$$

$$=(a+b)(b+c)\begin{vmatrix} 1 & -1 & -b \\ 1 & 1 & -a \\ -1 & 1 & a+b+c \end{vmatrix}$$

Then proceed

14. Replacing c_1 by $c_1 + c_3$ and $c_2\ c_2 \to c_2 - c_3$

$$\begin{vmatrix} a+b+c & b & a \\ a+b+c & c & b \\ a+b+c & a & c \end{vmatrix} = (a+b+c)\begin{vmatrix} 1 & b & a \\ 1 & c & b \\ 1 & a & c \end{vmatrix}$$

Replace R_2 by $R_2 - R_1$ and R_3 by $R_3 - R_1$

$$(a+b+c)\begin{vmatrix} 1 & b & a \\ 0 & c-b & b-a \\ 0 & a-b & c-a \end{vmatrix}$$

Then expand directly.

ADDENDUM

1. If a, b, c are in A.P then the value of

$$\begin{vmatrix} x+1 & x+2 & x+a \\ x+2 & x+3 & x+b \\ x+3 & x+4 & x+c \end{vmatrix} \text{ is}$$

(a) 3 (b) –3

(c) 0 (d) 1

2. If $p+q+r=0=a+b+c$ then the value of

$$\begin{vmatrix} pa & qb & rc \\ qc & ra & pb \\ rb & pc & qa \end{vmatrix} \text{ is}$$

(a) 0 (b) $pa+qb+rc$

(c) 1 (d) None of these

3. If $a>0, b>0, c>0$ are pth, qth & rth term of GP then the value of $\begin{vmatrix} \log a & p & 1 \\ \log b & q & 1 \\ \log c & r & 1 \end{vmatrix}$ is

(a) 1 (b) 0

(c) –1 (d) None of these

4. If A is an invertible matrix, then $\det\left(A^{-1}\right)$ equals to

(a) $\det A$ (b) $\dfrac{1}{\det A}$

(c) 1 (d) None of these

5. Determinant $\begin{vmatrix} a^2+x^2 & ab & ac \\ ab & b^2+x^2 & bc \\ ac & bc & c^2+x^2 \end{vmatrix}$ is divisible by

(a) x (b) x^2

(c) x^3 (d) x^4

6. The determinant $\begin{vmatrix} a & b & ax+b \\ b & c & bx+c \\ ax+b & bx+c & 0 \end{vmatrix} = 0$ if a, b, c are in

(a) AP

(b) GP

(c) HP

(d) x is a root of $ax^2+2bx+c=0$

7. If $a_1, a_2, a_3, ..., a_n$ are in AP then value of determinant

$$\begin{vmatrix} \log a_n & \log a_{n+1} & \log a_{n+2} \\ \log a_{n+3} & \log a_{n+4} & \log a_{n+4} \\ \log a_{n+5} & \log a_{n+6} & \log a_{n+7} \end{vmatrix} =$$

(a) 0 (b) 1

(c) 2 (d) None of these

8. If x, y, z are all distinct and

$$\begin{vmatrix} x & x^2 & 1+x^3 \\ y & y^2 & 1+y^3 \\ z & z^2 & 1+z^3 \end{vmatrix} = 0$$

then the value of xyz is

(a) –2 (b) –1

(c) –3 (d) None of these

9. If $\begin{vmatrix} b+c & c+a & a+b \\ a+b & b+c & c+a \\ c+a & a+b & b+c \end{vmatrix} = k \begin{vmatrix} a & b & c \\ c & a & b \\ b & c & a \end{vmatrix}$

then $k =$

(a) 1 (b) 2

(c) 3 (d) 4

10. If $a^2+b^2+c^2=0$ and

$$\begin{vmatrix} b^2+c^2 & ab & ac \\ ab & c^2+a^2 & bc \\ ac & bc & a^2+b^2 \end{vmatrix} = ka^2b^2c^2$$

then $k=$

(a) 2 (b) 1

(c) 4 (d) 3

11. If $a^{-1}+b^{-1}+c^{-1}=0$ &

$$\begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix} = \lambda \text{ then } \lambda =$$

(a) 0 (b) abc

(c) –abc (d) None of these

12. If A, B, C are angles of a triangle then value

of $\begin{vmatrix} -1 & \cos C & \cos B \\ \cos C & -1 & \cos A \\ \cos B & \cos A & -1 \end{vmatrix}$ is

(a) cos A cos B cos C

(a) sin A sin B sin C

(c) 0

(d) None of these

13. If $a \neq b \neq c$, the value of x satisfying the equation $\begin{vmatrix} 0 & x-a & x-b \\ x+a & 0 & x-c \\ x+b & x+c & 0 \end{vmatrix} = 0$ is

(a) a (b) b

(c) c (d) 0

14. The system of equations $x+y+z=1$, $x+ky+z=k$, $x+y+kz=k^2$ have no solution if $k=$

(a) 0 (b) 1

(c) –1 (d) –2

15. If B is non singular matrix and A is a square matrix then $\det\left(B^{-1}AB\right)$ is equal to

(a) det (B) (b) det (A)

(c) det (B^{-1}) (d) det (A^{-1})

16. The equation $\begin{vmatrix} x-a & x-b & x-c \\ x-b & x-c & x-a \\ x-c & x-a & x-b \end{vmatrix} = 0$ is satisfied for $a \neq b \neq c$.

(a) $x=0$

(b) $x=a$

(c) $x=\frac{1}{3}(a+b+c)$

(d) $x=a+b+c$

17. If a,b,c are non zero real numbers then

$\begin{vmatrix} bc & ca & ab \\ ca & ab & bc \\ ab & bc & ca \end{vmatrix} = 0$, when

(a) $\frac{1}{a}+\frac{1}{b}+\frac{1}{c}=0$ (b) $\frac{1}{a}-\frac{1}{b}+\frac{1}{c}=0$

(c) $\frac{1}{b}+\frac{1}{c}-\frac{1}{a}=0$ (d) $\frac{1}{b}-\frac{1}{c}-\frac{1}{a}=0$

18. $\begin{vmatrix} ka & k^2+a^2 & 1 \\ kb & k^2+b^2 & 1 \\ kc & k^2+c^2 & 1 \end{vmatrix} =$

(a) $k(a+b)(b+c)(c+a)$

(b) $kabc\left(a^2+b^2+c^2\right)$

(c) $k(a-b)(b-c)(c-a)$

(d) $k(a+b-c)(b+c-a)(c+a-b)$

19. Value of $\begin{vmatrix} 1 & \omega^3 & \omega^5 \\ \omega^3 & 1 & \omega^4 \\ \omega^5 & \omega^4 & 1 \end{vmatrix} =$

(a) $(1-\omega)^2$ (b) 3

(c) –3 (d) None of these

20. System of equations $x+ay+az=0$, $bx+y+bz=0$, $cx+cy+z=0$, where a,b,c are non-zero & non-unity has no trivial solution, then the value of $\frac{a}{1-a}+\frac{b}{1-b}+\frac{c}{1-c}$ is

(a) 0 (b) 1

(c) –1 (d) $\frac{abc}{a^2+b^2+c^2}$

21. Value of $\begin{vmatrix} x+1 & \omega & \omega^2 \\ \omega & x+\omega^2 & 1 \\ \omega^2 & 1 & x+\omega \end{vmatrix}$ is ___

(a) 1 (b) ω

(c) ω^2 (d) 0

22. If $a \neq p, b \neq q, c \neq r$ and $\begin{vmatrix} p & b & c \\ a & q & c \\ a & b & r \end{vmatrix} = 0$

then the value of

$\frac{p}{p-a}+\frac{q}{q-b}+\frac{r}{r-c}=$

(a) 0 (b) 1

(c) –1 (d) 2

23. $\begin{vmatrix} x & p & q \\ p & x & q \\ p & q & x \end{vmatrix} =$

(a) $(x+p)(x+q)(x-p-q)$

(b) $(x-p)(x-q)(x+p+q)$

(c) $(x-p)(x-q)(x-p-q)$

(d) $(x+p)(x+q)(x+p+q)$

24. $\begin{vmatrix} 1 & 1 & 1 \\ 1 & 1+x & 1 \\ 1 & 1 & 1+y \end{vmatrix} =$

(a) $x+y$ (b) xy

(c) $x-y$ (d) $1+x+y$

25. $\begin{vmatrix} \sin^2 x & \cos^2 x & 1 \\ \cos^2 x & \sin^2 x & 1 \\ -10 & 12 & 2 \end{vmatrix} =$

(a) 0

(b) $12\cos^2 x - 10\sin^2 x$

(c) $10\sin^2 x - 10\cos^2 x - 2$

(d) $10\sin 2x$

26. $\begin{vmatrix} x+a & b & c \\ a & x+b & c \\ a & b & x+c \end{vmatrix} = 0$, then $x=$

(a) $a+b+c$ (b) $-(a+b+c)$

(c) $0, a+b+c$ (d) $0, -(a+b+c)$

27. The system of equations $x+y+z=2$, $2x+y-z=3$, $3x+2y+kz=4$ has a unique solution of

(a) $k \neq 0$ (b) $-1<k<1$

(c) $-2<k<2$ (d) $k=0$

28. If A & B are square matrices of order 3 and |A| = –1, |B| = 3, then |3AB| =

(a) –9 (b) –81

(c) –27 (d) 81

29. $\begin{vmatrix} x & 4 & y+z \\ y & 4 & z+x \\ z & 4 & x+y \end{vmatrix} =$

(a) 4 (b) $x+y+z$

(c) xyz (d) 0

30. $\begin{vmatrix} 11 & 12 & 13 \\ 12 & 13 & 14 \\ 13 & 14 & 15 \end{vmatrix} =$

(a) 1 (b) 0

(c) –1 (d) 67

31. A root of the equation

$$\begin{vmatrix} 3-x & -6 & 3 \\ -6 & 3-x & 3 \\ 3 & 3 & -6-x \end{vmatrix} = 0 \text{ is}$$

(a) 6 (b) 3

(c) 0 (d) None of these

32. The value of $\begin{vmatrix} -1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{vmatrix} =$

(a) –4 (b) 0

(c) 1 (d) 4

33. If $1, \omega, \omega^2$ are cube roots of unity then

$$\begin{vmatrix} 1 & \omega^n & \omega^{2n} \\ \omega^{2n} & 1 & \omega^n \\ \omega^n & \omega^{2n} & 1 \end{vmatrix} =$$

(a) 0 (b) ω

(c) ω^2 (d) 0

34. The roots of $\begin{vmatrix} x-1 & 1 & 1 \\ 1 & x-1 & 1 \\ 1 & 1 & x-1 \end{vmatrix} = 0$ are

(a) 1, 2 (b) –1, 2

(c) 1, –2 (d) –1, –2

35. If k is a scalar and A is an $n \times n$ square matrix then |kA| =

(a) $k|A|^n$ (b) $k|A|$

(c) $k^n|A^n|$ (d) $k^n|A|$

ANSWER KEYS

1. Ans. (c) $R_2 \leftarrow 2R_2$

$$= \frac{1}{2}\begin{vmatrix} x+1 & x+2 & x+a \\ 2x+4 & 2x+6 & 2x+2b \\ x+3 & x+4 & x+c \end{vmatrix}$$

$R_2 \leftarrow R_2 \leftarrow (R_1 + R_3)$

$$= \frac{1}{2}\begin{vmatrix} x+1 & x+2 & x+a \\ 0 & 0 & 0 \\ x+2 & x+3 & x+c \end{vmatrix} \text{ (As 2b = a+c)}$$

2. Ans. (a)

As $\left.\begin{matrix} p+r+r=0 \\ a+b+c=0 \end{matrix}\right\} \Rightarrow p^3+q^2+r^3 = 3pqr$

& $a^3+b^3+c^3 = 3abc$

$$\begin{vmatrix} pa & qb & rc \\ qc & ra & pb \\ rb & pc & qa \end{vmatrix} = pqr\left(a^3+b^3+c^3\right)$$

$-abc\left(p^3+q^3+r^3\right)$

$= pqr(3abc) - abc(3pqr) = 0$

3. Ans. (b)

If A be the 1st term & R be the C.r of GP then

$$\begin{matrix} a = AR^{p-1} \\ b = AR^{q-1} \\ c = AR^{r-1} \end{matrix}\left|\begin{matrix} \log a = \log A + (p-1)\log R \\ \log b = \log A + (q-1)\log R \\ \log c = \log A + (r-1)\log R \end{matrix}\right.$$

$$\begin{vmatrix} \log a & p & 1 \\ \log b & q & 1 \\ \log c & r & 1 \end{vmatrix}$$

$C_1 \leftarrow C_1 - \log A C_3$ & taking logR common from C_1

$$= \log R\begin{vmatrix} p-1 & p & 1 \\ q-1 & q & 1 \\ r-1 & r & 1 \end{vmatrix}$$

$C_2 \leftarrow C_2 - C_3$ & $C_2 = C_3 = 0$

4. Ans. (b)

since A^{-1} exists therefore $AA^{-1} = I$

$\det\left(AA^{-1}\right) = \det\left(I\right)$

$\Rightarrow \det A.\det A^{-1} = 1$

$\Rightarrow \det\left(A^{-1}\right) = \dfrac{1}{\det A}$

5. All of (a), (b), (c), (d)

$C_1 \leftarrow aC_1$

$$\frac{1}{a}\begin{vmatrix} a^3+ax^2 & ab & ac \\ a^2b & b^2+x^2 & bc \\ a^2c & bc & c^2+x \end{vmatrix}$$

$C_1 \leftarrow C_1 + bC_2 + cC_3$ & taking common $\left(a^2+b^2+c^2+x^2\right)$ from C,

$$= \frac{1}{a}\left(a^2+b^2+c^2+x^2\right)\begin{vmatrix} a & ab & bc \\ b & b^2+x^2 & bc \\ c & bc & c^2+x^2 \end{vmatrix}$$

$C_2 \leftarrow C_2 - bC_1, C_3 \leftarrow C_3 - cC_1$

$$= \frac{1}{a}\left(a^2+b^2+c^2+x^2\right)\begin{vmatrix} a & 0 & 0 \\ b & x^2 & 0 \\ c & 0 & x^2 \end{vmatrix}$$

is div. by $x, x^2, x^3 \,\&\, x^4$

6. Ans. (b), (d)

$R_3 \leftarrow R_3 - xR_1 - R_2$

$$= \begin{vmatrix} a & b & ax+b \\ b & c & bx+c \\ 0 & 0 & -\left(ax^2+2bx+c\right) \end{vmatrix}$$

$= \left(b^2-ac\right)\left(ax^2+2bx+c\right) = 0$ if $b^2 = ac$

(a) $ax^2+2bx+c=0$

7. Ans. (a)

As $a_1, a_2, ..., a_n$ are in AP

$\Rightarrow a_{n+1}^2 = a_n a_{n+2}$

$2\log a_{n+1} - \log a_n - \log a_{n+2} = 0$

similarly

$2\log a_{n+4} - \log a_{n+3} - \log a_{n+5} = 0$

$2\log a_{n+7} - \log a_{n+6} - \log a_{n+8} = 0$

$C_1 \leftarrow C_1 + C_3 - 2C_2$, then $C_1 = 0$

so det = 0

8. Ans. (b)

$$\begin{vmatrix} x & x^2 & 1 \\ y & y^2 & 2 \\ z & z^2 & 1 \end{vmatrix} + \begin{vmatrix} x & x^2 & x^3 \\ y & y^2 & y^3 \\ z & z^2 & z^3 \end{vmatrix} = 0$$

$$\Rightarrow \left(1+xyz\right)\begin{vmatrix} x & x^2 & 1 \\ y & y^2 & 1 \\ z & z^2 & 1 \end{vmatrix} = 0$$

$\left(1+xyz\right)\left(x-y\right)\left(y-z\right)\left(z-x\right) = 0$

$\Rightarrow xyz = -1$ as $x \neq y \neq z$.

9. Ans. (b)

$C_1 \leftarrow C_1 + C_2 + C_3$ & taking common 2 from C_1

$$\text{LHS is } 2\begin{vmatrix} a+b+c & c+a & a+b \\ a+b+c & b+c & c+a \\ a+b+c & a+b & b+c \end{vmatrix}$$

$C_2 \leftarrow C_2 - C_1, C_3 \leftarrow C_3 - C_1$ & taking (–1) both from C_2 & C_3

$$= 2\begin{vmatrix} a+b+c & b & c \\ a+b+c & a & b \\ a+b+c & c & a \end{vmatrix}$$

$C_1 \leftarrow C_1 - \left(C_2 + C_3\right)$

$$= 2\begin{vmatrix} a & b & c \\ c & a & b \\ b & c & a \end{vmatrix}$$

10. Ans. (c)

$$\begin{vmatrix} -a^2 & ab & ac \\ ab & -b^2 & bc \\ ac & bc & -c^2 \end{vmatrix} \text{ as } \begin{matrix} a^2+b^2+c^2=0 \\ b^2+c^2=-a^2 \\ c^2+a^2=-p^2 \\ a^2+b^2=-c^2 \end{matrix}$$

$$abc\begin{vmatrix} -a & b & c \\ a & -b & c \\ a & b & -c \end{vmatrix}$$ (Taking common a, b, c from R_1, R_2, R_3)

$$=a^2b^2c^2\begin{vmatrix} -1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{vmatrix}$$ (Taking common a, b, c from C_1, C_2, C_3

$= 4a^2b^2c^2$

11. Ans. (a)

Taking a, b, c common from R_1, R_2 & R_3

$C_1 \leftarrow C_1 + C_2 + C_3$

det = 0 As $1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c}=0$

$\Rightarrow \lambda = 0$

12. Ans. (c)

$=-\left(1-\cos^2 A\right)-\cos C\left(-\cos C-\cos A\cos B\right)$

$+\cos B\left(\cos A\cos C+\cos B\right)$

$=-1+\cos^2 B+\cos^2 C+\cos A\cos B\cos C$

$+\cos A\cos B\cos C+\cos^2 B$

$=-1+1$ (As $\cos^2 A+\cos^2 B+\cos^2 C$ $+2\cos A\cos B\cos C=1$) = 0

13. Ans. (d)

$x=0$ satisfies $\begin{vmatrix} 0 & -a & -b \\ a & 0 & -c \\ b & c & 0 \end{vmatrix}=0$

14. Ans.(d)

system of equations is in consistent if

$$A=\begin{vmatrix} k & 1 & 1 \\ 1 & k & 1 \\ 1 & 1 & k \end{vmatrix}=0$$ & one of

$$\Delta_1=\begin{vmatrix} 1 & 1 & 1 \\ k & k & 1 \\ k^2 & 1 & k \end{vmatrix}, \Delta_2=\begin{vmatrix} k & 1 & 1 \\ 1 & k & 1 \\ 1 & k^2 & k \end{vmatrix},$$

$$\Delta_3=\begin{vmatrix} k & 1 & 1 \\ 1 & k & k \\ 1 & 1 & k^2 \end{vmatrix}$$ is non-zero.

$$\begin{matrix} \Delta=(k+2)(k-1)^2 \\ \Delta_1=-(k+1)(k-1)^2 \\ \Delta_2=-k(k-1)^2 \\ \Delta_3=(k+1)^2(k-1)^2 \end{matrix}\left|\begin{matrix} \text{for } k=-2 \\ \Delta=0 \\ \Delta_1,\Delta_2,\Delta_3 \\ \text{are non-zero} \end{matrix}\right.$$

15. Ans. (b)

$\det\left(B^{-1}AB\right)=\det\left(B^{-1}\right)\det A \det(B)$

$=\frac{1}{\det B}.\det A.\det B$

= det A

16. Ans. (c)

$C_1 \leftarrow C_1+C_2+C_3$ & taking common $3x-(a+b+c)$ from C_1

$$3x-(a+b+c)\begin{vmatrix} 1 & x-b & x-c \\ 1 & x-c & x-a \\ 1 & x-a & x-b \end{vmatrix}=0$$

$C_1 \leftarrow C_1-C_2, C_2 \leftarrow C_2-C_3$

$$=3x-(a+b+c)\begin{vmatrix} 1 & x-b & x-c \\ 0 & b-c & c-a \\ 0 & b-a & c-b \end{vmatrix}=0$$

$=\left\{2x-(a+b+c)\right\}\left(a^2+b^2+c^2-ab-bc-ca\right)=0$

$x=\frac{1}{3}(a+b+c)$

17. Ans. (a)

$$\begin{vmatrix} bc & ca & ab \\ ca & ab & bc \\ ab & bc & ca \end{vmatrix} = 0$$

$$\Rightarrow 3a^2b^2c^2 - \left[(ab)^3 + (bc)^3 + (ca)^3\right] = 0$$

$$\Rightarrow (ab)^3 + (bc)^3 + (ca)^3 - 3a^2b^2c^2 = 0$$

$$\Rightarrow (ab + bc + ca)(a^2b^2 + b^2c^2 + c^2a^2$$

$$-ab^2c - bc^2a - ca^2b) = 0$$

$$\Rightarrow ab + bc + ca = 0$$

$$\Rightarrow abc\left(\frac{1}{a} + \frac{1}{b} + \frac{1}{c}\right) = 0$$

$$\Rightarrow \frac{1}{a} + \frac{1}{b} + \frac{1}{c} = 0$$

18. Ans. (c)

$$\begin{vmatrix} ka & k^2 & 1 \\ kb & k^2 & 1 \\ kc & k^2 & 1 \end{vmatrix} + \begin{vmatrix} ka & a^2 & 1 \\ kb & b^2 & 1 \\ kc & c^2 & 1 \end{vmatrix} = 0 + k\begin{vmatrix} a & a^2 & 1 \\ b & b^2 & 1 \\ c & c^2 & 1 \end{vmatrix}$$

$$= k(a-b)(b-c)(c-a)$$

19. Ans. (b)

$$\begin{vmatrix} 1 & 1 & \omega^2 \\ 1 & 1 & \omega \\ \omega^2 & \omega & 1 \end{vmatrix} = 2\omega^3 - \omega^2 - \omega^4$$

$$= 2 - (\omega + \omega^2) = 2 - (-1) = 3$$

20. Ans. (c)

$C_1 \leftarrow C_1 - C_2, C_2 \leftarrow C_2 - C_3$

$$\begin{vmatrix} 1-a & 0 & a \\ b-1 & 1-b & b \\ 0 & c-1 & 1 \end{vmatrix} = 0$$

$$\Rightarrow (1-a)\left[(1-b) - b(c-1)\right] + a(b-1)(c-1) = 0$$

$$\Rightarrow (1-a)(1-b) - b(1-a)(c-1) + a(b-1)(c-1) = 0$$

$$\Rightarrow (a-1)(b-1) + b(a-1)(c-1) + a(b-1)(c-1) = 0$$

$$\Rightarrow \frac{1}{c-1} + \frac{b}{b-1} + \frac{a}{c-1} = 0$$

$$\Rightarrow \left(\frac{1}{c-1} + 1\right) + \frac{b}{b-1} + \frac{a}{c-1} = 1$$

$$\Rightarrow \frac{c}{c-1} + \frac{b}{b-1} + \frac{a}{a-1} = -1$$

21. Ans. (d)

$R_1 \leftarrow R_1 + R_2 + R_3$ & taking common x from R_1

$$x\begin{vmatrix} 1 & 1 & 1 \\ \omega & x + \omega^2 & 1 \\ \omega^2 & 1 & x + \omega \end{vmatrix} = 0$$

$C_2 \leftarrow C_2 - C_1,\ C_3 \leftarrow C_3 - C_1$

$$\Rightarrow x\begin{vmatrix} 1 & 0 & 0 \\ \omega & x + \omega^2 - \omega & 1 - \omega \\ \omega^2 & 1 - \omega^2 & x + \omega - \omega^2 \end{vmatrix} = 0$$

$$\Rightarrow x\left[\left(x + \omega^2 - \omega\right)\left(x + \omega - \omega^2\right) - (1-\omega)\left(1-\omega^2\right)\right] = 0$$

$$\Rightarrow x = 0$$

22. Ans. (d)

$R_3 \leftarrow R_3 - R_2, R_2 \leftarrow R_2 - R$

$$\begin{vmatrix} p & b & c \\ a-p & q-b & 0 \\ 0 & b-q & r-c \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} \frac{p}{p-a} & \frac{b}{q-b} & \frac{c}{r-c} \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{vmatrix} = 0$$

$$\Rightarrow \frac{p}{p-a} + \frac{b}{q-b} + \frac{c}{r-c} = 0$$

$$\Rightarrow \frac{p}{p-a} + \left(\frac{q}{q-b} - 1\right) + \left(\frac{r}{r-c} - 1\right) = 0$$

$$\Rightarrow \frac{p}{p-a} + \frac{q}{q-b} + \frac{r}{r-c} = 2$$

23. Ans. (b

$C_1 \leftarrow C_1 + C_2 + C_3$ & taking common $(x+p+q)$ from C_1

$$(x+p+q)\begin{vmatrix} 1 & p & q \\ 1 & x & q \\ 1 & q & x \end{vmatrix}$$

$R_2 \leftarrow R_2 - R_1, R_3 \leftarrow R_3 - R_1$

$$=(x+p+q)\begin{vmatrix} 1 & p & q \\ 0 & x-p & 0 \\ 0 & q-p & x-q \end{vmatrix}$$

$$=(x+p+q)(x-p)(x-q)$$

24. Ans. (b)

$R_2 \leftarrow R_2 - R_1$ & $R_3 \rightarrow R_3 - R$

25. Ans. (a)

$C_1 \rightarrow C_1 + C_2$ then $C_1 = C_3$

26. Ans. (d)

$C_1 \leftarrow C_1 + C_2 + C_3$

27. Ans. (a)

$$\begin{vmatrix} 1 & 1 & 1 \\ 2 & 1 & -1 \\ 3 & 2 & k \end{vmatrix} \neq 0 \Rightarrow k \neq 0$$

28. Ans. (a)

$|3AB| = 3|A|\,|B| = 3(-1)(3) = -9$

29. Ans. (d)

$C_1 \rightarrow C_1 + C_3$

Then $C_1 = C_2$ (By taking common $x+y+z$ & 6) from C_1 & C_2

30. Ans. (b)

$C_2 \rightarrow C_2 - C_1,\ C_3 \rightarrow C_3 - C_2$

then $C_2 = C_3$

31. Ans. (c)

$C_1 \rightarrow C_1 + C_2 + C_3$ & taking common $(-x)$ from C_1

$$-x\begin{vmatrix} 1 & -6 & 3 \\ 1 & 3-x & 5 \\ 1 & 3 & -6-x \end{vmatrix} = 0$$

$R_2 \rightarrow R_2 - R_1,\ R_3 \rightarrow R_3 - R_1$

$$-x\begin{vmatrix} 1 & -6 & 3 \\ 0 & 9-x & 0 \\ 0 & 9 & -9-x \end{vmatrix} = 0$$

$\Rightarrow x = 0, 9, -9$

32. Ans. (d)

$C_1 \leftarrow C_1 + C_2, C_2 \leftarrow C_2 + C_3$

33. Ans. (a)

$$1\left(1-\omega^{3n}\right) - \omega^n\left(\omega^{2n} - \omega^{2n}\right) + \omega^{2n}\left(\omega^{4n} - \omega^n\right)$$

$$= 0 - 0 + \omega^{2n}\left(\omega^n - \omega^n\right) = 0$$

34. Ans. (b)

$C_1 \rightarrow C_1 + C_2 + C_3$ & taking common $(x+1)$ from C_1

$$(x+1)\begin{vmatrix} 1 & 1 & 1 \\ 1 & x-1 & 1 \\ 1 & 1 & x-1 \end{vmatrix} = 0$$

$R_2 \rightarrow R_2 - R_1,\ R_3 \rightarrow R_3 - R$

$$(x+1)\begin{vmatrix} 1 & 1 & 1 \\ 0 & x-2 & 0 \\ 0 & 0 & (x-2) \end{vmatrix} = 0$$

$(x+1)(x-2)^2 = 0$

$x = -1, 2$

35. Ans. (d)

$|kA| = k^n\,|A|$

Unit - III

CALCULUS

CHAPTER - 1

CONTINUITY & DIFFERENTIABILITY

A. Multiple Choice Questions (MCQ)

1. $f(x)=[\sin x]$ is continuous at ____

(a) $\frac{\pi}{2}$ (b) π

(c) $\frac{3\pi}{2}$ (d) 2π

2. $f(x)=\left[\tan^{-1} x\right]$ is discontinuous at ____

(a) $\frac{\pi}{4}, -\frac{\pi}{4}, 0$

(b) $\frac{\pi}{3}, -\frac{\pi}{3}, 0$

(c) $\tan 1, -\tan 1, 0$

(d) None of these

3. $f(x)=\begin{cases}\sin\{x\}, & x<1\\ \cos x+a, & x\geq 1\end{cases}$ where {.} denotes the fractional part. If $f(x)$ is continuous at $x=1$ then a = ____

(a) sin 1 (b) a = cos1 - sin 1

(c) a = cos 1 (d) a = sin 1 - cos1

4. Value of $f(0)$ is so that $f(x)=\frac{1}{x^2}(1-\cos(\sin x))$ can be made continuous at $x=0$ is equal to ____

(a) 1/2 (b) 2

(c) 8 (d) 4

5. $f(x)=\begin{cases}[x], -2\leq x\leq 1/2\\ 2x^2-1, 1/2<x\leq 2\end{cases}$ then number of parts of discontinuity of $f(x)$ is ___

(a) 1 (b) 2

(c) 3 (d) None of these

6. $f(x)=\begin{cases}\frac{1-\cos x}{x}, & x\neq 0\\ k, & x=0\end{cases}$ is continuous at $x=0$ then k = ____

(a) 0 (b) 1/2

(c) 1/4 (d) –1/2

7. $f: R\to R$ defined as

$f(x)=\begin{cases}\frac{\cos 3x-\cos x}{x^2}, & x\neq 0\\ \lambda, & x=0\end{cases}$

If it is continuous at $x=0$ then $\lambda =$ ___

(a) –2 (b) –4

(c) 6 (d) –8

8. $f(x)=\begin{cases}\frac{\sin 5x}{x^2+2x}, & x\neq 0\\ k+\frac{1}{2}, & x=0\end{cases}$ is continuous at $x=0$ then k = ____

(a) 1 (b) –2

(c) 2 (d) 1/2

9. $f(x) = x + |x|$ is continuous for ____

(a) $x \in (-\infty, \infty)$

(b) $x \in (-\infty, \infty) - \{0\}$

(c) only $x > 0$

(d) no value of x

10. $f(x) = \begin{cases} \frac{1-\sqrt{2}\sin x}{\pi - 4x} & \text{if } x \neq \frac{\pi}{4} \\ a, \text{ at } x = \frac{\pi}{4} \end{cases}$

is continuous at $x = \frac{\pi}{4}$ then a = ____

(a) 4 (b) 2

(c) 1 (d) 1/4

11. $f(x) = \begin{cases} \frac{\tan x - \cot x}{x - \frac{\pi}{4}}, x \neq \frac{\pi}{4} \\ a, \quad x = \frac{\pi}{4} \end{cases}$

If $f(x)$ is continuous at $x = \frac{\pi}{4}$ then a = ____

12. The number of discontinuity of $f(x) = [x], x \in (-7/2, 100)$ is ____

(a) 104 (b) 100

(c) 102 (d) 103

13. If $f(x) = \begin{cases} x \sin 1/x, \ x \neq 0 \\ k, \quad x > 0 \end{cases}$ is continuous at $x = 0$ then k = ____

(a) 1 (b) –1

(c) 0 (d) 2

14. The points of discontinuity of $\tan x$ are ____

(a) $n\pi$ (b) $2n\pi$

(c) $(2n+1)\pi/2$ (d) None of these

15. $f(x) = \begin{cases} 2x, & x < 0 \\ 2x+1, & x \geq 0 \end{cases}$ then

(a) f is continuous at $x = 0$

(b) $f(|x|)$ is continuous at $x = 0$

(c) f is discontinuous at $x = 0$

(d) None of these

16. Let $(x) = \begin{cases} 5^{1/x}, & x < 0 \\ \lambda[x], & x \geq 0 \end{cases}$ and $\lambda \in R$

then at $x = 0$

(a) f is discontinuous

(b) f is continuous only if $\lambda = 0$

(c) f is continuous only whatever λ may be

(d) None of these

17. $f(x) = |x|$ is defined on [–2, 2] the points at which f is differentiable are

(a) –1, 0 (b) –1, 0, 1

(c) –1, 0, 1, 2 (d) None of these

18. $f(x) = [x]$ is continuous at ____

(a) 1 (b) 2.5

(c) 3 (d) –1

19. $f(x) = |x-1| + |x-2|$ then the number of points where $f(x)$ is not differenti able is ____

(a) one (b) three

(c) two (d) None of these

20. f is defined as $f(x) = \begin{cases} 0 \text{ if } x \text{ is rational} \\ 1 \text{ if } x \text{ is irrational} \end{cases}$

(a) continuous at every point

(b) discontinuous at every point

(c) differentiable at every point

(d) None of these

21. $f(x)=\begin{cases} \frac{\sin x}{x}, & x \neq 0 \\ 0, & x=0 \end{cases}$ is ___ at $x=0$.

(a) continuous (b) discontinuous

(c) differentiable (d) None of these

22. If $f(x)=e^{\ln x^3}$ then $f'(x)=$___

(a) $\frac{1}{x^3}$ (b) $3x^2$

(c) $e^{3\ln x}$ (d) None of these

23. If $f(x)=\ln e^{x^2}$ then $f'(x)=$____

(a) $\frac{1}{e^{x^2}}$ (b) $2x$

(c) x^2 (d) None of these

24. If $f(x)=\sec^{-1}\sqrt{x}+\operatorname{cosec}^{-1}\sqrt{x}$ then $f'(x)=$ ____

(a) 1 (b) 0

(c) 2 (d) $\pi/2$

25. The function $f(x)=2^{1/x}$ is not continuous at $x=$____

(a) 0 (b) 1

(c) –1 (d) None of these

26. $f(x)=[x]$ is discontinuous in ____

(a) set of all rational numbers

(b) set of all irrational numbers

(c) set of all integral points

(d) set of all prime numbers

27. $f(x)=\frac{|x|}{x}, x\neq 0$ may be continuous at origin if

(a) $f(0)=0$

(b) $f(0)=-1$

(c) $f(0)=1$

(d) can not be continuous for only value of $f(0)$

28. $f(x)=\sin x$ is continuous in ____

(a) $(-\infty, \infty)$ (b) (0, 1)

(c) (1, 2) (d) None of these

29. $f(x)=|x|$ at $x=0$ is ____

(a) continuous and differentiable

(b) continuous but not differentiable

(c) not continuous but differentiable

(d) None of these

30. $f(x)=|x+2|$ is not differentiable at ___

(a) $x=2$ (b) $x=-2$

(c) $x=-1$ (d) $x=1$

31. The set of points of discontinuity of the function $f(x)=\log|x|$ is ____

(a) {0} (b) ϕ

(c) {–1, 1} (d) None of these

32. The set of points of discontinuity of $f(x)=|\sin x|$ is ____

(a) $\{n\pi \mid n\in I\}$

(b) $\{(2n+1)\pi/2 \mid n\in I\}$

(c) ϕ

(d) None of these

33. The set of points of discontinuity of $f(x)=\frac{|\sin x|}{\sin x}$ is ____

(a) {0} (b) $\{n\pi \mid n\in I\}$

(c) ϕ (d) None of these

34. The set of points where the function $f(x)=|x-2|\cos x$ is differentiable is ___

(a) $(-\infty, \infty)$ (b) $(-\infty, \infty)|\{2\}$

(c) $(0, \infty)$ (d) None of these

35. $f(x)=|x-1|+|x|$ is not differentiable at ____

(a) 1, 0 (b) 1, 2

(c) 0, 2 (d) None of these

36. If $y=\sin^{-1}x+\cos^{-1}x$ then $\frac{dy}{dx}=$ ____

(a) 1 (b) 0

(c) –1 (d) 2

37. Derivative of $\tan^{-1}x$ w.r.t. $\cot^{-1}x$ is ___

(a) 1 (b) –1

(c) 0 (d) None of these

38. Derivative of $y=(1-x)(2-x)...(n-x)$ at $x=1$ is ____

(a) $(n-1)!$ (b) $-(n-1)!$

(c) $(n-2)!$ (d) None of these

39. If $y=\cos^{-1}\frac{2x}{1+x^2}$ then $\frac{dy}{dx}=$ ___

(a) $\frac{-2}{1+x^2}$

(b) $\frac{-2}{1+x^2}$ for all $|x|>1$

(c) $\frac{2}{1+x^2}$ for $|x|<1$

(d) None of these

40. $f(x)=[x^2]$ then $f'(3/2)=$ ___

(a) 0 (b) 2

(c) 3 (d) None of these

41. Derivative of $\sin^{-1}(\cos x)$ w.r.t x is ___

(a) 0 (b) –1

(c) 2 (d) 3

42. Derivative of $\tan^{-1}x$ w.r.t. $\cot^{-1}1/x$ is ___

(a) 1 (b) –1

(c) 0 (d) x

43. If $y=\sec^{-1}\frac{\sqrt{x}+1}{\sqrt{x}}+\sin^{-1}\frac{\sqrt{x}}{\sqrt{x}+1}$ then $\frac{dy}{dx}=$ ___

(a) 0 (b) $\frac{\pi}{2}$

(c) 1 (d) –1

44. If U is a constant and V is a variable then $\frac{du^v}{dv}=$ ___

(a) $u^v\ln u$ (b) v^{u-1}

(c) $u^v\ln v$ (d) None of these

45. $\frac{d}{dx}\ln\sin^{-1}\cos\left(\frac{\pi-2e^x}{2}\right)=$ ___

(a) $\frac{1}{\sin x}$ (b) 1

(c) 0 (d) none of these

ANSWER KEYS

1.	(c)	16.	(c)	31.	(a)
2.	(c)	17.	(d)	32.	(c)
3.	(b)	18.	(b)	33.	(b)
4.	(a)	19.	(c)	34.	(b)
5.	(b)	20.	(b)	35.	(a)
6.	(a)	21.	(b)	36.	(b)
7.	(b)	22.	(b)	37.	(b)
8.	(c)	23.	(b)	38.	(b)
9.	(a)	24.	(b)	39.	(d)
10.	(d)	25.	(a)	40.	(a)
11.	(b)	26.	(c)	41.	(b)
12.	(d)	27.	(d)	42.	(a)
13.	(c)	28.	(a)	43.	(a)
14.	(c)	29.	(b)	44.	(a)
15.	(c)	30.	(b)	45.	(b)

B. Long Answer Type Questions

1. If the derivative of the function $f(x)$ is everywhere continuous and is given by
$$f(x)=\begin{cases} bx^2+ax+4, & x\ge -1 \\ ax^2+b, & x<1 \end{cases}$$
then find a and b

2. If $f(x)=\begin{cases} ax^2+1, & x\le 1 \\ x^2+ax+b, & x>1 \end{cases}$
is differentiable at $x=1$ then find a and b.

3. Test the continuity and differentiability of
$$f(x)=\begin{cases} \dfrac{1}{e^{1/x+1}}, x\ne 0 \\ 0, \quad x=0 \end{cases} \text{ at } x=0$$

4. If $y=\tan^{-1}\left(\dfrac{\sqrt{1+x^2}-1}{x}\right)$ then find $\dfrac{dy}{dx}$

5. If $y=\tan^{-1}\left[\dfrac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}\right]$
then find $\dfrac{dy}{dx}$.

6. If $y=\cos^{-1}\left[\sqrt{\dfrac{\sqrt{1+x^2}+1}{2\sqrt{1+x^2}}}\right]$
then find $\dfrac{dy}{dx}$

7. If $\sin y = x\sin(a+y)$ then prove that
$$\frac{dy}{dx}=\frac{\sin^2(a+y)}{\sin a}$$

8. If $y=\sqrt{\sin x+y}$ then prove that
$$\frac{dy}{dx}=\frac{\cos x}{2y-1}$$

9. If $y=\left[x+\sqrt{1+x^2}\right]^n$ then find the value of $(1+x^2)\dfrac{d^2y}{dx^2}+x\dfrac{dy}{dx}$.

10. If $y=\tan\left[\dfrac{1}{2}\cos^{-1}\dfrac{1-u^2}{1+u^2}+\dfrac{1}{2}\sin^{-1}\dfrac{2u}{1+u^2}\right]$
and $x=\dfrac{2u}{1-u^2}$ then find $\dfrac{dy}{dx}$.

11. Find the derivative of $f(\tan x)$ w.r.t. $g(\sec x)$ at $x=\dfrac{\pi}{4}$ where $f'(1)=2$ and $g'(\sqrt{2})=4$.

12. If $\sec^{-1}\left(\dfrac{1+x}{1-y}\right)=a$
then show that $\dfrac{dy}{dx}=\dfrac{y-1}{x+1}$

13. If $f(x)=\cot^{-1}\left(\dfrac{x^x-x^{-x}}{2}\right)$
then find $f'(1)$.

14. Find the derivative of $\tan^{-1}\left(\dfrac{x}{a+\sqrt{a^2-x^2}}\right)$

15. If $y=\sin^{-1}\left(\dfrac{\sin x+\cos x}{\sqrt{2}}\right)$
then find $\dfrac{dy}{dx}$

16. Differentiate $\sin^{-1}\left(2ax\sqrt{1-a^2x^2}\right)$
w.r.t. $\sqrt{1-a^2x^2}$

17. If $\sin(x+y)=y\cos(x+y)$ then show that $\dfrac{dy}{dx}=-\dfrac{1+y^2}{y^2}$

18. If $y=\tan^{-1}\left(\sqrt{\dfrac{1+\sin x}{1-\sin x}}\right)$ then find $\dfrac{dy}{dx}$.

19. If $xe^{xy}=y+\sin^2 x$ then find $\dfrac{dy}{dx}$ at $x=0$.

20. If $x^y=y^x+\tan^{-1}\dfrac{\cos x}{1+\sin x}$ then find $\dfrac{dy}{dx}$

21. Differentiate $\tan^{-1}\left(\dfrac{1+\sin x}{1-\sin x}\right)^{1/2}$
w.r.t. $\ln\left(\dfrac{1+\cos x}{1-\cos x}\right)$

Answer Hints

1. Here $f'(x)=\begin{cases}2bx+a, & x\geq -1\\ 2ax, & x<-1\end{cases}$

$f(-1)=-2b+a,\ \lim_{x\to -1} 2bx+a=-2b+a$

$\lim_{x\to -1} -2ax=-2a$ taking $-2b+a=-2a$

$\Rightarrow 3a-2b=0 \Rightarrow a=2, b=3$

2. $f'(1-h)=\lim_{h\to 0}\dfrac{f(1-h)-f(1)}{1-h-1}$

$=\lim_{h\to 0}\dfrac{a(1-h)^2+1-(1+a)}{-h}=2a$

$f'(1+h)=\lim_{h\to 0}\dfrac{f(1+h)-f(1)}{1+h-1}$

$=\lim_{h\to 0}\dfrac{(1+h)^2+a(1+h)+b-(1+a)}{h}=2+a$

If $b=0 \Rightarrow 2a=2+a \Rightarrow a=2,\ b=0$

3. LHS $=\lim_{h\to 0} f(0-h)=\lim_{h\to 0}\dfrac{1}{1+e^{-1/h}}=\dfrac{1}{1+0}=1$

RHS $\lim_{h\to 0} f(0+h)=\lim_{h\to 0}\dfrac{1}{e^{1/h}+1}$

$=\lim_{h\to 0}\dfrac{e^{-1/h}}{1+e^{-1/h}}=0$

(Dividing numerator and dinominator by $e^{1/h}$)

As RHS $\neq$ LHS $\Rightarrow$ The functions is not continuous

$\Rightarrow$ The function is not differentiable.

4. Put $x=\tan\theta \Rightarrow \theta=\tan^{-1}x$

$y=\tan^{-1}\left[\dfrac{\sqrt{1+\tan^2\theta}-1}{\tan\theta}\right]$ then simplify

in $\sin x$ and $\cos x$ form

5. Put $x=\cos\theta$ and apply the formula

$1+\cos\theta=2\cos^2\theta/2,\ 1-\cos\theta=2\sin^2\theta/2$

6. Put $x=\tan\theta \Rightarrow \theta=\tan^{-1}x$

7. $x=\dfrac{\sin y}{\sin(a+y)} \Rightarrow \dfrac{dy}{dx}$

$\dfrac{\sin(a+y)\cos y-\cos(a+y).\sin y}{\sin^2(a+y)}$

$\dfrac{dx}{dy}=\dfrac{\sin(a+y-y)}{\sin^2(a+y)}=\dfrac{\sin a}{\sin^2(a+y)}$

$\Rightarrow \dfrac{dy}{dx}=\dfrac{\sin^2(a+y)}{\sin a}$

8. $y=\sqrt{\sin x+y} \Rightarrow y^2=\sin x+y$

then differentiable

9. $y_1=\dfrac{dy}{dx}=n\left[x+\sqrt{1+x^2}\right]^{n-1}$

$\left\{1+\dfrac{1}{\not 2}(1+x^2)^{1/2}.\not 2x\right\}$

$=n\left[x+\sqrt{1+x^2}\right]^{n-1}\times\left\{1+\dfrac{x}{\sqrt{1+x^2}}\right\}$

$=n\left[x+\sqrt{1+x^2}\right]^{n-1}\times\dfrac{\sqrt{1+x^2}+x}{\sqrt{1+x^2}}$

$=n\dfrac{\left[x+\sqrt{1+x^2}\right]^{n}}{\sqrt{1+x^2}}=\dfrac{ny}{\sqrt{1+x^2}}$

$y_1^2=\dfrac{n^2y^2}{1+x^2} \Rightarrow y^2(1+x^2)=n^2y^2$

Then differentiate twice.

10. Put $4=\tan\theta$ and simplify

Replace $\dfrac{1-\tan^2\theta}{1+\tan^2\theta}$ by $\cos 2\theta$ and $\dfrac{2\tan\theta}{1+\tan^2\theta}$ by $\sin 2\theta$

12. $\dfrac{1+x}{1-y}=\sec a \Rightarrow 1+x=\sec\alpha(1-y)$

$\Rightarrow 1+x=\sec a-y\sec a$ then differentiate

13. $f(x)=\cot^{-1}\left[\frac{x^{x^{-}}1/x^{x}}{2}\right]=\cot^{-1}\left[\frac{x^{2x}-1}{2x^{x}}\right]$

Then put $x^x=\tan\theta$ and differentiate

14. Put $x=a\sin\theta$ and differentiate

15. $y=\sin^{-1}\left(\sin x.\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2}}\cos x\right)$

$=\sin^{-1}\left[\sin x\cos\frac{\pi}{4}+\sin\frac{\pi}{4}\cos x\right]$

$=\sin^{-1}\sin\left(\frac{\pi}{4}+x\right)=\frac{\pi}{4}+x$

Then $\frac{dy}{dx}=1$

16. Put $ax=\sin\theta$ then differentiate

17. Here $\tan(x+y)=y$

$\Rightarrow\sec^2(x+y)\left[1+\frac{dy}{dx}\right]=\frac{dy}{dx}$

$\Rightarrow\sec^2(x+y)-\frac{dy}{dx}\left[\sec^2(x+y)-1\right]$

$\Rightarrow\frac{dy}{dx}=-\frac{\sec^2(x+y)}{\tan^2(x+y)}=-\frac{1+\tan^2(x+y)}{\tan^2(x+y)}$

$=-\frac{1+y^2}{y^2}$

18. Use $1+\sin x=\left(\cos\frac{x}{2}+\sin\frac{x}{2}\right)^2$ and

$(1-\sin x)=\left(\cos\frac{x}{2}-\sin\frac{x}{2}\right)^2$ then find $\frac{dy}{dx}$

19. $\tan^{-1}\frac{\cos x}{1+\sin x}=\tan^{-1}\left(\frac{\cos^2\frac{x}{2}-\sin^2\frac{x}{2}}{\left(\cos\frac{x}{2}+\sin\frac{x}{2}\right)^2}\right)$

$=\tan^{-1}\left[\frac{\cos\frac{x}{2}-\sin\frac{x}{2}}{\cos\frac{x}{2}+\sin\frac{x}{2}}\right]$

$=\tan^{-1}\left(\frac{1-\tan\frac{x}{2}}{1+\tan\frac{x}{2}}\right)$

(dividing each term by cos $x/2$)

$=\tan^{-1}\tan\left(\frac{\pi}{4}-\frac{x}{2}\right)=\frac{\pi}{4}-\frac{x}{2}$

$y=\frac{\pi}{4}-\frac{x}{2}\Rightarrow\frac{dy}{dx}=-\frac{1}{2}$ (1)

Let $v=x^y\Rightarrow\ln v=y\ln u$

$\Rightarrow\frac{1}{v}\frac{dv}{dx}=\frac{dy}{dx}\ln x+\frac{y}{x}$

$\Rightarrow\frac{dv}{dx}=v\left[\frac{y}{x}+\frac{dy}{dx}\ln x\right]$

$\frac{d}{dx}x^y=x^y\left[\frac{b}{x}+\frac{dy}{dx}\ln x\right]$ (2)

Let $\omega=y^x\Rightarrow\ln\omega=x\ln y$

$\frac{1}{\omega}\frac{d\omega}{dx}=\ln y+\frac{x}{y}\frac{dy}{dx}$

$\frac{d\omega}{dx}=\omega\left[\ln y+\frac{x}{y}\frac{dy}{dx}\right]$

$\frac{d}{dx}y^x=y^x\left[\ln y+\frac{x}{y}\frac{dy}{dx}\right]$ (3)

Here $\frac{d}{dx}x^y=\frac{d}{dx}y^x+\frac{d}{dx}\tan^{-1}\left(\frac{\cos x}{1+\sin x}\right)$

Then use (1), (2) and (3)

21. Replace $1+\cos x=2\cos^2\frac{x}{2}$,

$1-\cos x=2\sin^2\frac{x}{2}$

$1+\sin x=\left(\cos\frac{x}{2}+\sin\frac{x}{2}\right)^2$,

$1-\sin x=\left(\cos\frac{x}{2}-\sin\frac{x}{2}\right)^2$

Then differentiable.

ADDENDUM

1. $f(x)=[\sin x]$, where [ ] denotes the greatest integer function is continuous at ____

(a) $\frac{\pi}{2}$ (b) π

(c) $\frac{3\pi}{2}$ (d) 2π

2. $f(x)=\left[\tan^{-1} x\right]$ is discontinuous at

(a) $\frac{\pi}{4},-\frac{\pi}{4},0$ (b) $\frac{\pi}{3},-\frac{\pi}{3},0$

(c) tan 1, –tan1,0 (d) None of these

3. $f(x)=\begin{cases}\sin\{x\}, \ x<1\\ \cos x+a, \ x\geq 1\end{cases}$

where {.} denotes the fractional part of $f(x)$ is continuous at x = 1 then a = ____

(a) sin 1 (b) a = cos1–sin 1

(c) a = cos 1 (d) a = sin 1-cos1

4. Value of $f(0)$ is so that $f(x)=\frac{1}{x^2}$ $(1-\cos(\sin x))$ can be made continuous at $x=0$ is equal to

(a) 1/2 (b) 2

(c) 8 (d) 4

5. $f(x)=\begin{cases}x+a\sqrt{2}\sin x \quad 0\leq x<\pi/4\\ 2x\cot x+b \quad \pi/4\leq x<\pi/2\\ a\cos 2x-b\sin x \ \pi/2\leq x\leq\pi\end{cases}$ is continuous at $[0, \pi]$ then a = ___, b = ____

(a) $a=-\frac{\pi}{6},b=-\frac{\pi}{12}$

(b) $a=\frac{\pi}{6},b=\frac{\pi}{12}$

(c) $a=\frac{\pi}{6},b=-\frac{\pi}{12}$

(d) $a=-\frac{\pi}{6},b=\frac{\pi}{12}$

6. $f(x)=\begin{cases}[x],-2\leq x\leq 1/2\\ 2x^2-1,1/2<x\leq 2\end{cases}$

then number of points of discontinuity of $f(x)$ is

(a) 1 (b) 2

(c) 3 (d) None of these

7. If $f(x)=\begin{cases}\frac{1-\cos x}{x},x\neq 0\\ k, \quad x=0\end{cases}$ is continuous at $x=0$ then k = ____

(a) 0 (b) 1/2

(c) 1/4 (d) –1/2

8. If $f:R\to R$ is defined by

$f(x)=\begin{cases}\frac{\cos 3x-\cos 4}{x^2},x\neq 0\\ \lambda, \quad x=0\end{cases}$ and if f is continuous at $x=0$ then $\lambda =$ ____

(a) –2 (b) –4

(c) 6 (d) –8

9. $f(x)=\begin{cases}\frac{\sin 5x}{x^2+2x},x\neq 0\\ x+1/2,x=0\end{cases}$ is continuous at $x=0$ then the value of k is ____

(a) 1 (b) –2

(c) 2 (d) 1/2

10. $f(x) = x + |x|$ is continuous for

(a) $x \in (-\infty, \infty)$

(b) $x \in (-\infty, \infty) - \{0\}$

(c) only $x > 0$

(d) n value of x

11. $f(x) = \begin{cases} \frac{1-\sqrt{2}\sin x}{\pi - 4x} & \text{if } x \neq \frac{\pi}{4} \\ a, & x = \pi/4 \end{cases}$ then

(a) 4 (b) 2

(c) 1 (d) 1/4

12. $f(x) = \begin{cases} \frac{\tan x - \cot x}{x - \pi/4} & x \neq \pi/4 \\ a, & x = \pi/4 \end{cases}$ if $f(x)$ is continuous at $x = \pi/4$ then a = ____

(a) 2 (b) 4

(c) 3 (d) 1

13. Which of the following is not true always?

(a) if $f(x)$ is not continuous at x = a then it is not differentiable at x = a

(b) if $f(x)$ is continuous at x = a then it is differentiable at x = a.

(c) if $f(x)$ and $g(x)$ are differentiable at x = a then $f(x) + g(x)$ is differentiable at x = a

(d) If $f(x)$ is continuous at x = a then $\lim_{x \to a} f(x) = m^2 + 5$

14. The number of discontinuous of $x \to a$ $f(x) = [x], x \in [-7/2, 100)$ is ____

(a) 104 (b) 100

(c) 102 (d) 103

15. $f(x) = \begin{cases} mx + 1, & x \leq \pi/2 \\ \sin x + n, & x > \pi/2 \end{cases}$ is continuous at $x = \pi/2$ then

(a) m = 1, n = 0 (b) $m = \frac{n\pi}{2} + 1$

(c) $n = \frac{m\pi}{2}$ (d) $m = n = \pi/2$

16. $f(x) = \log_e \frac{(1 + x^2 \tan x)}{\sin x^3} x \neq 0$ is to be continuous at $x = 0$ then $f(x)$ must be defined as

(a) 1 (b) 0

(c) 1/2 (d) –1

17. If $f(x) = \begin{cases} x \sin 1/x, & x \neq 0 \\ k, & x > 0 \end{cases}$ is continuous at $x = 0$ then the value of k is

(a) 1 (b) –1

(c) 0 (d) 2

18. The parts of discontinuity of $\tan x$ are ____

(a) $n\pi, n \in I$

(b) $2n\pi, n \in I$

(c) $(2n+1)\pi/2, n \in I$

(d) None of the above

19. $f(x) = \begin{cases} 2x & , x < 0 \\ 2x + 1, & x \geq 0 \end{cases}$ then

(a) $f(x)$ is continuous at $x = 0$

(b) $f(|x|)$ is continuous at $x = 0$

(c) $f(x)$ is discontinuous at $x = 0$

(d) None of the above

20. The function $f(x) = x - |x - x^2|$ is ___

(a) continuous at $x = 1$

(b) discontinuous at $x = 1$

(c) not defined at $x = 1$

(d) none of the above

21. Let $f(x)=\begin{cases}5^{1/x}, x<0\\ \lambda(x), x\ge 0\end{cases}$ and $\lambda \in R$ then at x = 0

(a) f is discontinuous

(b) f is continuous only if $\lambda = 0$

(c) f is continuous only whatever λ may be

(d) None of these

22. If the derivative of the function $f(x)$ is everywhere continuous and is given by $f(x)=\begin{cases}bx^2+ax+4, & x\ge -1\\ ax^2+b, & x<-1\end{cases}$ then

(a) a = 2, b = –3 (b) a = 3, b = 2

(c) a = –2, b = –3 (d) a = –3, b = –2

23. The value of $f(0)$ so that $\dfrac{-e^x+2^x}{x}$ may be continuous at $x=0$ is

(a) log (1/2) (b) 0

(c) 4 (d) –1 + log2

24. $f(x)=\dfrac{1}{1-x}$ then the discontinuity of the function $f3n(x)$ wave $f^9 = f0f-$ of n times is are

(a) x = 2

(b) x = {0, 1}

(c) x = –1

(d) continuous everywhere

25. $f(x)=\left|x^{2n+1}\right|, n\in N$ then

(a) $f(x)$ is continuous but not differentiable at $x=0$

(b) $f(x)$ is differentiable at $x=0$

(c) $f(x)$ is discontinuous at $x=0$

(d) None of the above

26. $f(x)=\begin{cases}x, & x \text{ is rational}\\ 1-x, & x \text{ is irrational}\end{cases}$ then at $x=\dfrac{1}{2}$ $f(x)$ is

(a) continuous but not differentiable

(b) discontinuous

(c) differentiable

(d) none of the above

27. $f(x)=\begin{cases}\dfrac{x}{2x^2+|x|}, & x\ne 0\\ 1, & x=0\end{cases}$ then $f(x)$ is

(a) continuous but non-differentiable at x = 0

(b) differentiable at x = 0

(c) discontinuous at x = 0

(d) None of the above

28. $f(x)=\begin{cases}ax^2+1, & x\le 1\\ x^2+ax+b, & x>1\end{cases}$

is differentiable at $x=1$ then

(a) a = 1, b = 1 (b) a = 1, b = 0

(c) a = 2, b = 0 (d) a = 2, b = 1

29. $f(x)=[\sin x]+[\cos x],\ x\in[0, 2\pi]$ then total no of parts where $f(x)$ is not differentiable is

(a) 2 (b) 3

(c) 5 (d) 4

30. Let f and g are differentiable functions satisfying $g'(a)=2$, $g(a)=b$ and $fog = I$ then $f'(b)$ equals to ____

(a) 2 (b) 2/3

(c) 1/2 (d) None of these

30. $f(x)=\begin{cases}\dfrac{1}{e^{1/x}+1}, & x\neq 0\\ 0, & x=0\end{cases}$ then $f(x)$ is

(a) continuous as well as differentiable at $x=0$

(b) not continuous and not differentiable.

(c) differentiable but not continuous

(d) None of the above

31. $y=\sec^{-1}\dfrac{\sqrt{x}-1}{x+\sqrt{x}}+\sin^{-1}\dfrac{x+\sqrt{x}}{\sqrt{x}-1}$ then $\dfrac{dy}{dx}=$ ____

(a) x (b) 1

(c) 0 (d) None of these

32. $y=\tan^{-1}\left(\dfrac{\sqrt{1+x^2}-1}{x}\right)$ then $\dfrac{dy}{dx}=$ ___

(a) $\dfrac{1}{1+x^2}$ (b) $\dfrac{2}{1+x^2}$

(c) $\dfrac{1}{2(1+x^2)}$ (d) None of these

33. $y=\tan^{-1}\left[\dfrac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}\right]$ then $\dfrac{dy}{dx}=$

(a) $\dfrac{1}{\sqrt{1-x^2}}$ (b) $\dfrac{1}{2\sqrt{1-x^2}}$

(c) $\dfrac{2}{\sqrt{1-x^2}}$ (d) None of these

34. $y=\cot^{-1}\left(\sqrt{\dfrac{\sqrt{1+x^2}+1}{2\sqrt{1+x^2}}}\right)$ then $\dfrac{dy}{dx}=$

(a) $\dfrac{1}{1+x^2}$ (b) $\dfrac{1}{1-x^2}$

(c) $\dfrac{1}{2(1-x^2)}$ (d) None of these

35. If $\sin y = x\sin(a+y)$ then $\dfrac{dy}{dx}=$ ___

(a) $\dfrac{\sin a}{\sin a\sin^2(a+y)}$

(b) $\dfrac{\sin^2(a+y)}{\sin a}$

(c) $\sin a\sin^2(a+y)$

(d) $\dfrac{\sin^2(a-b)}{\sin a}$

36. $y=f(x^3), z=g(x^5), f'(x)=\tan x$, $g'(x)=\sec x$ then $\dfrac{dy}{dz}=$ ____

(a) $\dfrac{3}{5x^2}\dfrac{\tan x^3}{\sec x^3}$ (b) $\dfrac{5x^2}{3}\dfrac{\sec x^5}{\tan x^3}$

(c) $\dfrac{3x^2}{5}\dfrac{\tan x^3}{\sec x^5}$ (d) None of these

37. If $e^{x+y}=e^x+e^y$ then $dy/dx=$ ___ at (1,1)

(a) 0 (b) –1

(c) 1 (d) None of these

38. If $y=\sqrt{\sin x + y}$ then $\dfrac{dy}{dx}=$ ____

(a) $\dfrac{\cos x}{2y-1}$ (b) $\dfrac{\cos x}{1-2y}$

(c) $\dfrac{\sin x}{1-2y}$ (d) $\dfrac{\sin x}{2y-1}$

39. If $x^y=e^{x-y}$ then $\dfrac{dy}{dx}=$ ____

(a) $\dfrac{\log x}{(1+\log x)^2}$ (b) $\dfrac{x-y}{1+\log x}$

(c) $\dfrac{x-y}{(1+\log x)^2}$ (d) $\dfrac{1}{1+\log x}$

40. $y=\left[x+\sqrt{1+x^2}\right]^n$ then $(1+x^2)\frac{d^2y}{dx^2}+x\frac{dy}{dx}=$

(a) x^2y (b) $-x^2y$

(c) –y (d) $2x^2y$

41. Derivative of $\tan^{-1}\frac{2x\sqrt{1-x^2}}{1-2x^2}$ w.r.t. $\sec^{-1}\frac{1}{2x^2-1}$ at $x=\frac{1}{2}$ = 1

(a) 1/2 (b) –1/2

(c) –1 (d) None of these

42. If $y=\tan\left[\frac{1}{2}\cos^{-1}\frac{1-y^2}{1+x^2}+\frac{1}{2}\sin^{-1}\frac{dy}{1+y^2}\right]$ and $x=\frac{2y}{1-y^2}$ then $\frac{dy}{dx}=$

(a) –1 (b) 0

(c) 1 (d) None of these

43. Derivative of $f(\tan x)$ w.r.t. $g(\sec x)$ at $x=\frac{\pi}{4}$ where $f'(1)=2$ and $g'(\sqrt{2})=4$ is ____

(a) $\frac{1}{\sqrt{2}}$ (b) $\sqrt{2}$

(c) 1 (d) None of these

44. $y=e^{\tan x}$ then $\cos^2 x\frac{d^2y}{dx^2}=$

(a) $(1-\sin 2x)\frac{dy}{dx}$

(b) $-(1+\sin 2x)\frac{dy}{dx}$

(c) $(1+\sin 2x)\frac{dy}{dx}$

(d) None of these

45. Let f is a differentiable function $\forall x\in R$ and $f(x^3)=x^5x\neq 0$ $\forall x\in R$ then $f(2>)$ is ____

(a) 15 (b) 45

(c) 0 (d) None of these

46. Derivative of an odd function is always

(a) an eleven function

(b) an odd number

(c) does not exist

(d) None of these

47. $y=\cos^{-1}(\cos x)$ then $y'(x)$ equals to

(a) 1, $\forall x$

(b) –1, $\forall x$

(c) 1 in zero 3rd quadrants

(d) –1 in 3rd & 4th quadrant

48. $\frac{d}{dx}\left[\tan^{-1}\sqrt{\frac{1+\cos x/2}{1-\cos x/2}}\right]=$ ____

(a) –1/4 (b) 1/4

(c) –1/2 (d) 1/2

49. Let $g(x)$ be the inverse of $f(x)$ and $f'(x)=\frac{1}{1+x^3}$ then $g'(x)=$

(a) $\frac{1}{1+\{g(x)\}^3}$ (b) $\frac{1}{1+\{f(x)\}^3}$

(c) $1+\{g(x)\}^3$ (d) $1+\{f(x)\}^3$

50. If $f(x)=1+nx+\frac{n(n-1)}{2!}x^2+\frac{n(n-1)(n-2)}{3!}x^3+...+x^n$ then $f''(1)=$ ____

(a) $n(n-1)2^{n-1}$ (b) $(n-1)2^{n-1}$

(c) $n(n-1)2^{n-2}$ (d) $n(n-1)2^n$

51. If $\sec^{-1}\left(\frac{1+x}{1-y}\right)=a$ then $\frac{dy}{dx}=$ ____

(a) $\frac{y-1}{x+1}$ (b) $\frac{y+1}{x-1}$

(c) $\frac{x-1}{y-1}$ (d) $\frac{x-1}{y+1}$

52. $f(x)=\cot^{-1}\left(\frac{x^x-x^{-y}}{2}\right)$ then $f'(1)=$

(a) –1 (b) 1

(c) log 2 (d) –log2

53. $-y=\sin\left[\cos^{-1}\left\{\sin\left(\cos^{-1}x\right)\right\}\right]$ then $\frac{dy}{dx}$ at $x=\frac{1}{2}=$ ____

(a) 0 (b) –1

(c) $\frac{2}{\sqrt{3}}$ (d) 1

ANSWER KEYS

1.	(c)	15.	(c)	29.	(c)	42.	(c)
2.	(c)	16.	(a)	30.	(c)	43.	(a)
3.	(d)	17.	(c)	30.	(b)	44.	(c)
4.	(a)	18.	(c)	31.	(c)	45.	(a)
5.	(c)	19.	(c)	32.	(c)	46.	(a)
6.	(b)	20.	(a)	33.	(b)	47.	(d)
7.	(a)	21.	(c)	34.	(c)	48.	(a)
8.	(b)	22.	(c)	35.	(b)	49.	(a)
9.	(c)	23.	(d)	36.	(a)	50.	(c)
10.	(a)	24.	(b)	37.	(b)	51.	(a)
11.	(d)	25.	(b)	38.	(a)	52.	(a)
12.	(b)	26.	(a)	39.	(a)	53.	(d)
13.	(b)	27.	(c)	40.	(a)		
14.	(d)	28.	(c)	41.	(c)		

CHAPTER - 2

APPLICATIONS OF DERIVATIVE

A. Multiple Choice Questions (MCQ)

1. The slope of tangent to the curve $x = 2(\theta - 2\sin^2\theta)$ and $y = (1 - \cos\theta)$ at $\theta = \pi/4$ is ____

(a) $\sqrt{2} - 1$ (b) $1/\sqrt{2}$

(c) $\sqrt{2} + 1$ (d) $-\sqrt{2} - 1$

2. What is the point on the curve $x = a(\theta - \sin\theta)$, $y = a(1 - \cos\theta)$ at which the tangent is parallel to x-axis.

(a) $(a\pi, 2a)$ (b) (π, a)

(c) $(a\pi, a)$ (d) None of these

3. Find the open interval in which $f(x) = x^{1/x}, x > 0$ is decreasing.

(a) $(-\infty, e)$ (b) (e, ∞)

(c) $(-e, e)$ (d) None of these

4. Find the interval in which the function $y = \frac{\ln x}{x}, x > 0$ is increasing.

(a) $(-\infty, e)$ (b) $(0, \infty)$

(c) $(0, e)$ (d) None of these

5. Write the set of values of x for which the function $f(x) = \sin x - x$ is increasing.

(a) $x > \frac{\pi}{2}$ (b) $x < \frac{\pi}{2}$

(c) ϕ (d) None of these

6. For which values of $x, f(x) = 5 - 6x$ is increasing.

(a) $\forall x \in R$

(b) never increasing

(c) $x > 6$

(d) None of these

7. What is the value of a for which the function $f(x) = a\sin x + \frac{1}{3}\sin 3x$ has an extremum at $x = \frac{\pi}{3}$?

(a) 2 (b) 3

(c) 1 (d) None of these

8. What is the rate of change of the area of a circle with respect to its radius?

(a) 2π (b) $2r$

(c) $2\pi r$ (d) πr^2

9. A ballon which always remains spherical has a variable diameter $\frac{3}{2}(2x + 3)$. What is the rate of change of volume w.r.t. x?

(a) $\frac{\pi}{8}(2x + 3)^2$

(b) $2 > \pi(2x + 3)^2$

(c) $\frac{27\pi}{8}(2x + 3)^2$

(d) None of these

10. What is the equation of the normal to the curve $y=\sqrt{x}$ at $(1/4, 1/2)$?

(a) $4x+y-3=0$

(b) $4x+4y-3=0$

(c) $x+4y-3=0$

(d) None of these

11. Write the equation of the tangent to the curve $y=|x|$ at the point (–2, 2).

(a) $x-y=0$ (b) $x+y=0$

(c) $2x+y=0$ (d) $x+2y=0$

12. If the tangent to the curve $x=at^2, y=2at$ is perpendicular to x-axis then what is the point of contact ?

(a) (1, 1) (b) (2, 2)

(c) (0, 0) (d) None of these

13. Find equation of normal to the curve $y=\sin x$ at (0, 0).

(a) $x+y=0$ (b) $x-y=0$

(c) $y-x=0$ (d) None of these

14. For which value of x the function $f(x)=3x^2-x+3$ is minimum ?

(a) 1/4 (b) 1/5

(c) 1/6 (d) None of these

15. For which value of x the function $f(x)=4-x-x^2$ is maximum ?

(a) 1/2 (b) –1/2

(c) 1/3 (d) None of these

16. Mention the values of x for which $f(x)=x^3-12x$ is increasing

(a) $(-\infty, -2)$

(b) $(-2, \infty)$

(c) $(-\infty, -2)\cup(2, \infty)$

(d) None of these

17. Write the maximum value of $y=x^5$ in the interval [1, 5].

(a) 3224 (b) 3225

(c) 3235 (d) None of these

18. What is the slope of normal to the curve $2y=3-x^2$ at (1, 1)

(a) 1 (b) –1

(c) 2 (d) None of these

19. Find approximately the difference between the volumes of two cubes of sides 4cm and 4.03cm.

(a) 1.42 cubic cm

(b) 1.43 cubic cm

(c) 1.44 cubic cm

(d) None of these

20. Write the set of parts where the function $f(x)=x^3$ has relative extrema.

(a) at x = 6

(b) at x = 5

(c) no relative maxima of the function

(d) None of these

21. The slope of tangent to the curve $y=\sqrt{3}\sin x+\cos x$ at $(\pi/3, 2)$

(a) 1 (b) 0

(c) 2 (d) None of these

22. For what value of x the function $f(x)=3-2x^2$ maximum ?

(a) 1 (b) 2

(c) 0 (d) None of these

23. What is the radius of the sphere if the rate of increasing of its volume is twice that of the surface area ?

(a) 3 (b) 4

(c) 5 (d) None of these

24. If the rate of increase of the perimeter of a square is 3 then what is the rate of increase of its side ?

(a) 4/3 (b) 3/4

(c) –4/3 (d) None of these

25. If y = k is a tangent to the circle $x^2 + y^2 = 1$ then k = ___

(a) ±1 (b) ±2

(c) ±3 (d) ±4

26. Derivative of $f(x)$ is $x(x-1)$ then it increases for ___

(a) $0 \le x < 1$

(b) $0 < x \le 1$

(c) $0 < x < 1$

(d) $x > 1$ and $x < 0$

27. The function $f(x) = \cos x$ is decreasing on ___

(a) $(\pi, 3\pi/2)$ (b) $(0, \pi/2)$

(c) $(3\pi/2, 2\pi)$ (d) $(\pi/2, 3\pi/2)$

28. For the curve $y = xe^x$ the point of minimum is ___

(a) x = 0 (b) x = –1

(c) x = 1 (d) x = e

29. If $y = x^2, x = 10, dx = 0.1$ then $dy =$

(a) 1 (b) 2

(c) 3 (d) 4

30. If $4x = 2y - 3$, $dy = 0.5$ then $dx =$ ___

(a) 1/2 (b) 1/4

(c) 1/6 (d) 1/8

31. The function $f(x) = \sin^4 x + \cos^4 x$ increases in the internal ___

(a) $(0, \pi/8)$

(b) $(\pi/4, \pi/2)$

(c) $(3\pi/8, 5\pi/8)$

(d) $(5\pi/8, 3\pi/4)$

32. The greatest value of $f(x) = xe^{-x}$ in $(0, \infty)$ is ____

(a) 0 (b) 1/e

(c) –e (d) None of these

33. $f(x) = a - (x-3)^{89}$ then greatest value of $f(x)$ is at x = ___

(a) 3

(b) a

(c) no maximum value

(d) none of these

34. $f(x) = \log x - \tan^{-1} x$ increases in the interval ____

(a) $(-\infty, 0)$ (b) $(0, \infty)$

(c) $(-\infty, \infty)$ (d) None of these

ANSWER KEYS

1.	(b)	8.	(c)	15.	(b)	22.	(c)	29.	(b)
2.	(a)	9.	(c)	16.	(c)	23.	(b)	30.	(b)
3.	(b)	10.	(b)	17.	(b)	24.	(b)	31.	(b)
4.	(c)	11.	(b)	18.	(a)	25.	(a)	32.	(b)
5.	(c)	12.	(c)	19.	(c)	26.	(d)	33.	(a)
6.	(c)	13.	(a)	20.	(c)	27.	(b)	34.	(b)
7.	(a)	14.	(c)	21.	(b)	28.	(b)		

B. Long Answer Type Questions

1. Show that $2\sin x + \tan x \geq 3x$, for all $x \in (0, \pi/2)$.

2. Find the approximate value of $\sqrt[6]{63}$.

3. Find the local maximum and local minimum of the function $f(x) = \sin x - \cos x,\ 0 < x < 2\pi$.

4. Find the equation of tangent to the parabola $y^2 = 4ax$ at the point $(at^2, 2at)$.

5. Show that the two curves $x^3 - 3xy^2 = a$ and $3x^2y - y^3 = b$ where a and b are constants intersect orthogonally.

6. Find the maximum and minimum value of $x + \dfrac{1}{x}$.

7. Determine the points of extreme values on the following curve

   $y^3 = (x-1)^2(x+2)$

8. The whole surface of a cone is given. Prove that its volume is maximum when semi vertical angle is $\sin^{-1}\dfrac{1}{3}$

9. Show that the minimum distance of a point on the curve $\dfrac{a^2}{x^2} + \dfrac{b^2}{y^2} = 1$ from the origin is $a+b$.

10. Prove that the sum of the cubes of the intercepts on the coordinate axes of any tangent to the curve $x^{3/4} + y^{3/4} = a^{3/4}$ is a constant.

11. Show that the sum of the x-intercept and y-intercept of any tangent to the curve $\sqrt{x} + \sqrt{y} = \sqrt{a}$ is a constant.

12. Show that the length of the portion of the tangent to the curve $x^{2/3} + y^{2/3} = 4$ intercepted between the axes is a constant.

13. Find the equation of the normal to the curve given by $x = \cos^3\theta, y = \sin^3\theta$ at $\theta = \dfrac{\pi}{4}$.

14. Find the point on the curve $y^2 - x^2 + 2x - 1 = 0$ where the tangent is parallel to x-axis.

15. Show that the tangent to the curve $y = x^2 + 3x - 2$ at (1, 2) is parallel to the tangent at (–1, 1) to the curve $y = x^3 + 2x$.

16. Show that the curves $y = 2^x$ and $y = 5^x$ intersect at an angle $\tan^{-1}\left[\dfrac{\ln 5/2}{1 + \ln 2 \ln 5}\right]$

17. Find the altitude of the right circular cylinder of maximum volume that can be inscribed within a sphere of radius R.

18. Show that the semi vertical angle of a cone of given slant height $\tan^{-1}\sqrt{2}$ when its volume is maximum.

19. Prove that for all real $x, e \leq \dfrac{ex^2}{x^2}$.

20. Find two numbers x and y whose sum is 15 such that xy^2 is maximum.

21. Find the interval where $y = \sin x - \cos x$, $x \in [0, 2\pi]$ is increasing.

22. Find the maximum value of $y=(1+\cos x)\sin x, x\in[0,3\pi/4]$

23. Find the maximum value of $f(x)=(1/x)^x$.

24. If a is +ve then find the maximum value of $\frac{a+x}{\sqrt{ax}}$.

25. Find the maximum value of $f(x)=x^{1/x}, x>0$ and show that $e^{\pi}>\pi^{e}$.

26. If $y=a\ln x+6x^2+x$ has extremum at $x=-1$ and $x=2$ then find the values of a and b.

27. Find the approximate value of $(26.9)^{1/3}$.

ANSWER HINTS

1. $f(x)=2\sin x+\tan x-3x \Rightarrow f'(x)=2\cos x+\sec^2 x-3$

$f''(x)=-2\sin x+2\sec^2 x\tan x=-2\sin x\left(-1+\frac{1}{\cos^3 x}\right)=-2\sin x\left(\frac{1-\cos^3 x}{\cos^3 x}\right)$

In $(0,\pi/2), 0<\cos^3 x<1 \Rightarrow f''(x)>0$

$\Rightarrow f'(x)$ is an increasing function $\Rightarrow f'(x)>f(0)$

$\Rightarrow 2\cos x+\sec^2 x-3>0$

$\Rightarrow f'(x)>0 \Rightarrow f(x)$ is an increasing function. $\Rightarrow f(x)\geq f(0)$

$\Rightarrow 2\sin x+\tan x-3x\geq 0$
$\Rightarrow 2\sin x+\tan x\geq 3x$

2. Let $y=\sqrt[6]{63}\Rightarrow y+dy=\sqrt[6]{x+dx}$

$\Rightarrow \sqrt[6]{x}+\frac{1}{6}x^{-5/6}dx=\sqrt[6]{x+dx}$

when $x=64,\ dx=-1$ we have

$\sqrt[6]{64}+\frac{1}{6}(64)^{-5/6}(-1)=\sqrt[6]{64-1}$

$\Rightarrow 2-\frac{1}{6}(2^6)^{-5/6}=\sqrt[6]{63}$

$\Rightarrow 2-\frac{1}{6}.2^{-5}=\sqrt[6]{63}$

$\Rightarrow 2-\frac{1}{192}=\sqrt[6]{63}\Rightarrow 2-0.0005=\sqrt[6]{63}$

$\Rightarrow 1.995=\sqrt[6]{63}$

3. Let $f(x)=\sin x-\cos x\Rightarrow f'(x)=\cos x+\sin x$

For maximum

$f'(x)=0\Rightarrow \cos x+\sin x=0$

$\Rightarrow \sin x=-\cos x\Rightarrow \tan x=-1$

$x=\frac{3\pi}{4},\frac{7\pi}{4}$

$f''(x)-\sin x+\cos x$

At $x=\frac{3\pi}{4}$ $f''(x)=\sqrt{2}$

At $x=\frac{7\pi}{4}$ $f''(x)=\sqrt{2}$

so $f(x)$ has local maximum at $x=\frac{3\pi}{4}$

and local minimum at $\frac{7\pi}{4}$

4. The given curve is $y^2 = 4ax$ (1)

On differentiating $\frac{dy}{dx} = \frac{2a}{y}$

$\left.\frac{dy}{dx}\right]_{(at^2,2at)} = \frac{1}{t}$

Equation of tangent at $(at^2, 2at)$ is

$y - 2at = \frac{1}{t}(x - at^2)$

5. The two curves are $x^3 - 3xy^2 = 0$ (1)

$3x^2y - y^3 = b$ (2)

Differentiating eqn. (1)

$3x^2 - 3y^2 - 6xy\frac{dy}{dx} = 0$

$\Rightarrow \frac{dy}{dx} = \frac{x^2 - y^2}{2xy} \Rightarrow \left.\frac{dy}{dx}\right]_{(x_1,y_1)}$

$= \frac{x_1^2 - y_1^2}{2x_1y_1} = m_1$ (3)

Differentiating eqn. (2)

$6xy + 3x^2\frac{dy}{dx} - 3y^2\frac{dy}{dx} = 0$

$\Rightarrow \frac{dy}{dx} = \frac{-2xy}{x^2 - y^2}$

$\left.\frac{dy}{dx}\right]_{(x_1,y_1)} = \frac{-2x_1y_1}{x_1^2 - y_1^2} = m_2$ (4)

Now $m_1.m_2 = \frac{x_1^2 - y_1^2}{2x_1y_1} \times \frac{-2x_1y_1}{x_1^2 - y_1^2} = -1$

$\Rightarrow$ curves cut orthogonally.

6. The function is $y = x + \frac{1}{x} \Rightarrow \frac{dy}{dx}$

$= 1 - \frac{1}{x^2} = \frac{x^2 - 1}{x^2}$

$\frac{dy}{dx} = 0 \Rightarrow \frac{x^2 - 1}{x^2} = 0 \Rightarrow x^2 = 1 \Rightarrow x = \pm 1$

when $x = 1 \Rightarrow y = 2$

when $x = -1 \Rightarrow y = -2$

So 2 extreme values are 2 and –2

7. The given curve is $y^3 = (x-1)^2(x+2)$

on differentiating, $3y^2\frac{dy}{dx} = 2(x-1)$

$(x+2) + (x-1)^2$

$= (x-1)[2x + 4 + x - 1]$

$= 3(x-1)(x+1)$

$\frac{dy}{dx} = \frac{(x-1)(x+1)}{y^2}$

For extremum $\frac{dy}{dx} = 0$

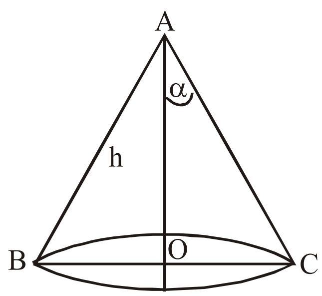
$\Rightarrow \frac{(x-1)(x+1)}{y^2} = 0 \Rightarrow x = -1, 1$

when $x = 1 \Rightarrow y^3 = 0 \Rightarrow y = 0$

when $x = -1$ $y^3 = 4 \Rightarrow y = \sqrt[3]{4} = 2^{2/3}$

The points of extremum are (1, 0) and $(-1, 2^{2/3})$

8. Let r be the radius of the base and h be the height, l is the slant height, s be the total surface area and v is the volume of the case.



$l^2 = r^2 + h^2 \Rightarrow h^2 = l^2 - r^2$

$h = \sqrt{l^2 - r^2}. v = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi r^2\sqrt{l^2 - r^2}$

$S = \pi rl + \pi r^2$

Here $v=\frac{1}{3}\pi r^2 h=\frac{1}{3}\pi r^2\sqrt{l^2-r^2}$

$$=\frac{1}{3}\pi r^2\sqrt{\frac{\left(s-\pi r^2\right)^2}{\pi^2 r^2}-r^2}$$

$$\Rightarrow v^2=\frac{1}{9}\pi^2 r^2\left[\frac{\left(s-\pi r^2\right)^2-\pi^2 r^4}{\pi^2 r^2}\right]$$

$$=\frac{S}{9}\left(Sr^2-2\pi r^4\right)$$

For maximum or minimum value of v,

$$\frac{dv^2}{dr}=0$$

$$\frac{d}{dr}v^2=\frac{s}{9}\left(2sr-8\pi r^3\right)=0$$

$$\Rightarrow s-4\pi r^2=0\Rightarrow r=\sqrt{s/4\pi}$$

For $r=\sqrt{s/4\pi}\ \frac{d^2v^2}{dr}=-\frac{4}{9}s^2<0$

so v^2 or v cs maximum when

$$r=\sqrt{s/4\pi}$$

Here $s=\pi rl+\pi r^2\Rightarrow 4\pi r^2=\pi rl+\pi r^2$

$$\Rightarrow \pi rl=3\pi r^2\Rightarrow l=3r\Rightarrow r/l=1/3$$

$$\sin\alpha=\frac{1}{3}\Rightarrow \alpha=\sin^{-1}\frac{1}{3}$$

9. Eqn of the curve is $\frac{a^2}{x^2}+\frac{b^2}{y^2}=1$ (1)

Let $P\left(a\sec\phi, b\cos ec\phi\right)$ be any point on the curve.

Let S = Distance of P from origin

$$=\sqrt{a^2\sec^2\phi+b^2\cos ec^2\phi}$$

$$\frac{ds}{d\phi}=\frac{a^2\sec^2\phi\tan\phi-b^2\cos ec^2\phi+\cot\phi}{\sqrt{a^2\sec^2\phi+b^2\cos ec^2\phi}}$$

Taking $\frac{ds}{d\phi}=0\Rightarrow a^2\sec^2\phi\tan\phi-b^2$

$\cos ec^2\phi\cot\phi=0$

On simplifying $\tan\phi=\frac{\sqrt{b}}{\sqrt{a}}\Rightarrow\frac{\sin\phi}{\cos\phi}=\frac{\sqrt{b}}{\sqrt{a}}$

$$\Rightarrow\frac{\sin\phi}{\sqrt{b}}=\frac{\cos\phi}{\sqrt{a}}=\frac{\sqrt{\sin^2\phi+\cos^2\phi}}{\sqrt{a+b}}=\frac{1}{\sqrt{a+b}}$$

$$\sin\phi=\frac{\sqrt{b}}{\sqrt{a+b}},\cos p=\frac{\sqrt{a}}{\sqrt{a+b}}$$

$$\Rightarrow\cos ec\phi=\frac{\sqrt{a+b}}{\sqrt{b}},\sec\phi=\frac{\sqrt{a+b}}{\sqrt{a}}$$

Then put the values of $\sec\phi$ and $\cos ec\phi$ in

$$s=\sqrt{a^2\sec^2\phi+b^2\cos ec^2\phi}$$

12. The given curve is $x^{2/3}y^{2/3}=4$ (1)

Differentiating (1) & (2)

$$\frac{2}{3}x^{-1/3}+\frac{2}{3}y^{-1/3}\frac{dy}{dx}=0$$

$$\Rightarrow\frac{dy}{dx}=-\frac{y^{1/3}}{x^{1/3}}$$

Now $\left.\frac{dy}{dx}\right]_{(x_1,y_1)}=-\frac{y_1^{1/3}}{x_1^{1/3}}$

Equation of tangent at $\left(x_1,y_1\right)$ is

$$y-y_1=\frac{-y_1^{1/3}}{x_1^{1/3}}\left(x-x_1\right)$$

Let P intersects x-axis at A and y-axis at B.

$$A\left(x_1^{1/3}a^{2/3},0\right)B\left(0,y_1^{1/3},a^{2/3}\right)$$

$A=2a^{2/3}$ which is a constant.

13. $x=\cos^3\theta \Rightarrow \dfrac{dx}{d\theta}=-3\cos^2\theta\sin\theta$

$y=\sin^3\theta \Rightarrow \dfrac{dy}{d\theta}=3\sin^2\theta\cos\theta$

$$\frac{dy}{dx}=\frac{3\sin^2\theta\cos\theta}{-3\cos^2\theta\sin\theta}\Rightarrow \left.\frac{dy}{dx}\right]_{\theta=\frac{\pi}{4}}=-1$$

when $\theta=\dfrac{\pi}{4}\Rightarrow x=\dfrac{1}{2\sqrt{2}},\ y=\dfrac{1}{2\sqrt{2}}$

Slope of normal = 1

Eqn. of normal at $\left(\dfrac{1}{2\sqrt{2}},\dfrac{1}{2\sqrt{2}}\right)$ with slope 1 is

$$y-\frac{1}{2\sqrt{2}}=1\left(x-\frac{1}{2\sqrt{2}}\right)\Rightarrow x-y=0$$

14. The eqn. of the curve is

$y^2-x^2+2x-1=0$

on differentiating $2y\dfrac{dy}{dx}-2x+2=0$

$\Rightarrow \dfrac{dy}{dx}=\dfrac{x-1}{y}$

Since tangent is $||^d$ to x-axis)

$\Rightarrow \dfrac{dy}{dx}=0\Rightarrow \dfrac{x-1}{y}=0$

$\Rightarrow x=1$ when $x=1$ then $y=0$

15. $y=2^x$ (1) $y=5^x$ (2)

Point of intersection (1) and (2) is (0,1)

Slope of tangent of curve (1) is

$2^x\ln 2=\dfrac{dy}{dx}$

$$\left.\frac{dy}{dx}\right]_{(0,1)}=5^0\ln 5=\ln 5=m_2$$

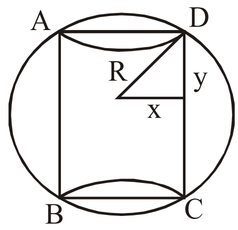
Let θ is the angle between the curves

$$\Rightarrow \tan\theta=\frac{\ln 5-\ln 2}{|+\ln 5\ln 2}=\frac{\ln 5/2}{|+\ln 5\ln 2}$$

17. Let R is the radius of the sphere.

Let ABCD be the inscribed cylynder.

Let 2x be the radius of the base and 2y be the height of the cylinder.



Here $R^2=x^2+y^2\Rightarrow x^2=R^2-y^2$

Let V be the volume of the cylinder

$V=\pi x^2y=\pi\left(R^2-y^2\right)y$

$=\pi\left(R^2y-y^3\right)$

$$\frac{dv}{dy}=\pi\frac{d}{dy}\left[R^2y-y^3\right]=\pi\left[R^2-3y^2\right]$$

$$\frac{d^2v}{dy^2}=\pi(-6y)=-6\pi y$$

For maximum

$$\frac{dv}{dy}=0\Rightarrow \pi\left[R^2-3y^2\right]=0$$

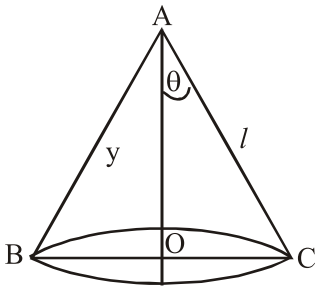
$\Rightarrow R^2-3y^2=0\Rightarrow 3y^2=R^2$

$\Rightarrow y^2=\dfrac{R^2}{3}\Rightarrow y=\dfrac{R}{\sqrt{3}}$

$$\left.\frac{d^2v}{dy^2}\right]_{y=R/\sqrt{2}}=-6.\pi\frac{R}{\sqrt{3}}<0$$

For maximum volume altitude of the cylinder $=2y=\dfrac{2R}{\sqrt{3}}$

18. Let ABC be a cone whose radius of the base is x and height is y. Let l is the slant height which is constant.



$y^2 + x^2 = l^2$

$\Rightarrow x^2 = l^2 - y^2$

Volume of the cone

$= \frac{1}{3}\pi x^2 y = \frac{1}{3}\pi\left(l^2 - y^2\right)y$

$= \frac{1}{3}\pi\left(l^2 y - y^3\right)$

$\frac{dv}{dy} = \frac{1}{3}\pi\left(l^2 - 3y^2\right) \Rightarrow \frac{d^2v}{dy^2} = \frac{1}{3}\pi(-6y) = -2\pi y$

For maximum or minimum $\frac{dv}{dy} = 0$

$\Rightarrow \frac{1}{3}\pi\left(l^2 - 3y^2\right) = 0 \Rightarrow l^2 - 3y^2 = 0$

$\Rightarrow y^2 = \frac{l^2}{3} \Rightarrow y = \frac{l}{\sqrt{3}}$

when $y = \frac{l}{\sqrt{3}}$ then $\frac{d^2v}{dy^2} = -2\pi\frac{l}{\sqrt{3}} < 0$

$\Rightarrow V$ is maximum when $y = \frac{l}{\sqrt{3}}$

$x^2 = l^2 - \frac{l^2}{3} \Rightarrow x = \frac{\sqrt{2}l}{3}$

Let θ is the semivertical angle of the cone.

$\tan\theta = \frac{x}{y} = \sqrt{2}\,\frac{l}{\sqrt{3}} / \frac{l}{\sqrt{3}} = \sqrt{2} \Rightarrow \theta = \tan^{-1}\sqrt{2}$

19. $f(x) = \frac{e^{x^2}}{x^2} \Rightarrow f'(x) = \frac{x^2 e^{x^2} 2x - e^{x^2}.2x}{x^4}$

$\Rightarrow f'(x) = \frac{2x.e^{x^2}\left(x^2 - 1\right)}{x^4} = \frac{2e^{x^2}\left(x^2 - 1\right)}{x^3}$

For maximum or minimum $f'(x) = 0$

$\Rightarrow \frac{2e^{x^2}\left(x^2 - 1\right)}{x^3} = 0$

$\Rightarrow x = \pm 1$

$f''(x) = \frac{2\left[2x^4 - 3\left(x^2 - 1\right)\right]}{x^4} e^{x^2}$

For $x = \pm 1$, $f''(x) > 0$ so the function has minimum at $x = \pm 1$

minimum value of $f(x) = e$

$\Rightarrow e \le \frac{e^{x^2}}{x^2}$

21. Let $f(x) = \sin x - \cos x \Rightarrow f'(x) = \cos x + \sin x$

For increasing

$f'(x) > 0 \Rightarrow \cos x + \sin x > 0 \Rightarrow \sin x > \cos x$

$\Rightarrow \tan x > -1$

$\Rightarrow x \in \left(0, \frac{\pi}{2}\right) \cup \left(\frac{\pi}{2}, \frac{3\pi}{4}\right) \cup \left(\frac{7\pi}{4} - 2a\right)$

23. $f(x) = \left(\frac{1}{x}\right)^x = y$

$\Rightarrow \ln y = \ln\left(\frac{1}{x}\right)^x = x\ln(1/x)$

$= x(\ln 1 - \ln x) = -x\ln x$

$(\because \ln 1 = 0)$

$$\frac{d}{dx}\ln y = \frac{d}{dx}(-x\ln x)$$

$$\Rightarrow \frac{1}{y}\frac{dy}{dx} = -\left[-x.\frac{1}{x} + \ln x\right] = -t + \ln x$$

$$\Rightarrow \frac{dy}{dx} = -y[\ln x - 1]$$

$$\frac{d^2y}{dx^2} = \frac{dy}{dx}[\ln x - 1] + \frac{y}{x}$$

For maximum or minimum take

$$\frac{dy}{dx} = 0$$

$$\Rightarrow \ln x = -1 \Rightarrow x = e^{-1} = \frac{1}{e}$$

when $x = \frac{1}{e}$ then $\frac{d^2y}{dx^2}$ is –ve

So y is maximum at $x = \frac{1}{e}$

maximum value of $f(x) = (e)^{1/e}$

27. Let $y = x^{1/3} \Rightarrow y + dy = (x + dx)^{1/3}$

$$\Rightarrow x^{1/3} + \frac{1}{3}x^{-2/3}dx = y + dy$$

Let $x = 27,\ dx = -0.1$

Now $(26.9)^{1/3} = (27 - 0.1)^{1/3} = 27^{1/3} + \frac{1}{3(27)^{2/3}}x - 0.1$

$$\Rightarrow (26.9)^{1/3} = 3 - \frac{0.1}{27} = 3 - 0.003 = 2.997$$

ADDENDUM

1. If the tangent at each point of the curve $y = \frac{2}{3}x^3 - 2ax^2 + 2x + 5$ makes an acute angle with the +ve direction of x-axis then
   (a) $a \geq 1$ (b) $-1 \leq a \leq 1$
   (c) $a \leq -1$ (d) None of these

2. Tangent are drawn from the origin to the curve $y = \sin x$ their part of contact lie on the curve
   (a) $x^2y^2 = x^2 + y^2$ (b) $x^2y^2 = x^2 - y^2$
   (c) $x^2y^2 = y^2 - x^2$ (d) None of these

3. Number of possible tangents to the curve $y = \cos(x + y)$ $-3\pi \leq x \leq 3\pi$ that are parallel to the line $x + 2y = 0$ is ____
   (a) 1 (b) 2
   (c) 3 (d) 4

4. The equation of tangent to the curve $y = \begin{cases} x^2 \sin\frac{1}{x}, & x \neq 0 \\ 0 & , x = 0 \end{cases}$ at origin is ____
   (a) $x = 0$ (b) $x = y$
   (c) $y = 0$ (d) None of these

5. The angle between the tangents at those points on the curve $y = (x+1)(x-3)$ where it meets x-axis is ____
   (a) $\pm\tan^{-1}\left(\frac{15}{8}\right)$ (b) $\pm\tan^{-1}\left(\frac{8}{15}\right)$
   (c) $\pm\frac{\pi}{4}$ (d) None of these

6. The two tangents to the curve $ax^2 + 2hxy + by^2 = 1$, $a > 0$ at the points where it crosses x-axis are
   (a) parallel (b) $\perp^r$
   (c) $\frac{\pi}{4}$ (d) None of these

7. The function $f(x) = \cot^{-1} x + x$ increases in the interval
   (a) $(1, \infty)$ (b) $(-1, \infty)$
   (c) $(-\infty, \infty)$ (d) $(0, \infty)$

8. The function $\frac{\sin x}{x} = f(x)$ is decreasing in the interval
   (a) $\left(-\frac{\pi}{2}, 0\right)$ (b) $\left(0, \frac{\pi}{2}\right)$
   (c) $\left(-\frac{\pi}{4}, 0\right)$ (d) None of these

9. The function $f(x) = \frac{\ln(\pi + x)}{\ln(e + x)}$ is ____
   (a) increasing on $(0, \infty)$
   (b) decreasing on $(0, \infty)$
   (c) increasing on $(0, \pi/e)$, decreasing on $(\pi/e, \infty)$
   (d) decreasing $(0, \pi/e)$, increasing on $(\pi/e, \infty)$

10. $f(x) = \cot^{-1}[g(x)]$ where $g(x)$ is an increasing function for $0 < x < \pi$. Then $f(x)$ is
    (a) increasing in $(0, \pi)$
    (b) decreasing in $(0, \pi)$
    (c) increasing in $(0, \pi/2)$ and decreasing in $(\pi/2, \pi)$
    (d) None of these

11. The tangent to the curve $3xy^2 - 2x^2y = 1$ at (1, 1) meets the curve again at ____
    (a) (–16/5, –1/2) (b) (16/5, –1/20)
    (c) (–16/5, 1/20) (d) None of these

12. If the tangent to a parabola $y^2 = 8x$ makes an angle $\frac{\pi}{4}$ with the st. line $y = 3x + 5$ then the part of contact is

(a) $\left(\frac{1}{2}, 2\right)$ (b) $\left(-\frac{1}{2}, 2\right)$

(c) $\left(\frac{1}{2}, -2\right)$ (d) None of these

13. Any tangent to the curve $y = 3x^7 + 5x + 3$

(a) is it to x-axis

(b) is it to y axis

(c) makes an acute angle with x-axis

(d) makes an obtuse angle with x-axis

14. If the normal to the curve $y = f(x)$ at the point (3, 4) makes an angle $\frac{3\pi}{4}$ with +ve x-axis equals to ____

(a) –1 (b) 3/4

(c) 4/3 (d) 1

15. If the line $ax + by + c = 0$ is a normal to the curve $xy = 1$ then

(a) $a > 0, b > 0$ (b) $a > 0, b < 0$

(c) $a < 0, b < 0$ (d) None of these

16. The point on the curve $9y^2 = x^3$, where the normal to the curve makes equal intercepts with the axes is ____

(a) $\left(4, \frac{8}{3}\right)$ (b) $\left(4, \frac{3}{8}\right)$

(c) $\left(-4, \frac{-8}{3}\right)$ (d) None of these

17. The line $\frac{x}{a} + \frac{y}{b} = 1$ touches the curve $y = be^{-x/a}$ at the point ____

(a) (–a, ba) (b) (a, a/b)

(c) (a, b/a) (d) None of these

18. If $f(x) = \begin{cases} x\sin\frac{\pi}{x}, & x > 0 \\ 0, & x = 0 \end{cases}$ then in the interval (0, 1) $f'(x)$ vanishes at

(a) exactly one point

(b) exactly 2 points

(c) at no part

(d) infinite number of parts

19. The tangent of the portion of the tangent at any point on the curve $x = a\cos^3\theta$, $y = a\sin^3\theta$ intercepted between the coordiante axis is ____

(a) a (b) 2a

(c) 3a (d) None of these

20. The length of the $\perp r$ from the origin to the tangent to the curve $y = e^{4x+2}$ drawn at the point $x = 0$ is

(a) $\frac{4}{\sqrt{7}}$ (b) $\frac{3}{\sqrt{17}}$

(c) $\frac{2}{\sqrt{17}}$ (d) None of these

21. $f(x) = 1 + [\cos x]x,\ 0 < x \le \frac{\pi}{2}$

(a) has a minimum 0

(b) has a maximum 2

(c) is continuous in [0, 2]

(d) is not differentiable at $x = \frac{\pi}{2}$

22. $f(x) = |x| + |x-1| + |x-2|$ then

(a) $f(x)$ has minima at $x = 1$

(b) $f(x)$ has maxima at $x = 1$

(c) $f(x)$ has neither maxima nor minima at $x = 1$

(d) None of these

23. $f(x)=\frac{x}{1+x\tan x}, x\in\left(0,\frac{\pi}{2}\right)$ then

(a) $f(x)$ has exactly one part of minima

(b) $f(x)$ has exactly one part of maxima

(c) $f(x)$ is increaing in $\left(0,\frac{\pi}{2}\right)$

(d) $f(x)$ is decreasing in $\left(0,\frac{\pi}{2}\right)$

24. $f(x)=a\sec x-b\tan x, a>b>0$ minimum value of $f(x)$ is

(a) a^2+b^2 (b) a^2-b^2

(c) $\sqrt{a^2+b^2}$ (d) $\sqrt{a^2-b^2}$

25. If $f(x)=\cos x+a^2x+b$ is an increasing function $\forall$ values of x then

(a) $a\in[-1,1]$

(b) $a\in(-\infty,-1)\cup[1,\infty]$

(c) $a\in[-1,\infty]$

(d) $a\in[-\infty,1]$

26. The function $f(x)=\log x-\frac{2x}{2+x}$ is increasing in the interval

(a) $(-\infty, 0)$ (b) $(0,\infty)$

(c) $(-\infty, 1)$ (d) None of these

27. $\log x-\tan^{-1}x$ increases in the interval ___

(a) $(-\infty, 0)$ (b) $(0,\infty)$

(c) $(-\infty,\infty)$ (d) None of these

28. $y=x^3(x-2)^2$ then the values of x for which y increases are ____

(a) $x<\frac{6}{5}$ or $x>3$ (b) $\frac{6}{5}<x<2$

(c) $x<\frac{6}{5}$ *or* $x>2$ (d) None of these

29. The function $f(x)=x^{1/x}$ is increasing in the interval

(a) (e,∞) (b) (0, e)

(c) (–e, e) (d) None of these

30. The function $f(x)=x-\log(1+x), x>-1$ is increasing in the internal ____

(a) $(0,\infty)$ (b) (–1, 0)

(c) $(-\infty, 0)$ (d) None of these

31. The range of values of x for which the function $f(x)=\frac{x}{\log x}$ $x>0, x\neq1$ may be decreasing is ____

(a) (0, e) (b) (e,∞)

(c) $(0,e)-\{1\}$ (d) None of these

32. $g(x)=f(x)+f(1-x)$ and $f''(x)<0$ for $0\le x\le1$ then

(a) $g(x)$ increases in $(-\infty,1/2)$

(b) $g(x)$ decreases in $(1/2,\infty)$

(c) $g(x)$ increases in (0, 1/2)

(d) None of these

33. $f(x)=\sin x+\cos x$ defined in $[0,2\pi]$ then $f(x)$.

(a) increases in $(\pi/4,\pi/2)$

(b) decreases in $(\pi/4, 5\pi/4)$

(c) increase in $(0,\pi/4)\cup(5\pi/4,2\pi)$

(d) decrease in $[\pi/6, 5\pi/6]$

34. $f(x)=a-(x-3)^{89}$ then greatest value of $f(x)$ is at $x=$ ____

(a) 3

(b) a

(c) no maximum value

(d) None of these

35. If $y = a\log x + bx^2 + x$ has its extreme values at $x = 1, x = 2$ then the values of a and b are ____

(a) $a = \frac{-1}{6}, b = \frac{4}{3}$

(b) $a = \frac{-9}{3}, b = \frac{-1}{6}$

(c) $a = \frac{4}{3}, b = \frac{-1}{6}$

(d) None of these

36. For the function $f(x) = x + 1/x$

(a) $x = 1$ is a part of maximum

(b) $x = -1$ is a part of minimum

(c) neither maximum nor minimum

(d) maximum value < minimum value

37. If $xz = 1$ where $x > 0$ then the least value of $x + z$ is ____

(a) 1 (b) 3

(c) 2 (d) None of these

38. When $\frac{dy}{dx} = 0, \frac{d^2y}{dx^2} = 0$ at $x = a$ then we have to obtain $\frac{d^3y}{dx^3}, \frac{dxy}{dx^4}$ and so on to ascertain the existence of parts of extremum.

For a function $y = f(x)$ if $\frac{dy}{dx} = 0, \frac{dy}{dx^2} = 0$ at a part $x = a$ then

(a) y must be maximum at $x = a$

(b) minimum at $x = a$

(c) y may not have a maximum or minimum at $x = a$

(d) It is a constant function

39. The greatest value of the function $f(x) = xe^{-x}$ in $[0, \infty)$ is ____

(a) 0

(b) 1/e

(c) –e

(d) None of these

40. The greatest value of $y = x(x-1)^2$ is ____

(a) 0

(b) 4/27

(c) –4

(d) None of these

41. The shortest distance of the part (0,0) from the curve $y = \frac{1}{2}(e^x + e^{-x})$ is ____

(a) 2

(b) 1

(c) 3

(d) None of these

42. On the interval [0, 1] the function $x^{25}(1-x)^{75}$ takes its maximum value at the part ____

(a) 0 (b) 1/4

(c) 1/2 (d) 1/3

43. The function $f(x) = \sin^4 x + \cos^4 x$ increases in the interval

(a) $(0, \pi/8)$

(b) $(\pi/4, \pi/2)$

(c) $(3\pi/8, 5\pi/8)$

(d) $(5\pi/8, 3\pi/4)$

Answer Hints

1.	(b)	12.	(c)	23.	(b)	34.	(a)
2.	(b)	13.	(c)	24.	(b)	35.	(b)
3.	(c)	14.	(d)	25.	(a)	36.	(d)
4.	(c)	15.	(b)	26.	(d)	37.	(c)
5.	(b)	16.	(a)	27.	(a)	38.	(c)
6.	(a)	17.	(d)	28.	(a)	39.	(b)
7.	(c)	18.	(d)	29.	(d)	40.	(b)
8.	(b)	19.	(a)	30.	(b)	41.	(b)
9.	(b)	20.	(b)	31.	(c)	42.	(b)
10.	(b)	21.	(a)	32.	(d)	43.	(b)
11.	(a)	22.	(a)	33.	(c)		

CHAPTER - 3

INTEGRATION

A. Multiple Choice Questions (MCQ)

1. Value of $\int_{-\pi/2}^{\pi/2} \log\left(\frac{2-\sin\theta}{2+\sin\theta}\right) d\theta =$ ___

(a) 1 (b) 0

(c) 2 (d) None of these

2. Value of $\int_{0}^{1000} e^{x-[x]} dx =$ ____

(a) 1000 (b) 1000e

(c) 1000 (e–1) (d) None of these

3. If $f(x) =$

$$\begin{vmatrix} \sin x + \sin 2x + \sin 3x & \sin 2x & \sin 3x \\ 3 + 4\sin x & 3 & 4\sin x \\ 1 + \sin x & \sin x & 1 \end{vmatrix}$$

then value of $\int_{0}^{\pi/2} f(x) dx =$ ____

(a) 2/3 (b) 1/3

(c) 4/3 (d) 5/3

4. $\int \frac{dx}{x\sqrt{1-x^3}} =$ ____

(a) $\frac{-1}{3} \ln\left(\frac{1-\sqrt{1-x^3}}{1+\sqrt{1-x^3}}\right) + C$

(b) $\frac{1}{3} \ln\left(\frac{1-\sqrt{1-x^3}}{1+\sqrt{1-x^3}}\right) + C$

(c) $-\frac{1}{3} \ln\left(\frac{1+\sqrt{1-x^3}}{1-\sqrt{1-x^3}}\right) + C$

(d) None of these

5. $\int \frac{dx}{x\left\{(\log x)^2 + 25\right\}} =$ ____

(a) $\frac{1}{5} \tan^{-1}\left(\frac{1}{5}\log x\right) + C$

(b) $\tan^{-1}\left(\frac{1}{5}\log x\right) + C$

(c) $\frac{1}{5} \tan^{-1}(\log x + C)$

(d) None of these

6. $\int \frac{3 + \cos x + \tan^2 x}{2x + \sin x + \tan x} dx =$ ____

(a) $\log(\sin x + \tan x) + c$

(b) $\log(2x + \tan x) + c$

(c) $\log(2x + \sin x + \tan x) + c$

(d) None of these

7. $\int \frac{\sin x - \cos x}{\sqrt{1-\sin 2x}} e^{\sin x} \cos x \, dx =$ ____

(a) $e^{\sin x} + c$ (b) $e^{\cos x} + c$

(c) $e^{\sin x + \cos x} + c$ (d) None of these

8. $\int 5^{5^{5^{5^x}}} . 5^{5^x} . 5^x \, dx =$ ____

(a) $\frac{5^{5^{5^x}}}{(\ln 5)^2} + c$ (b) $\frac{5^{5^{5^{5^x}}}}{(\ln 5)^3} + c$

(c) $\frac{5^{5^{5^x}}}{(\ln 5)^2} + c$ (d) None of these

9. Antiderivative of $2^{2^x} + x$ is ____

(a) $2^{2^x} + c$ (b) $\frac{2^{2^x}}{\ln 2} + c$

(c) $\frac{2^{2^x}}{(\ln 2)2} + c$ (d) None of these

10. $\int \frac{\cot x}{\ln \sin x} dx =$ _____

(a) $\ln \sin x + c$

(b) $\ln(\ln \sin x) + c$

(c) $\ln \cot x + c$

(d) None of these

11. $\int \frac{dx}{\cos^2 x \sin^2 x} =$ ____

(a) $\tan x - \cot x + c$

(b) $\cot x - \tan x + c$

(c) $\cot x + \tan x + c$

(d) None of these

12. $\int_{-\pi/4}^{\pi/4} \cos^4 x \sin^{99} x \, dx =$ ____

(a) 1 (b) 2

(c) 0 (d) None of these

13. What is the value of

$\int \frac{d}{dx}[f(x)]dx - \frac{d}{dx}\left[\int f(x)dx\right]$

(a) $f(x) + c$ (b) c

(c) $f'(x) + c$ (d) None of these

14. Value of $\int_{-1}^{1} \frac{dx}{1+x^2} =$ ____

(a) π (b) 2π

(c) $\pi/2$ (d) $\pi/4$

15. If f is an even function and $\int_{-2}^{0} f(t)dt = \frac{3}{2}$ then find $\int_{-2}^{2} f(x)dx$

(a) 2 (b) 3

(c) –3 (d) –2

16. $\int \frac{\cot^2 x - \cos ec^2 x}{x^2} dx =$ ____

(a) $\frac{1}{x} + c$ (b) $-\frac{1}{x} + c$

(c) $x + c$ (d) None of these

17. $\int \left(\sqrt{a^2 - x} + \frac{x^2}{\sqrt{a^2 - x^2}} \right) dx =$ ____

(a) $a^2 \sin^{-1} \frac{x}{a} + c$ (b) $a \sin^{-1} \frac{x}{a} + c$

(c) $\sin^{-1} \frac{x}{a} + c$ (d) None of these

18. $\int \frac{1 + 1/x^2}{x - 1/x + 4} dx =$ ___

(a) $\ln(n - 1/x + 4) + c$

(b) $\ln(x - 1/x) + c$

(c) $\ln(x + 1/x + c) + c$

(d) None of these

19. Value of $\int e^x \cos x dx + \int e^x \sin x dx =$ ___

(a) $e^x \sin x + c$ (b) $e^x \cos x + c$

(c) $e^x \sin^2 x + c$ (d) None of these

20. $\int e^{\ln(\cos ec^2 x - \cot^2 x)} dx =$ ____

(a) $-x + c$ (b) $x + c$

(c) $x\frac{2}{2} + c$ (d) None of these

21. $\int 2^x . 4^{-x/2} dx =$ ____

(a) $-x + c$ (b) $x + c$

(c) $x\frac{2}{2} + c$ (d) None of these

22. $\int e^{x^2}.2x\ dx =$ ____

(a) $e^{x^2}+c$ (b) $e^{x}+c$

(c) $e^{2x}+c$ (d) None of these

23. $\int_{-\pi/2}^{\pi/2} \sin^5 x \cos x\ dx =$ ____

(a) 0 (b) 1

(c) 2 (d) 3

24. $\int_{-\pi/2}^{\pi/2} (x^4 \sin x^3 + x\cos x^2)dx =$ ____

(a) 1 (b) 2

(c) 0 (d) 4

25. $\int_{-\pi/2}^{\pi/2} \sin^5 x \cos\frac{nx}{2} dx =$ ____

(a) 1 (b) –1

(c) 0 (d) 2

26. $\int_{0}^{1/2} 2y dy + \int_{1/2}^{1} 2y dy =$ ____

(a) 2 (b) 1

(c) –1 (d) 0

27. If $\int_0^1 f(t)dt = 2, \int_2^1 +(4)du = -1$ then

$\int_0^2 f(x)dx =$ ____

(a) 2 (b) –2

(c) 3 (d) –3

28. Value of $\frac{d}{dx}\int_{200}^{300}\left(x^4+5x^3\right)^2 dx =$ ____

(a) 2 (b) 3

(c) 0 (d) None of these

29. $\int_{-1}^{1}|x|dx =$ ____

(a) 1 (b) 2

(c) –1 (d) None of these

30. Value of $\int_1^3 \tan^{-1} xdx + \int_1^3 \cot^{-1} xdx =$ ____

(a) $\frac{\pi}{2}$ (b) $\frac{1}{\pi}$

(c) 2π (d) $\frac{3\pi}{2}$

31. $\int \log e^x dx =$ ____

(a) $\frac{x^2}{2}+c$ (b) $x+c$

(c) $\frac{-x^2}{2}+c$ (d) None of these

32. $\int_0^2 |x-2|dx =$

(a) –2 (b) 2

(c) 3 (d) None of these

33. $\int_0^1 \sin^2 xdx + \int_0^1 \cos^2 tdt - \int_0^1 dr =$ ____

(a) 1 (b) 0

(c) –1 (d) None of these

34. If $\int_0^1 f(1-x)dx = 2$ then value of

$\int_0^{1/2} f(2f)dt =$ ____

(a) –1 (b) 1

(c) 2 (d) –2

35. $\int \frac{x^5}{\left(x^3+1\right)^4}dx =$ ____

(a) $\frac{-1}{6\left(x^3+1\right)^2}$

(b) $\frac{1}{9\left(x^3+1\right)^3}$

(c) $\frac{-1}{6\left(x^3+1\right)^2}+\frac{1}{9\left(x^2+1\right)^3}+c$

(d) None of these

36. $\int \sec^4 x \operatorname{cos}ec^2 x\, dx$

(a) $\tan^2 x + 2\tan x - \cot x + c$

(b) $\dfrac{\tan^3 x}{3} + 2\tan x - \cot x + c$

(c) $\dfrac{\tan^2 x}{2} + \tan x - \cot x + c$

(d) $\dfrac{\tan^2 x}{2} + 2\tan x - \cot x + c$

37. If $f(x)$ is an even function then $\int_0^x f(t)\,dt$ is ____

(a) odd function

(b) even function

(c) neither even nor odd

(d) None of these

38. $\int_{\pi/2}^{3\pi/2} [\sin x]\,dx =$ ____

(a) $\dfrac{\pi}{2}$ (b) $-\dfrac{\pi}{2}$

(c) π (d) None of these

39. $\int_0^{\sqrt{n}} [t^2]\,dt =$ ____

(a) $n\sqrt{n} + (1 + \sqrt{2} + \sqrt{3} + ... + \sqrt{n})$

(b) $n\sqrt{n} - (1 + \sqrt{2} + \sqrt{3} + ... + \sqrt{n})$

(c) $-n\sqrt{n} + (1 + \sqrt{2} + \sqrt{3} + ... + \sqrt{n})$

(d) None of these

40. $\int_0^3 [\sqrt{x}]\,dx =$ ____

(a) 2 (b) 1

(c) –2 (d) –1

41. $\int_0^2 [x^2]\,dx =$ ____

(a) $2 - \sqrt{2}$ (b) $2 + \sqrt{2}$

(c) $\sqrt{2} - 1$ (d) $-\sqrt{2} - \sqrt{3} + 5$

42. $\int_{-1}^1 [x]\,dx =$ ____

(a) 0 (b) 1

(c) –1 (d) None of these

ANSWER KEYS

1.	(b)	12.	(c)	23.	(a)	34.	(b)
2.	(c)	13.	(b)	24.	(c)	35.	(c)
3.	(b)	14.	(c)	25.	(c)	36.	(b)
4.	(a)	15.	(b)	26.	(b)	37.	(a)
5.	(a)	16.	(a)	27.	(c)	38.	(b)
6.	(b)	17.	(a)	28.	(c)	39.	(b)
7.	(a)	18.	(c)	29.	(a)	40.	(a)
8.	(b)	19.	(a)	30.	(b)	41.	(d)
9.	(c)	20.	(b)	31.	(a)	42.	(c)
10.	(b)	21.	(a)	32.	(b)		
11.	(a)	22.	(a)	33.	(b)		

B. Long Answer Type Questions

1. $\int_{0}^{4}[8-3x]dx$

2. $\int \frac{\sin^8 x-\cos^8 x}{1-2\sin^2 x\cos^2 x}dx$

3. Evaluate $\int_{-\pi}^{\pi}\frac{2x(1+\sin x)}{1+\cos^2 x}dx$

4. $\int \frac{1}{\sin x+\sin 2x}dx$

5. $\int \frac{x^2-3x+1}{\sqrt{1-x^2}}dx$

6. $\int_{\pi/6}^{\pi/3}\frac{1}{1+\sqrt[4]{\cot x}}dx$

7. $\int_{0}^{\pi}\frac{x\sin x}{1+\cos^2 x}dx$

8. $\int_{0}^{1}\frac{\log(1+x)}{1+x^2}dx$

9. $\int x^2(\sin^4 x+\cos^4 x)$

10. $\int \frac{2\sin x+3\cos x}{3\sin x+4\cos x}dx$

11. $\int \frac{dx}{(x-2)\sqrt{3x^2-16x+24}}$

12. $\int \frac{\cos x}{\sin 2x+\sin x}dx$

13. $\int x^3\cos^2 x\ dx$

14. $\int \frac{1}{2\sin x+\cos x+3}\ dx$

15. $\int x^2(\sin^4 x+\cos^4 x)\ dx$

16. $\int \frac{x^2}{(x-1)^2(x-2)}\ dx$

17. $\int \frac{4x^2-x+3}{(x-1)(x^2+1)}\ dx$

18. $\int \sqrt{3+2x-x^2}\ dx$

19. $\int \frac{x^3dx}{x^4-x^2-2}$

20. $\int \frac{dx}{2-\sin x}$

21. $\int (x^2-2x+7).\sqrt{x+1}\ dx$

22. $\int e^x\cos x\ dx$

23. $\int \sec x\ \tan x\ \sqrt{\tan^2-3}\ dx$

24. $\int \frac{dx}{\sqrt{7+4x+x^2}}$

25. $\int \frac{2x+5}{(x+2)^{7/2}}dx$

26. $\int \frac{dx}{3e^x-1}$

27. $\int x^9\cos x^5dx$

28. $\int \frac{x+1}{(x+2)^2}e^xdx$

29. $\int \frac{e^x+e^{-x}+1}{e^x-e^{-x}+x}dx$

30. $\int \frac{\sin x}{\sin(\alpha+\beta)}dx$

31. $\int\left\{\frac{1}{\log x}-\frac{1}{(\log x)^2}\right\}dx$

32. $\int\frac{dx}{(1+x)\sqrt{1-x^2}}$

33. $\int_0^1\frac{\log(1+x)}{1+x^2}dx$

34. $\int_0^{\pi/2}\frac{\cos x\ dx}{(2+\sin x)(3+\sin x)}$

35. $\int_2^7\frac{dx}{\sqrt{x+2}+\sqrt{x-2}}$

36. $\int_0^{\pi/2}\frac{\sqrt{\tan x}}{\sqrt{\tan x}+\sqrt{\cot x}}dx$

37. $\int_0^{\pi/2}\frac{dx}{a^2\cos^2 x+b^2\sin^2 x}dx$

38. $\int_0^{\pi/4}\log(1+\tan x)dx$

39. $\int_0^{\pi/4}\frac{x\sin x\cos x}{\sin^4 x+\cos^4 x}dx$

40. $\int_0^{\pi/4}\frac{\cos x\ dx}{\sqrt{1-2\sin^2 x}}$

ANSWER HINTS

1. $|8-3x|=\begin{cases}8-3x & \text{when } x\le 8/3\\ 3x-8 & \text{when } x>8/3\end{cases}$

$$\int_0^4|8-3x|dx=\int_0^{8/3}|8-3x|dx+\int_{8/3}^4|8-3x|dx$$

$$=\int_0^{8/3}(8-3x)dx+\int_{8/3}^4(3x-8)dx$$

$$=\left(8x-\frac{3x^2}{2}\right)_0^{8/3}+\left(\frac{3x^2}{2}-8x\right)_{8/3}^4$$

Then simplify

2. $\int\frac{\sin^8 x-\cos^8 x}{1-2\sin^2 x\cos^2 x}dx$

$$\int\frac{(\sin^4 x+\cos^4 x)(\sin^4 x-\cos^4 x)}{1-2\sin^2 x\cos^2 x}dx$$

$$\int\frac{(\sin^4 x+\cos^4 x)(\sin^2 x-\cos^2 x)(\sin^2 x+\cos^2 x)}{1-2\sin^2 x\cos^2 x}dx$$

$$\int\frac{(\sin^4 x+\cos^4 x)(-\cos 2x)}{1-2\sin^2 x\cos^2 x}dx$$

$$=\int\frac{\left[(\sin^2 x+\cos^2 x)^2-2\sin^2 x\cos^2 x\right](-\cos 2x)}{1-2\sin^2 x\cos^2 x}dx$$

$$=\int\frac{(1-2\sin^2 x\cos^2 x)(-\cos 2x)}{(1-2\sin^2 x\cos^2 x)}dx$$

$$=\int-\cos 2x dx=-\frac{\sin 2x}{2}+c$$

3. $\int_{-\pi}^{\pi}\frac{2x(1+\sin x)}{1+\cos^2 x}dx$

$$=\int_{-\pi}^{\pi}\frac{2x}{1+\cos^2 x}dx+\int_{-\pi}^{\pi}\frac{2x\sin x}{1+\cos^2 x}dx$$

$$=0+2\int_0^{\pi}\frac{2x\ \sin x}{1+\cos^2 x}dx$$

($\because$ $\frac{2x}{1+\cos^2 x}$ is an odd function and $\frac{2x\sin x}{1+\cos^2 x}$ is an even function

$$=4\int_0^{\pi}\frac{x\sin x\,dx}{1+\cos^2 x} \qquad (1)$$

$$I=\int_0^{\pi}\frac{x\sin x}{1+\cos^2 x}dx$$

$$\int_0^{\pi}\frac{(\pi-x)\sin(\pi-x)}{1+\cos^2 x(\pi-x)}dx=\int_0^{\pi}\frac{(\pi-x)\sin x}{1+\cos^2 x}dx$$

$$I=\int_0^{\pi}\frac{\pi\sin x}{1+\cos^2 x}dx-\int_0^{\pi}\frac{x\sin x}{1+\cos^2 x}$$

$$I=\int_0^{\pi}\frac{\pi\sin x.dx}{1+\cos^2 x}\theta-I$$

$$\Rightarrow 2I=\pi\int_0^{\pi}\frac{\sin x}{1+\cos^2 x}dx$$

Putting $\cos x=t\Rightarrow \sin xdx=-dt$

we can set I. Then put the value of I in equation (4)

4. Let $I=\int\frac{dx}{\sin x+\sin 2x}=\int\frac{dx}{\sin x+2\sin x\cos x}$

$$=\int\frac{dx}{\sin x(1+2\cos x)}=\int\frac{\sin x\ dx}{\sin^2 x(1+2\cos x)}$$

$$=\int\frac{\sin x\ dx}{(1-\cos^2 x)(1+2\cos x)}dx$$

Let $t=\cos x\Rightarrow -dt=\sin x\ dx$

$$I=\int\frac{-dt}{(1+t)(1-t)(1+2t)}$$

Then apply method of partial fraction

5. $\int\frac{x^2-3x+1}{\sqrt{1-x^2}}dx=-\int\frac{-x^2+3x-1}{\sqrt{1-x^2}}dx$

$$=-\int\frac{-x^2+3x-2+1}{\sqrt{1-x^2}}dx$$

$$=-\int\frac{(1-x^2)+(3x-2)}{\sqrt{1-x^2}}dx$$

$$=-\int\sqrt{1-x^2}dx+\int\frac{2-3x}{\sqrt{1-x^2}}dx$$

$$=-\left[\frac{x}{2}\sqrt{1-x^2}+\frac{1}{2}\sin^{-1}x\right]$$

$$+\int\frac{2}{\sqrt{1-x^2}}dx-\int\frac{3x}{\sqrt{1-x^2}}dx$$

then integrate

6. $\int_{\pi/6}^{\pi/3}\frac{1}{1+\sqrt[4]{\cot x}}\ dx=\int_{\pi/6}^{\pi/3}\frac{\sqrt[4]{\sin x}}{\sqrt[4]{\sin x}+\sqrt[4]{\cos x}}dx \quad (1)$

$$I=\int_{\pi/6}^{\pi/3}\frac{\sqrt[4]{\sin\left(\frac{\pi}{6}+\frac{\pi}{3}-x\right)}}{\sqrt[4]{\sin\left(\frac{\pi}{6}+\frac{\pi}{3}-x\right)}+\sqrt[4]{\cos\left(\frac{\pi}{6}+\frac{\pi}{3}-x\right)}}dx$$

$$=\int_{\pi/6}^{\pi/3}\frac{\sqrt[4]{\cos x}\,dx}{\sqrt[4]{\cos x}+\sqrt[4]{\sin x}} \qquad (2)$$

Adding (1) and (2)

$$2I=\int_{\pi/6}^{\pi/3}\frac{\sqrt[4]{\sin x}+\sqrt[4]{\cos x}}{\sqrt[4]{\cos x}+\sqrt[4]{\sin x}}dx=\int_{\pi/6}^{\pi/3}dx$$

Then simplify

7. $\int_0^1\frac{\log(1+x)}{1+x^2}dx$ when

let
$$x=\tan\theta\Rightarrow dx=\sec^2\theta\ d\theta$$
$$x=0\Rightarrow\tan\theta=0\Rightarrow\theta=0$$
$$x=0\Rightarrow\tan\theta=1\Rightarrow\theta=\pi/4$$

$$I=\int_0^{\pi/4}\frac{\log(1+\tan\theta)}{1+\tan^2\theta}\sec^2\theta d\theta$$

$$=\int_0^{\pi/4}\log(1+\tan\theta)d\theta$$

$$I=\int_0^{\pi/4}\log\left[1+\tan(\pi/4-\theta)\right]d\theta$$

$$= \int_0^{\pi/4} \log\left[1+\frac{1-\tan\theta}{1+\tan\theta}\right] d\theta$$

$$= \int_0^{\pi/4} \log\left[\frac{2}{1+\tan\theta}\right] = \int_0^{\pi/4} \log 2 d\theta - \int_0^{\pi/4} \log(1+\tan\theta) d\theta$$

$$I = \log 2 \int_0^{\pi/4} d\theta - I$$

$$2I = \log 2 (\theta)_0^{\pi/4} = \frac{\pi}{4}\log 2$$

$$I = \frac{\pi}{8}\log 2$$

9. $\int x^2 \left(\sin^4 x + \cos^4 x\right) dx$

$$= \int x^2 \left[\left(\sin^2 x + \cos^2 x\right)^2 - 2\sin^2 x \cos^2 x)\right] dx$$

$$= \int x^2 \left[1 - \frac{1}{2} 4 \sin^2 x \cos^2 x\right] dx$$

$$= \int x^2 \left[1 - \frac{1}{2}\sin^2 2x\right] dx$$

$$= \int x^2 \left[1 - \frac{1}{4} 2\sin^2 2x\right] dx$$

$$= \int x^2 \left[1 - \frac{1}{4}(1 - \cos 4x)\right] dx$$

$$= \int x^2 \left(\frac{3}{4} + \frac{\cos 4x}{4}\right) dx$$

$$= \frac{3}{4}\int x^2 dx + \frac{1}{4}\int x^2 \cos 4x \; dx$$

Then apply integration by parts for the 2nd integral.

10. $I = \dfrac{2\sin x + 3\cos x}{3\sin x + 4\cos x} dx$

Let $2\sin x + 3\cos x$

$$= l(3\sin x + 4\cos x)\tan(3\cos x - 4\sin x)$$

Equating coefficients of $\sin x$ and $\cos x$ we have

$3l - 4m = 2$ (1) $4l + 3m = 3$ (2)

on solving $l = \dfrac{18}{25}$, $n = -\dfrac{1}{25}$

$$I = \int \frac{l(3\sin x + 4\cos x) + m(3\cos x - 4\sin x)}{3\sin x + 4\cos x} dx$$

$$l = \int dx + m\int \frac{3\cos x - 4\sin x}{3\sin x + 4\cos x}$$

Take t = $3\sin x + 4\cos x$ and integrate.

12. $\int \dfrac{\cos x}{\sin 2x + \sin x} dx = \int \dfrac{\cos x dx}{\sin x(2\cos x + 1)}$

$$= \int \frac{\cos x . \sin x \, dx}{\sin^2 x(2\cos x + 1)} = \int \frac{\cos x . \sin x \, dx}{(1-\cos^2 x)(2\cos x + 1)}$$

Let $\cos x \Rightarrow -dt = \sin x dx$

Required integral $= \int \dfrac{-t \; dt}{(1-t^2)(2t+1)}$

Then apply partial fraction.

13. $\int x^3 \cos^2 x = \dfrac{1}{2}\int x^3 2\cos^2 x d \; x$

$$= \frac{1}{2}\int x^3 (1 + \cos 2x) dx$$

$$= \frac{1}{2}\int x^3 dx + \frac{1}{2}\int x^3 \cos 2x \, dx$$

Then apply integration by parts for 2nd integral.

14. $\int \dfrac{1}{2\sin x + \cos x + 3} dx$ put

$$\sin x = \frac{2\tan x/2}{1+\tan^2 x/2} \quad \cos x = \frac{1-\tan^2 x/2}{1+\tan^2 x/2}$$

Then take $t = \tan\dfrac{x}{2}$

16. $\int \dfrac{x^2}{(x-1)^2 (x-2)} dx$

$$\int \frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{C}{x-2} \; dx$$

i.e. $\dfrac{x^2}{(x-1)^2(x-2)}=\dfrac{A}{(x-1)}+\dfrac{B}{(x-1)^2}\dfrac{C}{(x-2)}$

Then apply method of partial fraction

17. $\displaystyle\int\frac{4x^2-x+3}{(x-1)(x^2+1)}dx$

Resolving into partial fraction

$$\frac{4x^2-x+3}{(x-1)(x^2+1)}=\frac{A}{x-1}+\frac{Bx+c}{x^2+1}$$

$$=\frac{A(x^2+1)+(Bx+c)(x-1)}{(x-1)(x^2+1)}$$

$$\Rightarrow 4x^2-x+3=A(x^2+1)+(Bx+c)(x-1)$$

Then putting $x=1$ and equating coefficients of like terms.

18. $\displaystyle\int\sqrt{3+2x-x^2}\,dx=\int\sqrt{4-1+2x-x^2}\,dx$

$$=\int\sqrt{4-(x^2-2x+1)}\,dx$$

$$=\int\sqrt{4-(x-1)^2}\,dx=\int\sqrt{2^2-t^2}\,dt$$

Putting $t=x-1$ dt = dt

Then integrate

19. $\displaystyle\int\frac{x^3}{x^4-x^2-2}dx=\int\frac{x^2.x}{(x^2)^2-x^2-2}dx$

$$t=x^2, dt=2xdx\ \ dt/2=xdx$$

$$=\int\frac{t\ dt/2}{t^2-t-2}=\frac{1}{4}\int\frac{2t\ dt}{t^2-t-2}$$

$$=\frac{1}{4}\int\frac{(2t-1)+1}{t^2-t-2}dt$$

$$=\frac{1}{4}\left[\int\frac{2t-1}{t^2-t-2}dt+\int\frac{dt}{t^2-t-2}\right]$$

$$=\frac{1}{4}\ln(t^2-t-2)+\frac{1}{4}\int\frac{dt}{t^2+2.t-\frac{1}{2}+\frac{1}{4}-2-\frac{1}{4}}$$

$$=\frac{1}{4}\ln(t^2-t-2)+\frac{1}{4}\int\frac{dt}{(t-1/2)^2-9/4}$$

Then apply the formula.

21. $\displaystyle\int(x^2-2x+7)\sqrt{x+1}\ dx$

Let $x+1=t^2, dx=2tdt\ \ x=t^2-1$

$$\int\left[(t^2-1)^2-2(t^2-1)+7\right].t.2t.dt$$

Then integrate

23. $\displaystyle\int\sec x\tan x\sqrt{\tan x-3}\,dx$

$$=\int\sec x\tan x\sqrt{\sec^2 x-1-3}\,dx$$

$$=\int\sec x\tan x\sqrt{\sec^2 x-4}\,dx$$

then put $t=\sec x\ \ dt=\sec x\tan xdx$

24. $\displaystyle\int\frac{dx}{\sqrt{7+4x+x^2}}=\int\frac{dx}{\sqrt{x^2+2x.2+4+3}}$

$$=\int\frac{dx}{\sqrt{(x+2)^2+3}}$$

Then proceed

25. $\displaystyle\int\frac{2x+5}{(x+2)7/2}dx$

Let $x+2=t^2,\ dx=2tdt,\ x=t^2-2$

$$\int\frac{2(t^2-2)+5}{(t^2)^{7/2}}2t\,dt=4$$

26. $\displaystyle\int\frac{dx}{(3e^x-1)}=\int\frac{e^x dx}{e^x(3e^x-1)}$

Let $3e^x=t, 3e^x dx=dt,\ e^x dx=dt/3$

$$=\int\frac{dt/3}{t/3(t-1)}=\int\frac{dt}{t(t-1)}$$

$$\frac{1}{t(t-1)}=\frac{A}{t}+\frac{B}{t-1}$$

then apply partial fraction.

27. $\int x^9.\cos x^5 dx = \int x^4.x^5 \cos x^5 dx$

$t = x^5, dt = 5x^4 dx, dt/5 = x^4 dx$

$= \int t \cos t \frac{dt}{5}$.

Then apply partial fraction

28. $\int \frac{x+1}{(x+2)^2} d^x dx = \int \frac{x+2-1}{(x+2)^2} e^x dx$

$= \int e^x \left[\frac{1}{x+2} - \frac{1}{(x+2)^2} \right] dx$

Then apply

$\int e^x \left(f(x) + f'(x) \right) dx = e^x f(x) + c$

29. $\int \frac{e^x + e^{-x} + 1}{e^x - e^{-x} + x} dx$

Put $t = e^x - e^{-x} + x$, $dt = \left(e^x + e^{-x} + 1 \right) dx$

Then integrate

30. $\int \frac{\sin x}{\sin(x+\beta)} dx$

Let $t = x + \beta \Rightarrow dt = dx,\ x = t - \beta$

$\int \frac{\sin(t-\beta)}{\sin t} dt = \int \frac{\sin t \cos\beta - \sin\beta \cos t}{\sin t} dt$

Then integrate

31. $\int \left[\frac{1}{\log x} - \frac{1}{(\log x)^2} \right] dx = \int \frac{1}{\log x} dx - \frac{1}{(\log x)^2} dx$

Integrate the 1st integral by parts and keep the 2nd integral constant.

32. $\int \frac{dx}{(1+x)\sqrt{1-x^2}}$

Let $x = \sin\theta \Rightarrow dx = \cos\theta d\theta$

$\int \frac{\cos\theta d\theta}{(1+\sin\theta)\sqrt{1-\sin^2\theta}} d\theta$

$= \int \frac{d\theta}{1+\sin\theta} = \int \frac{d\theta}{1 + \frac{2\tan\theta/2}{1+\tan^2\theta/2}}$

Then integrate

34. $\int_0^{\pi/2} \frac{\cos x dx}{(2+\sin x)(3+\sin x)}$

Let $t = \sin x \Rightarrow dt = \cos x dx$

$x = 0 \Rightarrow t = 0,\ x = \pi/2 \Rightarrow t = 1$

$\int_0^1 \frac{dt}{(2+t)(3+t)}$ Apply partial fraction.

35. $\int_2^7 \frac{dx}{\sqrt{x+2} + \sqrt{x-2}}$

$= \int_2^7 \frac{\sqrt{x+2} - \sqrt{x-2}}{\left[\sqrt{x+2} + \sqrt{x-2}\right]\left[\sqrt{x+2} - \sqrt{x-2}\right]} dx$

$= \int_2^7 \frac{\sqrt{x+2} - \sqrt{x-2}}{(x+2)-(x-2)} dx = \frac{1}{4} \int \sqrt{x+2} - \sqrt{x-2}) dx$

Then integrate

36. Let $I = \int_0^{\pi/2} \frac{\sqrt{\tan x}}{\sqrt{\tan x} + \sqrt{\cot x}} dx$

$= \int_0^{\pi/2} \frac{\sqrt{\tan(\pi/2 - x)}}{\sqrt{\tan(\pi/2 - x)} + \sqrt{\cot x(\pi/2 - x)}} dx$

$= \int_0^{\pi/2} \frac{\sqrt{\cot x}}{\sqrt{\cot x} + \sqrt{\tan x}} dx$

$2I = \int_0^{\pi/2} \frac{\sqrt{\tan x}}{\sqrt{\tan x} + \sqrt{\cot x}} dx + \int_0^{\pi/2} \frac{\sqrt{\cot x}\, dx}{\sqrt{\tan x} + \sqrt{\cot x}}$

$2I = \int_0^{\pi/2} dx \Rightarrow I = \frac{1}{2} \int_0^{\pi/2} dx$

Then simplify

37. $\int_0^{\pi/2} \frac{dx}{a^2\cos^2 x + b^2 \sin^2 x}$

Dividing numerator and dinominator by $\cos^2 x$

Then put $t = \tan x, dt = \sec^2 x dx$

$x = 0 \Rightarrow t = 0,\ x = \frac{\pi}{2} \Rightarrow t = \infty$

$= \int_0^{\pi/2} \frac{\sec^2 x dx}{a^2 + b^2 \tan^2 x}$

$= \int_0^{\pi/2} \frac{dt}{a^2 + b^2 + 2}$

Then integrate

38. Let $I = \int_0^{\pi/4} \log(1+\tan x)dx$

$= \int_0^{\pi/4} \log\left[1+\tan(\pi/4)x\right]dx$

$= \int_0^{\pi/4} \log\left[1+\frac{1-\tan x}{1+\tan x}\right]dx$ Then proceed

39. Let $I = \int_0^{\pi/2} \frac{\sin x \cos x}{\sin^4 x + \cos^4 x}dx$

$= \int_0^{\pi/2} \frac{(\pi/2-x)\sin(\pi/2-x)\cos(\pi/2-x)}{\left[\sin(\pi/2-x)\right]^4\left[\cos(\pi/2-x)\right]^4}$

$= \int_0^{\pi/2} \frac{(\pi/2-x)\cos x \sin x}{\cos^4 x + \sin^4 x}dx$

$= \frac{\pi}{2}\int_0^{\pi/2} \frac{\sin x \cos x dx}{\cos^4 x + \sin^4 x} - \int_0^{\pi/2} \frac{x \sin x \cos x}{\sin^4 x + \cos^4 x}dx$

$I = \frac{\pi}{2}\int_0^{\pi/2} \frac{\sin x \cos x dx}{\sin^4 x + \cos^4 x} - I$

$2I = \frac{\pi}{2}\int_0^{\pi/2} \frac{\sin x \cos x}{\left(\sin^2 x + \cos^2 x\right)^2 - 2\sin^2 x \cos^2 x}dx$

$I = \frac{\pi}{4}\int_0^{\pi/2} \frac{\sin x \cos x}{1 - 2\sin^2 x \cos^2 x}$

$= \frac{\pi}{8}\int_0^{\pi/2} \frac{\sin 2x\ dx}{1 - 1/2\sin^2 x\ 2x}$

$= \frac{\pi}{8}\int_0^{\pi/2} \frac{\sin 2x\ dx}{1 - 1/2(1-\cos^2 2x)}$

Put $t = \cos 2x, dt = -2\sin^2 x dx$

$-dt/2 = \sin 2x dx$

when $\begin{array}{l} x = 0 \Rightarrow t = 1 \\ x = \pi/2 \Rightarrow t = \cos\pi = -1 \end{array}$

$= \frac{\pi}{8}\int_1^{-1} \frac{-dt/2}{\frac{1}{2}\left(1+t^2\right)} = \frac{\pi}{8}\int_{-1}^{1} \frac{dt}{1+t^2}$

Then integrate

CHAPTER - 4

APPLICATIONS OF INTEGRALS

A. Multiple Choice Questions (MCQ)

1. The area bounded by $x = e^y, x = 0,$ $y = 0, y = 1$ is ____

   (a) e (b) 1

   (c) e – 1 (d) None of these

2. The area bounded by $y = x, x = 0$ and $x = 1$ is ____

   (a) 1 (b) 1/2

   (c) 2 (d) None of these

3. The area bounded by $y = -2x, y = 0$, $x = 1$ and $x = 3$ is ____

   (a) 8 (b) –8

   (c) 4 (d) 6

4. The area of trapezium bounded by the sides $y = x, x = 0, y = 3, y = 4$ is ____

   (a) 9/2 (b) 7/2

   (c) 15/2 (d) None of these

5. The area enclosed by the curve $y^2 = x$ and the straight lines $x = 0, y = 1$ is ___

   (a) 2/3 (b) 4/3

   (c) 1/3 (d) 5/3

6. The solution of a 2nd order differential equation contains ____ arbitrary constants.

   (a) no (b) 1

   (c) 2 (d) 3

7. If $\dfrac{d^2s}{dt^2} = 0$ then s is a ___ function of t

   (a) linear (b) quadratic

   (c) cubic (d) constant

8. ____ is not a solution of $\dfrac{d^2y}{dx^2} = 1$.

   (a) $\dfrac{dy}{dx} = x$

   (b) $2x = x^2$

   (c) $y = \dfrac{x^2}{2} + 1$

   (d) $3x^2 - 2y + 4 = 0$

9. $\left(\dfrac{d^2y}{dx^2}\right)^2 = \left[1 + \left(\dfrac{dy}{dx}\right)^3\right]^2$ is of ____

   (a) 1st order, 2nd degree

   (b) 1st order, 6th degree

   (c) 2nd order, 2nd degree

   (d) 2nd order, 6th degree

10. The differential equation whose solution is $y = 3x + k$ is ____

    (a) $\dfrac{dy}{dx} = 3$ (b) $\dfrac{dy}{dx} = k$

    (c) $\dfrac{dy}{dx} = 0$ (d) None of these

11. The order of the differential equation whose solution is $y = a\cos x + b\sin x + ce^{-x}$ is ____

    (a) 2 (b) 3

    (c) 4 (d) None of these

12. Degree of the differential equation $\left(\frac{dy}{dx}\right)^4 + 5^5 = \frac{d^3y}{dx^3}$

(a) 3 (b) 1

(c) 2 (d) None of these

13. How many arbitrary constants does the general solution of the differential equation $\frac{d^2y}{dx^2} = \sin x + \cos x$ contains ?

(a) 1 (b) 2

(c) 3 (d) None of these

14. Write the solution of $\frac{dy}{dx} = 8x$ given $y = 2$ when $x = 1$.

(a) $y = 4x^2 - 2$ (b) $y = 4x^2 + 2$

(c) $y = x^2 + 2$ (d) None of these

15. Order and degree of the differential equation $\left(\frac{dy}{dx}\right)^8 + \left(\frac{d^2y}{dx^2}\right) = 0$.

(a) 2, 8 (b) 2, 1

(c) 8, 1 (d) None of these

16. Write the solution of $\frac{d^2y}{dx^2} = 0$

(a) $y = cx$ (b) $y = d$

(c) $y = cx + d$ (d) None of these

17. The differential equation whose general solution is $y = 3x + k$.

(a) $\frac{dy}{dx} = 0$ (b) $\frac{dy}{dx} = 3$

(c) $\frac{dy}{dx} = 0$ (d) None of these

18. Write the differential equation of the parabola $y^2 4x + 12$.

(a) $ydx - 2dx = 0$ (b) $2y\frac{dy}{dx} = 0$

(c) $\frac{d^2y}{dx^2} = 0$ (d) None of these

19. Write the particular solution of $\frac{dy}{dx} = \frac{1}{1+x^2}$ given that when $x = 0, y = 1$?

(a) $y = \tan^{-1} x$

(b) $y = \tan^{-1} x + 1$

(c) $y = \tan^{-1} x + 2$

(d) None of these

20. Write the differential equation whose solution is $y = e^{x+a}$

(a) $\frac{dy}{dx} = e^a$ (b) $\frac{dy}{dx} = y$

(c) $\frac{dy}{dx} = 0$ (d) None of these

21. General solution of $\frac{dy}{dx} = x + xy$ is ___

(a) $y = ce^{x^2/2}$ (b) $1 + y = ce^{x^2/2}$

(c) $1 = ce^{x^2/2}$ (d) None of these

22. Form the differential equation representing the family of curves $y = A\cos(A + B)$.

(a) $\frac{d^2y}{dx^2} = 0$ (b) $\frac{d^2y}{dx^2} + y = 0$

(c) $\frac{dy}{dx} = y$ (d) None of these

23. Form the differential equation $y = \sec x$ by eliminating arbitrary constants.

(a) $\frac{dy}{dx} = y\sec x$ (b) $\frac{dy}{dx} = c\sec x$

(c) $\frac{dy}{dx} = y\tan x$ (d) None of these

24. The differential equation of the family of straight lines parallel to y-axis.

(a) $\frac{dy}{dx} = 0$ (b) $\frac{dx}{dy} = 0$

(c) $\frac{dy}{dx} = c$ (d) None of these

25. Write the solution of $xdx + ydy = 0$

(a) $x^2 + y^2 = c$ (b) $xy = c$

(c) $\frac{x}{y} = c$ (d) None of these

26. Write the particular solution of the equation $\frac{dy}{dx} = \sin x$ for which $y = 2$ when $x = \pi$.

(a) $y = -\cos x$

(b) $y = -\cos x + 1$

(c) $y = -\cos x + 2$

(d) None of these

ANSWER HINTS

1.	(c)	8.	(d)	15.	(b)	22.	(b)
2.	(b)	9.	(c)	16.	(c)	23.	(a)
3.	(a)	10.	(a)	17.	(b)	24.	(a)
4.	(b)	11.	(b)	18.	(a)	25.	(a)
5.	(c)	12.	(a)	19.	(b)	26.	(b)
6.	(c)	13.	(b)	20.	(b)		
7.	(a)	14.	(a)	21.	(b)		

B. Long Answer Type Questions

1. Find the area enclosed by the parabola $y^2 = 4x$ and the line $y = 2x$.

2. Determine the area of the region bounded by the curve $y^4 = x^3$ and the double ordinate through (2, 0).

3. Show that the area bounded by the parabolas $y^2 = 4x$ and $x^2 = 4y$ is equal to the area bounded by the curve $x^2 = 4y$ and the lines $y = 0, x = 4$.

4. Determine the area included between the parabola $y^2 = x$ and the circle $x^2 + y^2 = 2x$.

5. Find the area of the region between the curve $y = \cos x$ and $y = \sin x$, $x \in [0, \pi/4]$.

6. Find the area of the portion of the ellipse $\frac{x^2}{12} + \frac{y^2}{16} = 1$ bounded by the major axes and the double ordinate $x = 3$.

7. Find the area of the smaller portion of the circle $x^2 + y^2 = 4$ cut off by the line $x = 1$.

8. Find the area included between the line $y = x$ and the parabola $x^2 = 4y$.

9. Find the area enclosed by 2 curves given by $y^2 = x + 1$ and $y^2 = -x + 1$.

10. Find the area of the region bounded by the curves $y = x^2 + 2, y = x, x = 0$ and $x = 3$.

11. Find the area bounded by the curves $y^2 = 8x$ and $x^2 = 8y$.

12. Find the area enclosed between the parabola $y = x^2 - x + 2$ and the line $y = x + 2$.

13. Solve the differential equation

$$x \sin \frac{y}{x}\, dy = \left(y \sin \frac{y}{x} - x \right) dx.$$

14. Find the general solution of

$$\left[2\sqrt{xy} - x \right] dy + y dx = 0.$$

15. Solve the differential equation $\left(x + 2y^3 \right) \frac{dy}{dx} = y$.

16. Find the solution of

$$2x^2 y \frac{dy}{dx} = \tan\left(x^2 y^2 \right) \quad -2xy^2 y(1) = \sqrt{\frac{\pi}{2}}$$

17. Find the solution of $x^3 \frac{dy}{dx} + 4x^2 + axy$ $= e^x \sec y$ if $y(1) = 0$.

18. Find the general solution of $\frac{dy}{dx} =$ $y \tan x - y^2 \sec x$

19. Find the solution of $\frac{dy}{dx} = e^{x-y}\left(1 - e^y \right)$

20. Find the solution of

$$\left(\frac{x+y-1}{x+y-2} \right) \frac{dy}{dx} = \left(\frac{x+y+1}{x+y+2} \right)$$

21. Find the solution of

$$xdx + ydy + \frac{xdy - ydx}{x^2 + y^2} = 0$$

22. Find the general solution of

$$y\left(x^2 y + e^x \right) dx - e^x dy = 0$$

23. Solve $\frac{2+\sin x}{y+1}\left(\frac{dy}{dx}\right)=-\cos x$,

with $y(0)=1$

24. Find the general solution of

$x\left(\frac{dy}{dx}\right) \quad y\ln\left(\frac{y}{x}\right)$

25. Find the solution of

$\frac{dy}{dx}=\frac{xy+y}{xy+x}$

26. Find the solution of the differential equation $x\frac{dy}{dx}=2y+x^3e^x$ with $y(1)=0$]

27. Solve $x\frac{dy}{dx}=y(\log y-\log x+1)$

28. $e^y\frac{dy}{dx}+\frac{e^y}{x+1}=\frac{e^x}{x+1}$

29. $e^y\frac{d^2y}{dx^2}=2x$

30. $\frac{dy}{dx}=\frac{x\sin x}{3y^2+4y}$

31. $\frac{dy}{dx}-y\cot x=xy$

32. $\frac{dy}{dx}+2y\tan x=\sin x, y\left(\frac{\pi}{3}\right)=0$

ANSWER HINTS

1. Let $y^2=4x(1)\ y=2x(2)$

From (1) and (2)

$\Rightarrow 4x^2=4x\Rightarrow 4x(x-1)=0$

$\Rightarrow x=0,1$

when $x=0$ then $y=0$

when $x=1\Rightarrow y=2$

Required area $=\int_0^1\left(\sqrt{4x}-2x\right)dx$

Then simplify

2. The given curve is $y^4=x^3\Rightarrow y=\frac{x^{3/4}}{}$ (1)

The area bounded by the curve and the double ordinate through

$(2,0)=\int_0^2 ydx=\int_0^2 x^{3/4}dx$.

Then simplify.

3. The equation of 2 parabolas are $y^2=4x(1), x^2=4y(2)$ solving (1) and (2) $x=0,4$.

when $x=0\Rightarrow y=0$, when $x=4$ then $y=4$.

The parts of intersection of 2 curves are (0, 0), (4, 4).

Required area $=\int_0^4 \sqrt{4x}-\frac{x^2}{4}|\,dx$ = 16/3 squares unit unity.

Then simplify.

Equation of the curve is $x^2=4y$ (3)

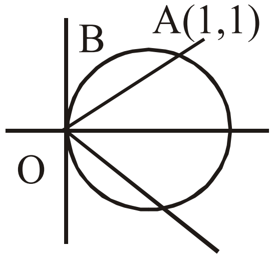
The given line is $y=0$ (4)

Area bounded between curves (3) and (4)

$=\int_0^4 ydx=\int_0^4\frac{x^2}{4}dx=\frac{16}{3}$ squares.

Hence the proof.

4. The given parabola is $y^2 = x$ (1)



The equation of the circle is

$x^2 + y^2 = 2x$ (2)

Solving (1) and (2) $x^2 + x = 2x$

$\Rightarrow x^2 - x = 0 \Rightarrow x = 0,1$

Parts of intersection are (0, 0), (1, 1), (1, –1).

Required area $= 2\left[\int_0^1 \left(\sqrt{2x - x^2} - \sqrt{x}\right) dx\right]$

5. Required area $= \int_0^{\pi/4} (\cos x - \sin x)\, dx$

6. The given ellipse is $\frac{x^2}{9} + \frac{y^2}{16} = 1$ (1)

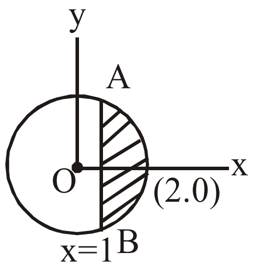
$\Rightarrow \frac{y^2}{16} = 1 - \frac{x^2}{9} = \frac{9 - x^2}{9} \Rightarrow y^2 = \frac{16}{9}(9 - x^2)$

$\Rightarrow y = \frac{4}{3}\sqrt{9 - x^2}$

Required area $= 4\int_0^3 \frac{4}{3}\sqrt{9 - x^2}\, dx$

when $y = 0 \Rightarrow x^2 = 9 \Rightarrow x = \pm 3$

7. The circle $d^3x^2 + y^2 = 4 \Rightarrow y^2 = 4 - x^2$



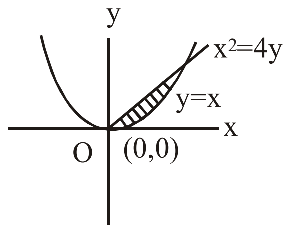
$y = \sqrt{4 - x^2}$

Required area $= 2\int_1^2 \sqrt{4 - x^2}\, dx$

8. Given parabola is $x^2 = 4y$ (1)

and the line is $y = x$ (2)

On solving (1) and (2) $x^2 = 4x$



$\Rightarrow x(x - y) = 0 \Rightarrow x = 0, 4$

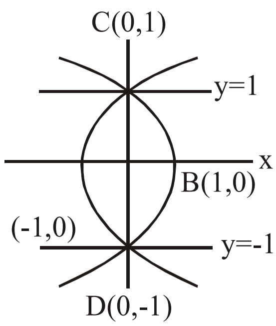
Line $y = x$ cuts the parabola at 0(0,0) and B(4,4) whose x coordinate is 4.

Required area $= \left|\int_0^4 \left(x - \frac{x^2}{4}\right) dx\right|$

$= \left|\left(\frac{x^2}{2}\right)_0^4 - \frac{1}{4}\left(\frac{x^3}{3}\right)_0^4\right| = \frac{1}{2}(16 - 0) - \frac{1}{12}(64 - 0)$

$= 8 - \frac{16}{3} = \frac{8}{3}$ sq. unit.

9. Given curves are $y^2 = x + 1$ (1)



$y^2 = -x + 1$ (2)

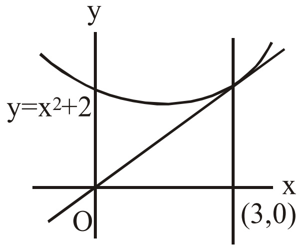
curve (1) is the parabola having axis $y = 0$ and vertex (–1, 0) curve (2) is the parabola having axis $y = 0$ and vertex (+1, 0) on solving (1) and (2) $2x = 0 \Rightarrow x = 0$.

From eqn. (1) $x=0 \Rightarrow y=\pm 1$

Required Area $= \int_{-1}^{1}\left[\left(1-y^2\right)-\left(y^2-1\right)\right]dy$

$= \int_{-1}^{1}(2-2y)\,dy =$ then simplify

10. The equation of the given curves are



$y=x^2+2(1)\, y=x(2)\, x=0(3)$

Eqn.(1) can be written as $x^2=y-2$

$\Rightarrow (x-0)^2 = y-2(4)$

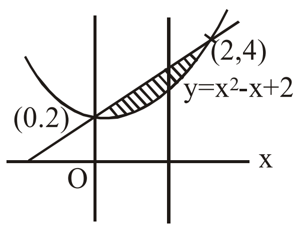
The parabola opens upwards. The line $y=x$ is a line passing through origin and makes 45^0 angle with x-axis, x = 0 is y-axis and x = 3 is a line parallel to y-axis at a distance 3 units from it.

Required area $= \int_{0}^{3}\left[\left(x^2+2\right)-x\right]dx$

Then simplify.

12. $y=x^2-x+2=\left(x-\frac{1}{2}\right)^2+\frac{7}{4}$

$\Rightarrow \left(x-\frac{1}{2}\right)^2 = y-\frac{7}{4}$ (1)



The given line is $y=x+2$ (2)

On solving (1) and (2) $x=0,2$

Required area

$= \left|\int_{0}^{2}(x+2)-\left(x^2-x+2\right)dx\right|$

13. We have

$\frac{dy}{dx} = \frac{y\frac{\sin y}{x}-x}{x\frac{\sin y}{x}}$ Put $y=vx \Rightarrow y/x=v$, $\frac{dy}{dx}=v+y\,dv/dx$

$\Rightarrow V+x\frac{dv}{dx} = \frac{Vx\sin v - x}{x.\sin v}\frac{V\sin v-1}{\sin v}$

$\Rightarrow V+x\frac{dv}{dx} = V-\frac{1}{\sin v} \Rightarrow x\frac{dv}{dx} = -\frac{1}{\sin v}$

$\Rightarrow \int -\sin v dv = \int\frac{dx}{x}$

14. $\left[2\sqrt{xy}-x\right]dy+ydx=0$

$\Rightarrow \frac{dy}{dx} = \frac{y}{x-2\sqrt{xy}}$

Put $y=vx$ $\frac{dy}{dx}=v+x\frac{dv}{dx}$

$\Rightarrow V+x\frac{dv}{dx} = \frac{vx}{x-2\sqrt{xvx}} = \frac{V}{1-2\sqrt{v}}$

$\Rightarrow x\frac{dv}{dx} = \frac{V}{1-2\sqrt{v}} - V = \frac{V-v+2v^{3/2}}{1-2\sqrt{v}} =$

$\Rightarrow \int\frac{\left(1-2\sqrt{v}\right)}{2v^{3/2}}dv = \int\frac{dx}{x}$

15. $\left(x+2y^3\right)\frac{dy}{dx}=y$

$\Rightarrow y\frac{dx}{dy}=x+2y^3$

$\Rightarrow y\frac{dx}{dy}-x=2y^3$

$\Rightarrow \frac{dx}{dy}-\frac{x}{y}=2y^2$

which is a linear equation

If $e^{\int -1/y\,dy}=e^{-\log y}=\frac{1}{y}$

$\frac{1}{y}\frac{dx}{dy}-\frac{1}{y^2}x=\frac{2y^2}{y}\Rightarrow \frac{d}{dy}\left(\frac{x}{y}\right)=2y$

$\Rightarrow \frac{x}{y}=\int 2ydy=y^2+c\Rightarrow x=y\left(y^2+c\right)$

16. The equation can be writeen as

$x^2 2y\frac{dy}{dx}+y^2 2x=\tan\left(x^2y^2\right)$

$\Rightarrow \frac{dy}{dx}\left(x^2y^2\right)=\tan\left(x^2y^2\right)$

$\frac{dz}{dx}=\tan z\Rightarrow \int \cot zdz=\int dx$

$x^2y^2=z$

Let $\frac{d}{dx}\left(x^2y^2\right)=\frac{dz}{dx}$

$\ln\left|\sin z\right|+c=x\Rightarrow \ln\left(x^2y^2\right)+c=x$

when $x=1, y=\sqrt{\frac{\pi}{2}}$

$1=\ln\left(\sin\frac{\pi}{2}\right)+c\Rightarrow c=1$

$\Rightarrow x=\ln\sin\left(x^2y^2\right)+1$

17. Here $\frac{dy}{dx}+\frac{y}{x}\tan y=\frac{e^x\sec y}{x^3}$

Dividing by $\sec y\ \cos y$

$\frac{dy}{dx}+\frac{y}{x}\sin y=\frac{e^x}{x^3}$ $\left| \begin{array}{l} t=\sin y \\ \cos y\frac{dy}{dx}=\frac{dt}{dx} \end{array}\right.$

$\Rightarrow \frac{dt}{dx}+\frac{y}{x}t=\frac{e^x}{x^3}$ which is linear

If $=\int_e \frac{y}{x}dx=e^{y\log x}=x^4$

$x^y\frac{dt}{dx}+x^y\frac{y}{x}t=\frac{e^x}{x^3}.x^y$

$x^y\frac{dt}{dx}+yx^3t=e^x.x$

$\frac{d}{dx}\left(tx^y\right)=xe^x\Rightarrow tx^y=\int xe^xdx$

18. $\frac{dy}{dx}-y\tan x=-y^2\sec x$

$\Rightarrow \frac{1}{y^2}\frac{dy}{dx}-\frac{1}{y}\tan x=-\sec x$

Let $z=-\frac{1}{y}\Rightarrow \frac{dz}{dx}=\frac{1}{y^2}\frac{dy}{dx}$

Now $\frac{dz}{dx}+z\tan x=-\sec x$

$\frac{d}{dx}(z\sec x)=-\sec^2 x$

$\Rightarrow z\sec x=\int -\sec^2 x\,dx$

Then simplify.

19. $\frac{dy}{dx}=\frac{e^x}{e^y}\left(1-e^y\right)$

$\Rightarrow e^y\frac{dy}{dx}=e^x-e^x.e^y$

$\Rightarrow e^y\frac{dy}{dx}=e^xe^y=e^x$

Let $e^y=V\Rightarrow e^y\frac{dy}{dx}=\frac{dv}{dx}$

$\frac{dv}{dx}+e^xV=e^x$. Here IF $=\int e^{e^x dx}=e^{e^x}$

$e^{e^x}\frac{dv}{dx}+e^{e^x}e^xv=e^{e^x}.e^x$

Then integrate

20. Let $x+y=y\Rightarrow 1+\frac{dy}{dx}=\frac{dy}{dx}\Rightarrow\frac{dy}{dx}=\frac{dy}{dx}-1$

Now $\left(\frac{y-1}{y-2}\right)\left(\frac{dy}{dx}-1\right)=\frac{y+1}{y+2}$

$\frac{dy}{dx}-1=\frac{y+1}{y+2}\frac{y-2}{y-1}=\frac{y^2-y-2}{y^2+y-2}$

$\frac{dy}{dx}=\frac{y^2-y-2}{y^2+y-2}+1$

$\Rightarrow\int\frac{y^2+y-2}{2y^2-y}dy=\int dx$

Then simplify.

21. $\frac{1}{2}\frac{d}{dx}\left(x^2+y^2\right)+\frac{d}{dx}\tan^{-1}\left(y/x\right)=0$

On integrating $\frac{1}{2}\left(x^2+y^2\right)+\tan^{-1}\frac{y}{x}=\frac{c}{2}$

then simplify

22. The equation can be written as

$x^2y^2dx+ye^xdx-e^xdy=0$

$\Rightarrow x^2dx+\frac{ye^xdx-e^xdy}{y^2}=0$

$\Rightarrow x^2dx+\frac{d}{dx}\left(e^{x/y}\right)=0$

Then integrate

23. $\frac{dy}{y+1}=\frac{-\cos x}{2+\sin x}dx$

The integrate $\int\frac{dy}{y+1}=\int\frac{-\cos x\ dx}{2+\sin x}$

24. $\frac{dy}{dx}=\frac{y}{x}\ln\left(\frac{y}{x}\right)$ | Let $y=vx$, $\frac{dy}{dx}=v+x\frac{dv}{dx}\Rightarrow\frac{y}{x}=V$

$V+x\frac{dv}{dx}=v\ln v$

$\frac{xdv}{dx}=v\left(\ln v-1\right)\Rightarrow\frac{dv}{\left(\ln v-1\right)}=\int\frac{dx}{x}$

25. $\frac{dy}{dx}=\frac{y\left(x+1\right)}{x\left(y+1\right)}\Rightarrow\int\frac{\left(y+1\right)dy}{y}=\int\frac{dx}{x}$

26. $x\frac{dy}{dx}-2y=x^3e^x$

$\Rightarrow\frac{dy}{dx}-\frac{2y}{x}=x^2e^x$ which is linear

I.F. $=e^{\int -2/x}=\frac{1}{x^2}$

multiplying with IF $\frac{1}{x^2}\frac{dy}{dx}-\frac{2y}{x^3}=e^x$

$\Rightarrow\frac{d}{dx}\left(\frac{y}{x^2}\right)=e^x$

$\Rightarrow\frac{y}{x^2}=\int e^x\ dx=e^x+c$

Then proceed

27. $\frac{dy}{dx}=\frac{y}{x}\left(\log\frac{y}{x}+1\right)$

Let $\frac{y}{x}=v \Rightarrow y=vx,\ \frac{dy}{dx}=v+x\frac{dv}{dx}$

$v+x\frac{dv}{dx}=v(\log v+1)=v\log v+v$

$\Rightarrow x\frac{dv}{dx}=v\log v \Rightarrow \int\frac{dv}{v\log v}=\int\frac{dx}{x}$

29. $\frac{d^2y}{dx^2}=e^{-x}2x \Rightarrow \frac{d}{dx}\left(\frac{dy}{dx}\right)=e^{-x}2x$

$\Rightarrow \frac{dy}{dx}=\int e^{-x}2x\,dx$.

Then apply integration by parts and proceed.

31. $\frac{dy}{dx}-y\cot x=xy^4$

$\Rightarrow \frac{1}{y^4}\frac{dy}{dx}-\frac{1}{y^3}\cot x=x$

$z=-\frac{1}{y^3} \Rightarrow \frac{dz}{dx}=\frac{3}{y^4}\frac{dy}{dx}$

$\Rightarrow \frac{1}{3}\frac{dz}{dx}=\frac{1}{y^4}\frac{dy}{dx}$

$\frac{1}{3}\frac{dz}{dx}-z\cot x=x$

which is linear then proceed.

CHAPTER - 5

DIFFERENTIAL EQUATION

A. Multiple Choice Questions (MCQ)

1. An ordinary differential equation is one which involves only ______ independent variables.
   (a) 2 (b) 1
   (c) 3 (d) None of these

2. A differential equation is called linear if every dependent variable and derivatives involved occur to the ______ degree only.
   (a) 3 (b) 2
   (c) 1 (d) None of these

3. Order of the differential equation $\left(\frac{dy}{dx}\right)^3 = \sqrt{\left(\frac{dy}{dx}\right)^2 + 1}$
   (a) 1 (b) 2
   (c) 3 (d) 4

4. Degree of differential equation $\frac{d^3y}{dx^3} - 3\left(\frac{dy}{dx}\right)^7 + \sqrt{x} = 0$ is ______
   (a) 1 (b) 2
   (c) 7 (d) None of these

5. Degree of differential equation $\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^{\frac{1}{2}} = x$ is ______
   (a) 1 (b) 2
   (c) 7 (d) None of these

6. Order of differential equation $\frac{d^3y}{dx^3} + \left(\frac{dy}{dx}\right)^4 = \sin x$ is ______
   (a) 1 (b) 4
   (c) 3 (d) None of these

7. Degree of differential equation $e^{\frac{dy}{dx}} = x$ is ______
   (a) 1 (b) 2
   (c) 3 (d) None of these

8. Order of differential equation whose solution $y = a\sin x + b\,e^x + c\,e^{2x}$ is ______
   (a) 1 (b) 2
   (c) 3 (d) None of these

9. The general solution of differential equation $\frac{dy}{dx} = \frac{y}{x}$ is ______
   (a) $\frac{x^2}{2} - \frac{y^2}{2} = c$ (b) $y = e^{cx}$
   (c) $y = cx$ (d) None of these

10. The differential equation whose primitive is $y = Ae^{2x} + Be^{-2x}$ is ______
    (a) $\frac{d^2y}{dx^2} = 4y$ (b) $\frac{d^2y}{dx^2} = 4$
    (c) $\frac{dy}{dx} = 4$ (d) None of these

11. Particular Solution of $\frac{dy}{dx} = \frac{1}{1+x^2}$, given when x = 0, y = 1 is ______
    (a) $y = \tan^{-1} x$
    (b) $y = 2\tan^{-1} x + 1$
    (c) $y = 2\tan^{-1} x - 1$
    (d) None of these

12. From the differential equation from $y = C \sec x$ by eliminating the arbitrary constant is ______

(a) $\frac{dy}{dx} = y \sec x$

(b) $\frac{dy}{dx} = y \tan x$

(c) $\frac{dy}{dx} = C \sec^2 \tan x$

(d) None of these

13. The differential equation of the family of straight lines parallel to y-axis is ______

(a) $\frac{dx}{dy} = 0$ (b) $\frac{dy}{dx} = 0$

(c) $\frac{dy}{dx} = 1$ (d) None of these

14. Order of the differential equation $\ln\left(d^2y / dx^2\right) = y$ is ______

(a) 1 (b) 2

(c) 3 (d) None of these

15. Solution of $x\,dx + y\,dy = 0$ is ______

(a) $x^2 + y^2 = c$ (b) $x + y = c$

(c) $\frac{x^2}{2} + \frac{y^2 2}{} = 0$ (d) None of these

16. Order of the differential equation whose general solution is $y = ax^2 + b$ where a and be are arbitary constants is ______

(a) 1 (b) 3

(c) 2 (d) None of these

17. The particular solution of the equation $\frac{dy}{dx} = \sin x$ for $y = 2$ when $x = \pi$ is ______

(a) $y = \cos x + 1$

(b) $y = -\cos x + 1$

(c) $y = \cos x - 1$

(d) None of these

18. The order and degree of the differential equation $\frac{d^3y}{dx^3} = \left(\frac{d^2y}{dx^2}\right)^2 + \left(\frac{dy}{dx}\right)^4 + y$ is ______

(a) 3, 1 (b) 3, 2

(c) 3, 4 (d) None of these

19. Particular solution of $\frac{dy}{dx} = (1+x)^4$ when $x = -1$, $y = 0$ is ______.

(a) $y = \frac{(1+x)^5}{5} + 2$

(b) $y = \frac{(1+x)^5}{5} + 1$

(c) $y = \frac{(1+x)^5}{5}$

(d) None of these

20. The differential equation of all straight lines passing through origin is ______.

(a) $x\,dy + y\,dx = 0$

(b) $x\,dy = y\,dx$

(c) $\frac{x}{y} = c$

(d) None of these

21. The differential equation whose solution is $y = e^{x+c}$ is ______

(a) $\frac{dy}{dx} = x$

(b) $\frac{dy}{dx} = c$

(c) $\frac{dy}{dx} = y$

(d) None of these

22. If the homogeneous form of the differential equation $\frac{dy}{dx} = \frac{x+y+1}{x-y+1}$ is $\frac{dY}{dX} = \frac{X+Y}{X-Y}$ then the relation between Y and y is ______

(a) $Y = 2y$ (b) $Y = y$
(c) $Y = 3y$ (d) None of these

23. The differential equation of the parabola $y^2 = 4x + 2$ is ______

(a) y dx - 2 dy = 0

(b) $y\,dy - 2dx = 0$

(c) $2y\,dx = dy$

(d) None of these

24. The differential equation whose general solution is y = 3x + K is ______

(a) $\frac{dy}{dx} = 3$ (b) $\frac{dx}{dy} = 3$

(c) dx = dy (d) None of these

25. Solution of $\frac{d^2y}{dx^2} = 0$ is ______

(a) $x = cy + d$ (b) $y = cx + d$

(c) $\frac{y}{x} = C$ (d) None of these

26. The number of arbitrary constants in the general solution of the differential equation $\frac{d^2y}{dx^2} = \operatorname{Sin} x + \operatorname{Cos} x$ is ______

(a) 1 (b) 2

(c) 3 (d) None of these

27. Solution of a 2nd Order differential equation contains ______ arbitrary constants.

(a) No (b) 1

(c) 2 (d) 3

28. Order and degree of

$$\frac{d^2y}{dx^2} = \frac{3y + \frac{dy}{dx}}{\sqrt{\frac{d^2y}{dx^2}}}$$ is ______

(a) 2, 3 (b) 1, 2

(c) 2, 2 (d) None of these

29. Order and degree of $\frac{\frac{dy}{dt}}{y + \frac{dy}{dt}} = \frac{yt}{\frac{dy}{dt}}$

(a) 2, 1 (b) 1, 3

(c) 2, 2 (d) None of these

ANSWER HINTS

1.	(b)	7.	(a)	13.	(a)	19.	(c)	26.	(b)
2.	(c)	8.	(c)	14.	(b)	20.	(b)	27.	(c)
3.	(a)	9.	(c)	15.	(a)	21.	(a)	28.	(a)
4.	(a)	10.	(a)	16.	(c)	22.	(b)	29.	(b)
5.	(b)	11.	(c)	17.	(b)	23.	(b)		
6.	(c)	12.	(b)	18.	(a)	24.	(a)		
						25.	(b)		

B. Long Answer Type Questions

1. Find the particular solution of the differential equation $\frac{d^2y}{dx^2} = 6x$, given that $y = 1$ and $\frac{dy}{dx} = 2$ when $x = 0$.

2. Solve $(x + y) + (x - y)dx = 0$

3. Solve $\frac{dy}{dx} = \frac{1}{x^2 - 7x + 12} = \frac{1}{(x-4)(x-3)}$

4. Solve $\frac{dy}{dt} = \frac{\tan^{-1} t\, e^{\tan - 1t}}{1+t^2}$

5. Solve $(1+x^2)\frac{dy}{dx} + 2xy = \text{Cos}\, x$

6. Solve $\frac{dy}{dt} = e^{2t+3y}$

7. Solve (x + tan y) dy = sin 2y dx

8. Solve $\frac{dy}{dx} + y = e^{-x}$

9. Find the particular solutionof the following differential equation

   $\frac{dy}{dx} + \frac{1+y^2}{1+x^2} = 0,\ y(-1) = \sqrt{3}$.

10. Solve (x + tan y) dy = tan y dx.

11. Solve (x - ln y)

    $(x - \ln y)\frac{dy}{dx} = -y \ln y$

12. Solve $\text{Cos}\, x \frac{dy}{dx} + y \sin x = 5$

13. $(1+x^2)\tan^{-1} y\, dy = (1+y^2)\tan^{-1} x\, dx$.

14. Solve $\frac{dy}{dx} = \frac{y(4+x^2)}{x(4+y^2)}$

15. Solve $x\frac{dy}{dx} + y = xy^2$

16. Solve $\frac{d^2y}{dx^2} = 3x^2 + 1$, given that y = 2, $\frac{dy}{dx} = 4$ when x = 0.

17. Solve $\frac{dy}{dx} = \frac{y^2}{xy - x^2}$

18. Solve $\frac{dy}{dx} = \frac{3x - 7y + 7}{3y - 7x - 3}$

19. Solve $\frac{dy}{dx} = \frac{1}{2}\left(\frac{y}{x}\right) + \frac{y^2}{x^2}$

20. Solve (2x + y + 1) dx + (4x + 2y - 1)dy=0

21. Solve $\frac{dy}{dx} - y\,\text{Cot}\, x = x\, y^4$.

22. Solve $\frac{dy}{dx} = \frac{y^2 + xy}{x^2 - xy}$

23. Solve $\frac{dy}{dx} + 2y \tan x = \sin x$

    (linear equation)

24. $\frac{dy}{dx} = \frac{y - x + 1}{y + x + 5}$

25. $\frac{dy}{dx} + y = \frac{1}{1 + ex}$

26. Solve $\frac{dy}{dx} = (x + y)^2$

27. Solve $e^y \frac{dy}{dx} + \frac{e^y}{x+1} = \frac{ex}{x+1}$

28. Solve $\frac{dy}{dx} = \frac{x \sin x}{(3y^2 + 4y)}$

29. Solve $\log\left(\frac{d^2y}{dx^2}\right) + x = 0$ if y=0, $\frac{dy}{dx} = 0$

    When x = 0.

ANSWER HINTS

1. Here $\frac{d}{dx}\left(\frac{dy}{dx}\right) = 6x$

$\Rightarrow \frac{dy}{dx} = \int 6x\ dx$

$\Rightarrow \frac{dy}{dx} = 6\frac{x^2}{2} + C_1 = 3x^2 + C_1$

When x = 0 then $\frac{dy}{dx} = 2$

$\Rightarrow 2 = C_1$

Now $\frac{dy}{dx} = 3x^2 + 2$.

$\int dy = \int (3x^2 + 2)dx$

$y = \frac{3x^3}{3} + 2x + C_2$

When x = 0 then y = 1, 1 = C_2.

Now particular solution is $y = x^3 + 2x + 1$.

2. $(x + y)dy = (y - x)dx$

$\Rightarrow \frac{dy}{dx} = \frac{y - x}{x + y}$ | Put $y = vx$, $\frac{dy}{dx} = v + x\frac{dv}{dx}$

Above equation is homogeneous.

$V + x\frac{dv}{dx} = \frac{Vx - x}{x + Vx} = \frac{V - 1}{V + 1}$

$\Rightarrow x\frac{dv}{dx} = \frac{V - 1}{V + 1} - V$

$= \frac{V - 1 - V^2 - V}{V + 1} = -\frac{V^2 + 1}{V + 1}$

$\Rightarrow \int \frac{V + 1}{V^2 + 1}dv = -\int \frac{dx}{x}$

$\Rightarrow \int \frac{V}{V^2 + 1}dv = -\int \frac{1}{V^2 + 1}dv$

$= -\log x + \log c$

$\frac{1}{2}\log(V^2 + 1) + \tan^{-1} V + \log x = \log C.$

$\Rightarrow \frac{1}{2}\log\left(\frac{y^2}{x^2} + 1\right) + \tan^{-1}\frac{y}{x} + \log x = \log C$.

3. By resolving into partical fraction

$\frac{1}{(x - 3)(x - 4)} = \frac{A}{x - 3} + \frac{B}{x - 4}$.

$1 = A(x - 4) + B(x - 3)$

When x = 3 $\Rightarrow$ A = -1

When x = 4 $\Rightarrow$ B = 1

Now $\int dy = -\int \frac{1}{x - 3}dx + \int \frac{1}{x - 4}dx$.

$\Rightarrow y = -\log(x - 3) + \log(x - 4) + C$.

4. $dy = \frac{\tan^{-1} t\ e^{\tan^{-1} t}}{1 + t^2}dt$

Integrating both sides

$\int dy = \int \frac{\tan^{-1} t\ e^{\tan^{-1} t}}{1 + t^2}dt$ | Let $u = \tan^{-1} t$, $\frac{du}{dt} = \frac{1}{1 + t^2}$, $du = \frac{1}{1 + t^2}dt$

$\Rightarrow y = \int u.e^u.du$

$\Rightarrow y = u.\int e^u du - \int \frac{d}{du}u.\int e^u du\ du$

$\Rightarrow y = 4e^u - \int e^u du = u\,e^u - e^u + c$

$\Rightarrow y = \tan^{-1} t\ e^{\tan^{-1} t} - e^{\tan^{-1} t} + c.$

5. The equation can be written as

$$\frac{d}{dx}[(1+x^2)y] = \text{Cos}\, x$$

$$\Rightarrow y(1+x^2) = \int \text{Cos}\, x\, dx = \sin x + C.$$

6. $dy = e^{2t+3y}dt$

$$\Rightarrow dy = e^{2t}.e^{3y}dt$$

$$\Rightarrow \frac{dy}{e^{3y}} = e^{2t}dt$$

$$\Rightarrow \int e^{-3y}dy = \int e^{25}dt$$

$$\Rightarrow \frac{e^{3y}dy}{-3} = \frac{e^{2t}}{2} + C.$$

7. $\Rightarrow \frac{x+\tan y}{\sin 2y} = \frac{dx}{dy}$

$$\Rightarrow \frac{x}{\sin 2y} + \frac{\tan y}{\frac{2\tan y}{1+\tan^2 y}} = \frac{dx}{dy}$$

$$\Rightarrow \frac{dx}{dy} - \frac{x}{\sin 2y} = \frac{1}{2}\sec^2 y$$

Which is a linear equation.

$$\text{I.F.} = e\int -\frac{1}{\sin 2y}dy = e\int -\text{cos ec}2y\, des$$

$$= e\ln\, (\text{cos ec}2y - \cot 2y)^{-\frac{1}{2}}$$

$$= (\text{cos ec}2y - \cot 2y)^{-\frac{1}{2}}$$

$$= \left(\frac{1}{\sqrt{\frac{1}{\sin 2y} - \frac{\cos 2y}{\sin^2 y}}}\right) = \left(\frac{1}{\sqrt{\frac{1-\text{Cos}^2 y}{\sin 2y}}}\right)$$

$$= \left(\frac{1}{\sqrt{\frac{2\sin^2 y}{2\sin y\cos y}}}\right) = \frac{1}{\sqrt{\tan y}}$$

$$\text{Now } \frac{1}{\sqrt{\tan y}}\frac{dx}{dy} - \frac{\text{Cos ec}2y}{\sqrt{\tan y}}x$$

$$= \frac{\tan y}{2\sqrt{\tan y}}\sec^2 y$$

$$\frac{d}{dy}\left[\frac{x}{\sqrt{\tan y}}\right] = \frac{\sqrt{\tan y}\sec^2 y}{2}$$

$$\Rightarrow \frac{d}{\sqrt{\tan y}} = \frac{1}{2}\int \tan y \sec^2 y\, des$$

Putting $t^2 = \tan y$ on rhs we can proceed.

8. The given eqn is a linear equation.

IF = $e\int 1\, dx = e^x$. Multiplying with IF.

$$e^x\frac{dy}{dx} + e^x y = e^x - e^{-x}.$$

$$\Rightarrow e^x = x + C.\ \frac{d}{dx}(e^x y) = 1$$

9. Here $\frac{dy}{dx} = -\frac{1+y^2}{1+x^2}$

$$\Rightarrow \frac{dy}{1+y^2} = -\frac{dx}{1+x^2}$$

$$\Rightarrow \frac{dy}{1+y^2} + \frac{dx}{1+x^2} = 0$$

$$\Rightarrow \int \frac{dy}{1+y^2} + \int \frac{dx}{1+x^2} = 0$$

$$\Rightarrow \tan^{-1} y + \tan^{-1} x = \tan^{-1} C.$$

$$\Rightarrow \tan 1\frac{x+y}{1-xy} = \tan^{-1} C.$$

$$\Rightarrow \frac{x+y}{1-xy} = C.$$

As $y(-1) = \sqrt{3}$

$$\Rightarrow \frac{-1+\sqrt{3}}{1+\sqrt{3}} = C$$

So particular solution is

$$\frac{x+y}{1-xy} = \frac{\sqrt{3}-1}{\sqrt{3}+1}$$

10. The given equation can be written as

$$\frac{x+\tan y}{\tan y}=\frac{dx}{dy}$$

$$\Rightarrow \frac{x}{\tan y}+1=\frac{dx}{dy}$$

$$\Rightarrow \frac{dx}{dy}-x\cot y=1$$ which is linear

IF = $e^{\int -\cot y des}=e^{-\ln \sin y}=e^{\ln \cos ec y}$

$=\operatorname{Cos} ec\, y$

Multiplying with IF

$$\operatorname{Cos} ec\, y\frac{dx}{dy}-x\cos ec\, y\cot y=\cos ec\, y$$

$$\Rightarrow \frac{d}{dy}[x\cos ec\, y]=\operatorname{Cos} ec\, y$$

$$\Rightarrow x\operatorname{Cos} ec\, y=\int \operatorname{Cos} ec\, y^{dy}$$

Then proceed.

11. Here $x-\ln y=-y\ln y\frac{dx}{dy}$

$$\Rightarrow y\ln y\frac{dx}{dy}+x=\ln y$$

$$\Rightarrow \frac{dx}{dy}+\frac{1}{y\ln y}x=\frac{1}{y}$$ which is linear

IF $=e^{\int \frac{1}{y\ln y}dy}=e^{\int \frac{1}{t}dt}=e^{\ln t}=t$ (Taking

$=\ln y$ $\ln y=t$

$\frac{1}{y}ay=dt$)

Now $\ln y\frac{dx}{dy}+\frac{1}{y}x=\frac{\ln y}{y}$

$$\Rightarrow \frac{d}{dy}[x\ln y]=\frac{\ln y}{y}$$

$$\Rightarrow x\ln y=\int \frac{\ln y}{y}des$$

Then proceed.

12. Equation can be written as

$$\frac{dy}{dx}+y\frac{\sin x}{\cos x}=\frac{5}{\cos x}$$ which is linear

IF = $e^{\int \frac{\sin x}{\cos x}dx}=e^{-\ln \cos x}=e^{\ln \sec x}=\sec x$

Multiplying with

IF $\operatorname{Sec} x\frac{dy}{dx}+y\sec x\tan x=5\sec^2 x$

$$\frac{d}{dx}[y\sec x]=5\operatorname{Sec}^2 x$$

$$\Rightarrow y\operatorname{Sec} x=5\int \sec^2 x dx=5\tan x+C.$$

13. Here $\frac{\tan^{-1} y\, dy}{1+y^2}=\frac{\tan^{-1} x}{1+x^2}dx$

$$\Rightarrow \int \frac{\tan^{-1} y\, dy}{1+y^2}=\int \frac{\tan^{-1} x}{1+x^2}dx$$

Then proceed.

14. Here $\frac{4+y^2}{y}dy=\frac{(4+x^2)dx}{x}$

$$\Rightarrow \int\left(\frac{4}{y}+y\right)dy=\int\left(\frac{4}{x}+x\right)dx$$

Then proceed.

15. Here $\frac{dy}{dx}+\frac{y}{x}=y^2$

$$\Rightarrow \frac{1}{y^2}\frac{dy}{dx}+\frac{1}{y}x=1$$

Let $z=-\frac{1}{y}$

$$\Rightarrow \frac{dz}{dx}=\frac{1}{y^2}\frac{dy}{dx}$$

Now $\frac{dz}{dx}-\frac{z}{x}=1$ Which is linear.

$$IF = e^{\int -\frac{1}{x}dx} = e^{-\ln x} = \frac{1}{x}$$

Multiplying with IF $\frac{1}{x}\frac{dz}{dx} - \frac{z}{x^2} = \frac{1}{x}$

$$\Rightarrow \frac{z}{x} = \int \frac{1}{x}dx = \log x + C.$$

16. We have $\frac{d}{dx}\left(\frac{dy}{dx}\right) = 3x^2 + 1$

$$\Rightarrow \frac{dy}{dx} = \int 3x^2 + 1)dx = 3\frac{x^3}{3} + x + C_1$$

When $x = 0$, $\frac{dy}{dx} = 4$.

$$\Rightarrow 4 = 0 + 0 + C_1 \Rightarrow C_1 = 4.$$

Now $\frac{dy}{dx} = x^3 + x + 4$.

$$\Rightarrow \int dy = \int (x^3 + x + 4)dx$$

$$\Rightarrow y = \frac{x^4}{4} + \frac{x^2}{2} + 4x + C_2.$$

When $x = 0,\ y = 2$

$$\Rightarrow 2 = 0 + 0 + 0 + C_2 = C_2$$

Required particular solution

$$y = \frac{x^4}{4} + \frac{x^2}{2} + 4x + 2.$$

17. The given eqn is homogeneous.

Let $y = Vx \Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$

Now $v + x\frac{dv}{dx} = \frac{v^2x^2}{x.vx - x^2} = \frac{v^2}{v-1}$

$$x\frac{dv}{dx} = \frac{v^2}{v-1} - v = \frac{v^2 - v^2 + v}{v-1}$$

$$\Rightarrow x\frac{dv}{dx} = \frac{v}{v-1} \Rightarrow \frac{v-1}{v}dv = \frac{dx}{x}.$$

$$\Rightarrow \int\left(1 - \frac{1}{v}\right)dv = \int\frac{dx}{x}.$$

$$\Rightarrow v - \ln v = \ln x + C$$

$$\Rightarrow \frac{y}{x} - \ln\frac{y}{x} = \ln x + C \qquad \left(\begin{array}{l}\therefore y = vx \\ \Rightarrow v = \frac{y}{x}\end{array}\right)$$

18. The given equation is not homogeneous so to make it homogeneous let's put

$x = X + h,\ y = Y + K$

Where hand x and K are constants to be determined.

$dx = dX,\ dy = dY$

$$\Rightarrow \frac{dy}{dx} = \frac{dY}{dX}$$

$$\frac{dY}{dX} = \frac{3(X+h) - 7(Y+K) + 7}{3(Y+K) - 7(X+h) - 3}$$

$$= \frac{3X - 7Y + (3h - 7K + 7)}{3Y - 7X + (3K - 7h - 3)}$$

Taking $3h - 7K + 7 = 0$ ... (1)

$3K - 7h - 3 = 0$ ... (2)

On Solving (1) and (2) h = 0, K = 1

So x = X, y = Y+1 $\Rightarrow Y = y - 1$

$$\frac{dY}{dX} = \frac{3X - 7Y}{3Y - 7X}$$

Let $Y = VX \Rightarrow \frac{dY}{dX} = V + X\frac{dV}{dX}$

$$V + X\frac{dV}{dX} = \frac{3X - 7VX}{7X + 3VX} = \frac{3 - 7V}{-7 + 3V}$$

$$\Rightarrow X\frac{dV}{dX} = \frac{3 - 7V}{3V - 7} - V$$

$$= \frac{3-7V-3V^2+7V}{3V-7}$$

$$\Rightarrow X\frac{dV}{dX} = -\frac{3(-1+V^2)}{3V-7}$$

$$\Rightarrow \int \frac{3V-7dV}{(V^2-1)} = \int -3\frac{dX}{X}$$

$$\Rightarrow 3\int \frac{V}{(V^2-1)}dV - 7\int \frac{dV}{V^2-1}$$

$$= -3\ln X + \ln C$$

$$\Rightarrow \frac{3}{2}\ln(V^2-1) - \frac{7}{2}\ln\frac{V-1}{V+1}$$

$$= -3\ln x + \ln C$$

$$\Rightarrow \frac{3}{2}\ln\left(-1+\frac{Y^2}{X^2}\right) - \frac{7}{2}\ln\frac{\frac{Y}{X}-1}{\frac{Y}{X}+1}$$

$$= -3\ln X + \ln C$$

$$\Rightarrow \ln\left(\frac{Y^2}{X^2}-1\right)^{\frac{3}{2}} - \ln\left(\frac{Y-X}{Y+X}\right)^{\frac{7}{2}} = \ln\frac{C}{X^3}.$$

Cohere $X = x, Y = y + 1$

19. $\frac{dy}{dx} - \frac{1}{2}\frac{y}{x} = \frac{y^2}{x^2}$

$$\Rightarrow \frac{1}{y^2}\frac{dy}{dx} - \frac{1}{2}\frac{1}{xy} = \frac{1}{x^2}$$

Let $Z = -\frac{1}{y} \Rightarrow \frac{dz}{dx} = \frac{1}{y^2}\frac{dy}{dx}$

Given equation reduces to

$$\frac{dz}{dx} + \frac{z}{x} = \frac{1}{x^2}$$

Which is linear

Then proceed.

20. The given eqn can be written as

$(2x + y + 1) = - (4x + 2y - 1)\, dy$

$$\Rightarrow \frac{2x+y+1}{4x+2y-1} = -\frac{dy}{dx}$$

$$\Rightarrow \frac{(2x+y)+1}{2(2x+y)-1} = -\frac{dy}{dx}$$

Let $V = 2x + y$

$$\frac{dv}{dx} = 2 + \frac{dy}{dx}$$

$$-\frac{dy}{dx} = 2 - \frac{dv}{dx}$$

$$\Rightarrow 2 - \frac{dv}{dx} = \frac{V+1}{2V-1}$$

$$\Rightarrow -\frac{dv}{dx} = \frac{V+1}{2V-1} - 2 = \frac{V+1-4v+2}{2V-1}$$

$$\Rightarrow -\frac{dV}{dx} = \frac{-3V+3}{2V-1} = \frac{3(1-V)}{2V-1}$$

$$\Rightarrow \int dx = -\int \frac{2V-1}{3(1-V)}$$

Then proceed.

21. Equation can be written as

$$\frac{1}{y^4}\frac{dy}{dx} - \frac{1}{y^3}Cotx = x$$

Let $z = -\frac{1}{y^3} \Rightarrow \frac{dz}{dx} = \frac{3}{y^4}\frac{dy}{dx}$

Given eqn reduces to $\frac{1}{3}\frac{dz}{dx} - ZCot\,x = x$

$\Rightarrow \frac{dz}{dx} - 3z\,Cot\,x = 3x$ which is linear

Then proceed.

22. Eqn is homogeneaus, so put $y = vx$

$$\Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$$

$$v + x\frac{dv}{dx} = \frac{v^2x^2 + x.vx}{x^2 - xvx} = \frac{v^2 + v}{1 - v}$$

$$\Rightarrow x\frac{dv}{dx} = \frac{v^2 + v}{1 - v} - v = \frac{v^2 + v - v + v^2}{1 - v}$$

$$\Rightarrow x\frac{dv}{dx} = \frac{2v^2}{1 - v}$$

$$\Rightarrow \frac{1 - v}{2v^2}dv = \frac{dx}{x}$$

$$\Rightarrow \frac{1}{2}\int \frac{1 - v}{v^2}dv = \int \frac{dx}{x}$$

Then proceed.

23. Do yourself.

24. It is a homogeneous equation do yourself.

25. Linear equation do youself.

26. $\frac{dt}{dx} - 1 = t^2$

$$\Rightarrow \frac{dt}{dx} = 1 + t^2$$

Let $x + y = t$

$1 + \frac{dy}{dx} = \frac{dt}{dx}$

$\frac{dy}{dx} = \frac{dt}{dx} - 1$

$$\Rightarrow \int \frac{dt}{1 + t^2} = \int dx$$

$$\Rightarrow \tan^{-1} t = x + c$$

$$\Rightarrow \tan^{-1}(x + y) = x + c.$$

27. Let $z = ey$

$$\Rightarrow \frac{dz}{dx} = ey\frac{dy}{dx}$$

$$\frac{dz}{dx} + \frac{1}{x + 1}z = \frac{ex}{x + 1}$$

Which is linear then proceed.

28. $\frac{dy}{dx} = \frac{x \sin x}{(3y^2 + 4y)}$

$$\Rightarrow (3y^2 + 4y)dy = x \sin x\, dx$$

$$\Rightarrow \int (ey^2 + 4y)dy = \int x \sin x\, dx$$

Then proceed.

29. The given equation can be written as

$$\log\left(\frac{d^2y}{dx^2}\right) = -x \qquad \frac{d^2y}{dx^2} = e^{-x}$$

$$\Rightarrow \frac{dy}{dx} = \int e^{-x}dx = -e^{-x} + c$$

When $x = 0$ then $\frac{dy}{dx} = 0$

$$\Rightarrow 0 = -e^0 + c_1 \qquad \Rightarrow c_1 = 1$$

So $\frac{dy}{dx} = -e^{-x} + 1 \Rightarrow \int dy = \int (-e^{-x} + 1)dx$

$$\Rightarrow y = e^{-x} + x + c_2$$

When $x = 0 \Rightarrow y = 0$

$$0 = e^{-0} + 0 + c_2 \Rightarrow c_2 = -1$$

So particular solution is $y = e^{-x} + x = 1$.

Unit - IV

VECTORS AND THREE-DIMENSIONAL GEOMETRY

CHAPTER - 1

VECTORS

A. Multiple Choice Questions (MCQ)

11. $\vec{a} = 3\hat{i} + 3\hat{j} + \hat{k}$ and $\vec{b} = -2\hat{i} + \hat{j} - 2\hat{k}$ then what is the unit vector parallel to $\vec{a} + \vec{b}$?

(a) $\frac{\hat{i} + 4\hat{j} - \hat{k}}{3\sqrt{2}}$ (b) $\frac{\hat{i} - \hat{k}}{3\sqrt{2}}$

(c) $\frac{\hat{i} - 4\hat{k}}{3}$ (d) None of these

2. What is the angle between $\hat{i} + \hat{j}$ and $\hat{i} - \hat{j}$?

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{3}$

(c) $\frac{\pi}{2}$ (d) π

3. What is the angle between $\vec{a}$ and $\vec{b}$ with magnitude 2 and 1 respectively such that $\vec{a}.\vec{b} = \sqrt{3}$.

(a) $\frac{\pi}{3}$ (b) $\frac{\pi}{6}$

(c) $\frac{5\pi}{6}$ (d) $\frac{3\pi}{6}$

4. What is the value of $\left|\vec{a} \times \vec{b} + \vec{b} \times \vec{a}\right|$

(a) 1 (b) 2

(c) 0 (d) None of these

5. The component of the vector $\vec{b} = 8\hat{i} + \hat{j}$ in the direction of the vector $\vec{a} = \hat{i} + 2\hat{j} - 2\hat{k}$ is ____

(a) $\frac{9}{10}\left(\hat{i} + 2\hat{j} - 2\hat{k}\right)$

(b) $\frac{10}{9}\left(\hat{i} + 2\hat{j} - 2\hat{k}\right)$

(c) $\frac{10}{3}\left(\hat{i} + 2\hat{j} - 2\hat{k}\right)$

(d) None of these

6. Find the values of μ for which the vector $\vec{a} = \mu\left(6\hat{i} + 2\hat{j} - 3\hat{k}\right)$ will be of unit length.

(a) $\pm\frac{1}{7}$ (b) $\pm\frac{2}{7}$

(c) $\pm\frac{3}{7}$ (d) $\pm\frac{4}{7}$

7. How many directions a null vector has?

(a) 1 (b) 2

(c) 3 (d) arbitrary

8. If $\vec{a}.\vec{b} = 0$ and $\vec{a} \times \vec{b} = \vec{0}$ then the conclusion is ____

(a) $\perp^r$ vectors

(b) parallel vectors

(c) any one of $\vec{a}$ or $\vec{b}$ is zero

(d) None of these

9. Value of $\left(\hat{i}-\hat{j}+2\hat{k}\right).\left(\hat{i}+\hat{j}\right)$ = ____

(a) 1 (b) 2
(c) 0 (d) 4

10. If $\hat{a}.\hat{b}=\frac{1}{2}$ then angle between $\hat{a}$ and $\hat{b}$ is ____

(a) $\frac{\pi}{6}$ (b) $\frac{2\pi}{3}$
(c) $\frac{\pi}{3}$ (d) $\frac{\pi}{4}$

11. If $\vec{a}=2\hat{i}+3\hat{j}-6\hat{k}$ and $\vec{b}=\alpha\hat{i}-\hat{j}+2\hat{k}$ are parallel then find α.

(a) $\frac{2}{3}$ (b) $-\frac{2}{3}$
(c) $\frac{3}{2}$ (d) $-\frac{3}{2}$

12. The unit vector perpendicular to the vectors $\hat{i}-\hat{j}$ and $2\hat{i}-3\hat{j}$ is ____

(a) $\hat{k}$ (b) $-\hat{k}$
(c) $-2\hat{k}$ (d) $-4\hat{k}$

13. If $\vec{a}$ and $\vec{b}$ are unit vectors such that $\vec{a}\times\vec{b}$ is a unit vector then what is the angle between $\vec{a}$ and $\vec{b}$?

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{2}$
(c) $\frac{2\pi}{3}$ (d) $\frac{3\pi}{2}$

14. What is the projection of $\hat{i}+\hat{j}+\hat{k}$ upon vector $\hat{i}$?

(a) 2 (b) 1
(c) –1 (d) –2

15. The no. of vectors of unit length perpendicular to $\vec{a}=\hat{i}+\hat{j}$, $\vec{b}=\hat{j}+\hat{k}$ is ____

(a) 1 (b) 2
(c) 4 (d) Infinite

16. The vectors of x for which the angle between the vectors $\vec{a}=x\hat{i}-3\hat{j}-\hat{k}$ and $\vec{b}=2x\hat{i}+x\hat{j}-\hat{k}$ is acute.

(a) 1 (b) –2
(c) 3 (d) 4

17. Let $\vec{a},\vec{b},\vec{c}$ are 3 vectors of same length and taken pairwise they form equal angles. If $\vec{a}=\hat{i}+\hat{j}$ and $\vec{b}=\hat{j}+\hat{k}$ then find $\vec{c}$.

(a) $\hat{i}+\hat{k}$
(b) $\hat{i}+2\hat{j}+3\hat{k}$
(c) $-\hat{i}+\hat{j}+2\hat{k}$
(d) $\frac{1}{3}\left(-\hat{i}+4\hat{j}-\hat{k}\right)$

18. Let $\vec{a},\vec{b},\vec{c}$ are 3 non-zero vectors, no two of which are collinear. If vector $\vec{a}+2\vec{b}$ is collinear with $\vec{c}$ and $\vec{b}+3\vec{c}$ is collinear with $\vec{a}$ then $\vec{a}+2\vec{b}+\vec{c}$ is ____

(a) $\lambda\vec{a}$ (b) $\lambda\vec{b}$
(c) $\lambda\vec{c}$ (d) $\vec{0}$

19. Let $\overrightarrow{OA}=\hat{i}+3\hat{j}-2\hat{k}$ and $\overrightarrow{OB}=3\hat{i}+\hat{j}-2\hat{k}$. The vector $\overrightarrow{OC}$ bisecting the angle AOB and C being a point on the line AB is ____

(a) $4\left(\hat{i}+\hat{j}-\hat{k}\right)$ (b) $2\left(\hat{i}+\hat{j}-\hat{k}\right)$
(c) $\hat{i}+\hat{j}-\hat{k}$ (d) None of these

20. Given 2 vectors $\hat{i}-\hat{j}$ and $\hat{i}+2\hat{j}$, the unit vector coplanar with 2 vectors and $\perp^r$ to the 1st is ____

(a) $\frac{1}{\sqrt{2}}\left(\hat{i}+\hat{j}\right)$ (b) $\frac{1}{\sqrt{5}}\left(2\hat{i}+\hat{j}\right)$
(c) $\frac{1}{\sqrt{2}}\left(\hat{i}+\hat{j}\right)$ (d) None of these

21. For any vector $\vec{a}$, $|\vec{a}\times\hat{i}|^2+|\vec{a}\times\hat{j}|^2+|\vec{a}\times\hat{k}|^2$ is ____

(a) $|\vec{a}|^2$ (b) $2|\vec{a}|^2$

(c) $3|\vec{a}|^2$ (d) None of these

22. The vectors $\vec{a}=3\hat{i}-2\hat{j}+3\hat{k}$ and $\vec{b}=-\hat{i}-2\hat{k}$ are adjacent sides of a parallelogram. Then angle between its diagonals is ____

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{3}$

(c) $\frac{3\pi}{4}$ (d) $\frac{2\pi}{3}$

23. If ABCD is a rhombus whose diagonals cut at the origins then $\overrightarrow{OA}+\overrightarrow{OB}+\overrightarrow{OC}+\overrightarrow{OD}=$ _____

(a) $\overrightarrow{AB}+\overrightarrow{AC}$ (b) $\vec{O}$

(c) $2(\overrightarrow{AB}+\overrightarrow{AC})$ (d) $\overrightarrow{AC}+\overrightarrow{BD}$

24. If G is the centroid of a triangle ABC then $\overrightarrow{GA}+\overrightarrow{GB}+\overrightarrow{GC}=$ ____

(a) $\vec{O}$ (b) $3\overrightarrow{GA}$

(c) $3\overrightarrow{GB}$ (d) $3\overrightarrow{GC}$

25. If $(x,y,z)\neq(0,0,0)$ and $(\hat{i}+\hat{j}+3\hat{k})x+(3\hat{i}-3\hat{j}+\hat{k})y+(-4\hat{i}+5\hat{j})z=a(x\hat{i}+y\hat{j}+z\hat{k})$, then a = ____

(a) 0, –2 (b) 2, 0

(c) 0, –1 (d) 1, 0

26. Two vectors $\vec{a}=2\hat{i}+\hat{j}+3\hat{k}$, $\vec{b}=4\hat{i}-\lambda\hat{j}+6\hat{k}$ are parallel if $\lambda=$ ____ .

(a) 2 (b) –3

(c) 3 (d) –2

27. If $\vec{a},\vec{b},\vec{c}$ are 3 mutually $\perp^r$ vectors. Each of magnitude unity then $|\vec{a}+\vec{b}+\vec{c}|=$ ____

(a) 3 (b) 1

(c) $\sqrt{3}$ (d) None of these

28. If $|\vec{a}|=|\vec{b}|$ then

(a) $|\vec{a}+\vec{b}|$ is $\|^c$ to $(\vec{a}-\vec{b})$

(b) $(\vec{a}+\vec{b})$ is $\perp^r$ to $(\vec{a}-\vec{b})$

(c) $(\vec{a}+\vec{b}).(\vec{a}-\vec{b})=2|\vec{a}|^2$

(d) None of these

29. If $\vec{a}+\vec{b}+\vec{c}=\vec{0}$, $|\vec{a}|=3, |\vec{b}|=5, |\vec{c}|=7$ then angle between $\vec{a}$ and $\vec{b}$ is ___

(a) $\frac{\pi}{6}$ (b) $\frac{2\pi}{3}$

(c) $\frac{5\pi}{3}$ (d) $\frac{\pi}{3}$

30. $\vec{a}=2\hat{i}-\hat{j}+\hat{k}$, $\vec{b}=\hat{i}+2\hat{j}-\hat{k}$ and $\vec{c}=\hat{i}+\hat{j}-2\hat{k}$ be 3 vectors. A vector in the plane of $\vec{b}$ and $\vec{c}$ whose projection on $\vec{a}$ is of magnitude $\sqrt{2/3}$ is ____

(a) $2\hat{i}+3\hat{j}-3\hat{k}$ (b) $2\hat{i}+3\hat{j}+3\hat{k}$

(c) $-2\hat{i}-\hat{j}+5\hat{k}$ (d) $2\hat{i}+\hat{j}+5\hat{k}$

31. If $\vec{a},\vec{b},\vec{c}$ are unit vectors such that $\vec{a}+\vec{b}+\vec{c}=\vec{0}$ then the value of $\vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a}$ is ____

(a) 1 (b) 3

(c) –3/2 (d) None of these

32. If $\vec{a} = 4\hat{i} + 6\hat{j}$, $\vec{b} = 3\hat{i} + 4\hat{k}$ the vector form of component of $\vec{a}$ along $\vec{b}$ is

(a) $\frac{18}{10\sqrt{3}}\left(3\hat{j} + 4\hat{k}\right)$

(b) $\frac{18}{25}\left(3\hat{i} + 4\hat{k}\right)$

(c) $\frac{18}{\sqrt{3}}\left(3\hat{j} + 4\hat{k}\right)$

(d) $4\hat{j} + 4\hat{k}$

33. If $|\vec{a}| = 7, |\vec{b}| = 11, |\vec{a} + \vec{b}| = 10\sqrt{3}$ then $|\vec{a} - \vec{b}| =$ ____

(a) 10 (b) $\sqrt{10}$

(c) $2\sqrt{10}$ (d) 20

34. The unit vector $\perp^r$ to vectors $\hat{i} - \hat{j}$ and $\hat{i} + \hat{j}$ forming a right handed system is ____

(a) $\hat{k}$ (b) $-\hat{k}$

(c) $\frac{1}{\sqrt{2}}\left(\hat{i} - \hat{j}\right)$ (d) $\frac{1}{\sqrt{2}}\left(\hat{i} + \hat{j}\right)$

35. If $\hat{n}_1$ and $\hat{n}_2$ are 2 unit vectors and θ is the angle between them then $\cos\frac{\theta}{2}$ is ____

(a) $\frac{1}{2}\left|\hat{n}_1 + \hat{n}_2\right|$ (b) $\frac{1}{2}\left|\hat{n}_1 - \hat{n}_2\right|$

(c) $\frac{1}{2}\left|\hat{n}_1 . \hat{n}_2\right|$ (d) $\frac{\hat{n}_1 . \hat{n}_2}{2|\hat{n}_1||\hat{n}_2|}$

36. If $\vec{a}$ is a vector $\perp^r$ to the vectors $\vec{b} = \hat{i} + 2\hat{j} + 3\hat{k}$ and $\vec{c} = -2\hat{i} + 4\hat{j} + \hat{k}$ and satisfies the condition $\vec{a}.(\hat{i} - 2\hat{j} + \hat{k}) = -6$ then $\vec{a} =$ ____ .

(a) $5\hat{i} + \frac{7}{2}\hat{j} - 4\hat{k}$

(b) $10\hat{i} + 7\hat{j} - 8\hat{k}$

(c) $5\hat{i} - \frac{7}{2}\hat{j} + 4\hat{k}$

(d) None of these

37. If $\left(\vec{a} \times \vec{b}\right)^2 + \left(\vec{a} . \vec{b}\right)^2 = 144$ and $|\vec{a}| = 4$ then $|\vec{b}| =$ ____

(a) 16 (b) 8

(c) 3 (d) 12

38. If $|\vec{a}| = 3, |\vec{b}| = 4$ and $|\vec{a} + \vec{b}| = 5$ then $|\vec{a} - \vec{b}| =$ ____

(a) 6 (b) 5

(c) 4 (d) 3

39. If $\vec{r} \times \vec{b} = \vec{c} \times \vec{b}$ and $\vec{r} . \vec{a} = 0$ where $\vec{a} = 2\hat{i} + 3\hat{j} - \hat{k}$, $\vec{b} = 3\hat{i} - \hat{j} + \hat{k}$ and $\vec{c} = \hat{i} + \hat{j} + 3\hat{k}$ then $\vec{r} =$ ____

(a) $\frac{1}{2}\left(\hat{i} + \hat{j} + \hat{k}\right)$ (b) $2\left(\hat{i} + \hat{j} + \hat{k}\right)$

(c) $2\left(-\hat{i} + \hat{j} + \hat{k}\right)$ (d) $\frac{1}{2}\left(\hat{i} - \hat{j} + \hat{k}\right)$

40. If $|\vec{a}| = 3$ and $-1 \le k \le 2$ then $|k\vec{a}|$ lies in the interval.

(a) [0, 6] (b) [–3, 6]

(c) [3, 6] (d) [1, 2]

41. If $|\vec{a}| = 8, |\vec{b}| = 3$ and $|\vec{a} \times \vec{b}| = 12$ then $\vec{a} . \vec{b} =$ ____

(a) $6\sqrt{3}$ (b) $8\sqrt{3}$

(c) $12\sqrt{3}$ (d) None of these

42. If $\vec{a}$ and $\vec{b}$ are vectors of magnitudes $\sqrt{3}$ and 4 respectively and $\vec{a}.\vec{b} = 2\sqrt{3}$ then angle between $\vec{a}$ and $\vec{b}$ is ____

(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{3}$

(c) $\frac{\pi}{2}$ (d) $\frac{5\pi}{2}$

43. The vector in the direction of $\hat{i} - 2\hat{j} + 2\hat{k}$ having magnitude 9 is ____

(a) $\hat{i} - 2\hat{j} + 2\hat{k}$ (b) $\frac{\hat{i} - 2\hat{j} + 2\hat{k}}{3}$

(c) $3\left(\hat{i} - 2\hat{j} + 2\hat{k}\right)$ (d) $9\left(\hat{i} - 2\hat{j} + 2\hat{k}\right)$

44. If θ is the angle between 2 unit vectors $\hat{e}_1$ and $\hat{e}_2$ then $\frac{1}{2}\left|\hat{e}_1 - \hat{e}_2\right| =$ _____

(a) $\cos\theta_2$ (b) $\sin\theta_2$

(c) $\cos\theta$ (d) $\sin\theta$

45. If $\vec{a}, \vec{b}, \vec{c}$ are non coplanar vectors then the vector $5\vec{a} + 6\vec{b} + 7\vec{c}$, $7\vec{a} - 8\vec{b} + 9\vec{c}$ and $3\vec{a} + 20\vec{b} + 5\vec{c}$ are ____

(a) collinear (b) coplanar

(c) non coplanar (d) None of these

ANSWER KEYS

1.	(a)	12.	(b)	23.	(b)	34.	(a)
2.	(c)	13.	(b)	24.	(a)	35.	(a)
3.	(b)	14.	(b)	25.	(c)	36.	(a)
4.	(c)	15.	(b)	26.	(d)	37.	(c)
5.	(b)	16.	(b)	27.	(c)	38.	(b)
6.	(a)	17.	(a)	28.	(b)	39.	(c)
7.	(d)	18.	(d)	29.	(d)	40.	(a)
8.	(c)	19.	(b)	30.	(c)	41.	(c)
9.	(c)	20.	(b)	31.	(c)	42.	(b)
10.	(c)	21.	(b)	32.	(b)	43.	(c)
11.	(b)	22.	(a)	33.	(c)	44.	(b)
						45.	(b)

B. Long Answer Type Questions

1. Find the p.v. of points which divide the join of the points $2\vec{a}-3\vec{b}$ and $3\vec{a}-2\vec{b}$ internally & externally in the ratio 2:3.

2. Let $\vec{a}, \vec{b}, \vec{c}$ be the p.v. of three distinct points A, B, C. If there exists scalars x, y, z (not all zero) such that $x\vec{a}+y\vec{b}+z\vec{c}=0$ and $x+y+z=0$ then show that A, B and C lie on a line.

3. Show that the points with p.v. $\vec{a}-2\vec{b}+3\vec{c}$, $-2\vec{a}+3\vec{b}-\vec{c}$ and $4\vec{a}-7\vec{b}+7\vec{c}$ are collinear.

4. If ABCD is quadrillateral and E are F are mid points of AC & BD then prove that $\overrightarrow{AB}+\overrightarrow{AB}+\overrightarrow{CB}+\overrightarrow{CD}=4\overrightarrow{EF}$

5. Show that the vectors $2\vec{a}-\vec{b}+3\vec{c}$, $\vec{a}+\vec{b}-2\vec{c}$ and $\vec{a}+\vec{b}-3\vec{c}$ are non-coplaner vectors.

6. Prove that four points $2\vec{a}+3\vec{b}-\vec{c}$, $\vec{a}-2\vec{b}+3\vec{c}$, $3\vec{a}+4\vec{b}-2\vec{c}$ and $\vec{a}-6\vec{b}+6\vec{c}$ are coplanar.

7. Find the condition that two non-zero non-collinear vectors are linearly independent.

8. Does there exists scalars u, v, w such that $u\vec{e}_1+v\vec{e}_2+w\vec{e}_3=\hat{i}$, where $\vec{e}_1=k, \vec{e}_2=\hat{j}+\hat{k}$, $\vec{e}_3=-\hat{j}+2\hat{k}$?

9. Show that the vectors $\hat{i}-3\hat{j}+2\hat{k}$, $2\hat{i}-4\hat{j}-4\hat{k}$, $3\hat{i}+2\hat{j}-\hat{k}$ are linearly independent.

10. Show that the vectors $\hat{i}+\hat{j}+\hat{k}$, $2\hat{i}+3\hat{j}-\hat{k}$ and $-\hat{i}-2\hat{j}+2\hat{k}$ are linearly dependent.

11. Forces of magnitudes 5 and 3 units acting in the directions $6\hat{i}+2\hat{j}+3\hat{k}$ and $3\hat{i}-2\hat{j}+6\hat{k}$ respectively act on a particle which is displaced from the point (2, 2, –1) to (4, 3, 1). Find the work done by the forces.

12. Prove that any two opposite edges in a regular tetrahedron are perpendicular.

13. If $\vec{a}, \vec{b}, \vec{c}$ are three mutually $\perp^r$ vectors of equal magnitude, then prove that $\vec{a}+\vec{b}+\vec{c}$ is equally inclined to $\vec{a}, \vec{b}$ & $\vec{c}$.

14. Show that the projection of $\vec{a}$ on $\vec{b} \neq \vec{0}$ is $\left(\dfrac{\vec{a}.\vec{b}}{|\vec{b}|^2}\right)\vec{b}$.

15. Prove that $\left(\vec{a}\times\vec{b}\right)^2+\left(\vec{a}.\vec{b}\right)^2=\left|\vec{a}\right|^2+\left|\vec{b}\right|^2$

16. If $\vec{a}, \vec{b}, \vec{c}$ are p.v. of vertices A, B, C of a ΔABC, then show that area of ΔABC is $\frac{1}{2}\left|\vec{a}\times\vec{b}+\vec{b}\times\vec{c}+\vec{c}\times\vec{a}\right|$.

17. Show that the $\perp^r$ distance of a point $\vec{c}$ from the line joining $\vec{a}$ & $\vec{b}$ is $\dfrac{\left|\vec{b}\times\vec{c}+\vec{c}\times\vec{a}+\vec{a}\times\vec{b}\right|}{\left|\vec{b}-\vec{a}\right|}$

18. For any three vectors $\vec{a}, \vec{b}, \vec{c}$ show that $\vec{a}\times(\vec{b}+\vec{c})+\vec{b}\times(\vec{c}+\vec{a})+\vec{c}\times(\vec{a}+\vec{b})=\vec{0}$

19. If $\vec{a}\times\vec{b}=\vec{c}\times\vec{d}$ and $\vec{a}\times\vec{c}=\vec{b}\times\vec{d}$ then show that $\vec{a}-\vec{d}$ is parallel to $\vec{b}-\vec{c}$ where $\vec{a}\neq\vec{d}$ and $\vec{b}\neq\vec{c}$.

20. If $\vec{a}, \vec{b}, \vec{c}$ are unit vectors such that $\vec{a}.\vec{b} = \vec{a}.\vec{c} = 0$, angle between $\vec{b}$ & $\vec{c}$ is $\pi/6$. Prove that $\vec{a} = \pm 2(\vec{b} \times \vec{c})$.

21. If $\vec{a}, \vec{b}, \vec{c}$ are three vectors such that $\vec{a} + \vec{b} + \vec{c} = \vec{0}$ then prove that $\vec{a} \times \vec{b} = \vec{b} \times \vec{c} = \vec{c} \times \vec{a}$

22. If $\vec{a}, \vec{b}, \vec{c}$ are three non-zero vectors such that $\vec{a}.(\vec{b} \times \vec{c}) = 0$ and $\vec{b}$ & $\vec{c}$ are parallel vectors then prove that $\vec{a} = \lambda \vec{b} + \mu \vec{c}$ where λ and μ are scalars.

23. Prove that the normal to the plane containing three points whose p.v. are $\vec{a}, \vec{b}, \vec{c}$ lies in the direction of $\vec{b} \times \vec{c} + \vec{c} \times \vec{a} + \vec{a} \times \vec{b}$

24. If $\vec{a} \times \vec{b} = \vec{a} \times \vec{c}$, $\vec{a} \neq \vec{0}, \vec{b} \neq \vec{c}$ then show that $\vec{b} = \vec{c} + t\vec{a}$ for some scalar t.

25. If $\vec{a}, \vec{b}, \vec{c}$ are vectors such that $\vec{a}.\vec{b} = \vec{a}.\vec{c}$, $\vec{a} \times \vec{b} = \vec{a} \times \vec{c}, \vec{a} \neq \vec{0}$ then show that $\vec{b} = \vec{c}$.

26. If $\vec{a}, \vec{b}, \vec{c}$ are three non-zero vectors such that $\vec{a} \times \vec{b} = \vec{c}$ and $\vec{b} \times \vec{c} = \vec{a}$ then prove that $\vec{a}, \vec{b}, \vec{c}$ are mutually $\perp^r$ and $|\vec{b}| = 1$ and $|\vec{c}| = |\vec{a}|$.

27. A force is represented in magnitude and direction by the line joining the points A(1, –2, 4) to the point B(5, 2, 3). Then find its moment about the point (–2, 3, 5).

28. Find the moment of the couple consisting of his force $\vec{F} = 3\hat{i} + 2\hat{j} - \hat{k}$ acting through the points $\hat{i} - \hat{j} + \hat{k}$ and $-\vec{F}$ acting through the point $2\hat{i} - 3\hat{j} - \hat{k}$

29. If $\vec{a} = -\hat{i} + \hat{j} + \hat{k}, \vec{b} = 2\hat{i} + \hat{k} + x\hat{j}$ then find the vector $\vec{X}$ satisfying the condition that $\vec{a}$ is $\perp^r$ to $\vec{b}$.

30. If $\vec{u}, \vec{v}, \vec{w}$ are vectors such that $\vec{u} + \vec{v} + \vec{w} = \vec{0}$, $|\vec{u}| = 3$, $|\vec{v}| = 4$, $|\vec{w}| = 5$, then find $\vec{u}.\vec{v} + \vec{v}.\vec{w} + \vec{w}.\vec{v}$.

ANSWER HINTS

1. If p.v. of A & B are $2\vec{a}-3\vec{b}$ & $3\vec{a}-2\vec{b}$. Let P & Q which divides AB internally & externally in the ratio 2 : 3 internally & externally then p.v. of

$$P=\frac{3\left(2\vec{a}-3\vec{b}\right)+2\left(3\vec{a}-2\vec{b}\right)}{3+2}=\frac{1}{5}\left(12\vec{a}-13\vec{b}\right)$$

p.v. of

$$Q=\frac{3\left(2\vec{a}-3\vec{b}\right)-2\left(3\vec{a}-2\vec{b}\right)}{3-2}=-5\vec{b}$$

2. $x\vec{a}+y\vec{b}+z\vec{c}=\vec{0}$

$$\Rightarrow \vec{c}=-\frac{\left(x\vec{a}+y\vec{b}\right)}{z}$$

As $x+y+z=0$

$$\Rightarrow z=-(x+y)$$

$$\Rightarrow \vec{c}=\frac{x\vec{a}+y\vec{b}}{x+y}$$

$\Rightarrow$ C divides the line joining the points A & B in the ratio y : x.
Hence A, B, C lie on the same line.

3. Let P, Q, R be points with p.v. $\vec{a}-2\vec{b}+3\vec{c}$, $-2\vec{a}+3\vec{b}-\vec{c}$ and $4\vec{a}-7\vec{b}+7\vec{c}$

$\overrightarrow{PQ}$ = p.v. of Q - p.v. of $P=-3\vec{a}+5\vec{b}-4\vec{c}$

$\overrightarrow{QR}$ = p.v. of R - p.v. of $Q=6\vec{a}-10\vec{b}+8\vec{c}$

$\Rightarrow \overrightarrow{QR}=-2\overrightarrow{PQ}$

$\overrightarrow{PQ}\,||^{r'}\,\overrightarrow{QR}$ & Q is the common point

$\Rightarrow$ P, Q, R are collinear.

4. Since F is mid point of BD

In ΔABD

$$\left(\overrightarrow{AB}\right)+1\left(\overrightarrow{AD}\right)=(1+1)\overrightarrow{AF}$$

$$\overrightarrow{AB}+\overrightarrow{AD}=2\overrightarrow{AF} \quad (1)$$

In ΔRCD

$$1\left(\overrightarrow{CB}\right)+1\left(\overrightarrow{CD}\right)=(1+1)\overrightarrow{CF}$$

$$\overrightarrow{CB}+\overrightarrow{CD}=2\overrightarrow{CF} \quad (2)$$

(1) + (2)

$$\overrightarrow{AB}+\overrightarrow{AD}+\overrightarrow{CB}+\overrightarrow{CD}=2\overrightarrow{AF}+2\overrightarrow{CF}$$

$$=-2\left(\overrightarrow{FA}+\overrightarrow{FC}\right)$$

$=-2\left(2\overrightarrow{FE}\right)$ (E is mid pot. of AC)

$$=4\overrightarrow{EF}$$

5. If possible the given vectors be coplanaer

Then $2\vec{a}-\vec{b}+3\vec{c}=x\left(\vec{a}+\vec{b}-2\vec{c}\right)$

$+y\left(\vec{a}+\vec{b}-3\vec{c}\right)$ for some scalars x & y

$\Rightarrow 2\vec{a}-\vec{b}+3\vec{c}$

$=(x+y)\vec{a}+(x+y)\vec{b}+(-2x-3y)\vec{c}$

$\Rightarrow 2=x+y, -1=x+y, 3=-2x-3y$

$\Rightarrow 2\neq -1$. So given vector are not coplanar.

6. Let p.v. of $2\vec{a}+3\vec{b}-\vec{c}, \vec{a}-2\vec{b}+3\vec{c}$, $3\vec{a}+4\vec{b}-2\vec{c}$ & $\vec{a}-6\vec{b}+6\vec{c}$ of P, Q, R & S then

$\overrightarrow{PQ}=-\vec{a}-5\vec{b}+4\vec{c}$

$\overrightarrow{PR}=\vec{a}+\vec{b}-\vec{c}$

$\overrightarrow{PS}=-\vec{a}-9\vec{b}+7\vec{c}$

Let $\overrightarrow{PQ}=x\overrightarrow{PR}+y\overrightarrow{PS}$

$\Rightarrow -\vec{a}-5\vec{b}+4\vec{c}=(x-y)\vec{a}+(x-9\vec{y})\vec{b}$

$+(-x+7\vec{c})\vec{c}$

$\Rightarrow x-y=-1, x-9y=-5, -x+7y=4$

$\Rightarrow x=-1/2,\ y=1/2$ (unique value of x & y)

Hence 4 given points are coplanar.

7. Let $\vec{a}, \vec{b}$ be two collinear vectors

$\Rightarrow \vec{b}=x\vec{a}$ for some scalar x

$\Rightarrow x\vec{a}+(-1)\vec{b}=\vec{0}=x\vec{a}+y\vec{b}$ (Let)

$\Rightarrow y=-1\neq 0$

Hence $\vec{a}$ & $\vec{b}$ are linearly dependent

8. We have $u\vec{e}_1+v\vec{e}_2+w\vec{e}_3=\hat{i}$

$\Rightarrow u\hat{k}+v(\hat{j}+\hat{k})+w(-\hat{j}+2\hat{k})=\hat{i}$

$\Rightarrow (-1)\hat{i}+(v-w)\hat{j}+(u+v+2w)\hat{k}=\vec{0}$

$\Rightarrow -1=0$ (not possible)

Hence there does not exists scalars u, v, w Satisfying the given equation

9. Let x, y, z be scalars such that

$x(\hat{i}-3\hat{j}+2\hat{k})+y(2\hat{i}-4\hat{j}-4\hat{k})$

$z(3\hat{i}+2\hat{j}-\hat{k})=\vec{0}$

$\Rightarrow (x+2y+3z)\hat{i}+(-3x-4y+2z)\hat{j}$

$+(2x-4y-z)\hat{k}=\vec{0}$

$\Rightarrow x+2y+3z=0$

$-3x-4y+2z=0$

$2x-4y-z=0$

$$\Rightarrow \begin{vmatrix} 1 & 2 & 3 \\ -3 & -4 & 2 \\ 2 & -4 & -1 \end{vmatrix} = 78 \neq 0$$

so the system of equation has only trivial soln $x=y=z=0$

Hence the given set of vectors is linearly independent.

10. Let x, y, z be scalars such that

$x(-\hat{i}-2\hat{j}+2\hat{k})+y(2\hat{i}+3\hat{j}-\hat{k})$

$+z(\hat{i}+\hat{j}+\hat{k})=\vec{0}$

$\Rightarrow (-x+2y+z)\hat{i}+(-2x+3y+z)\hat{j}$

$+(2x-y+z)\hat{k}=\vec{0}$

$\Rightarrow -x+2y+z=0$

$-2x+3y+z=0$

$2x-y+z=0$

$$\Rightarrow \begin{vmatrix} -1 & 2 & 1 \\ -2 & 3 & 1 \\ 2 & -1 & 1 \end{vmatrix} = 0$$

Hence the given set of vectors are linearly determinant

11. Let $\vec{F}$ be the resultant force & $\vec{d}$ be the displacement vector.

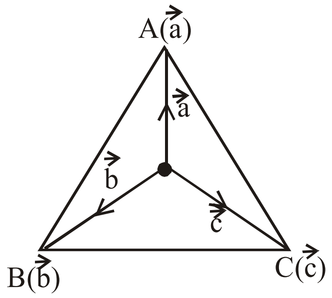
Then $\vec{F}=\dfrac{5(6\hat{i}+2\hat{i}+3\hat{k})}{\sqrt{36+4+9}}+3\dfrac{(3\hat{i}-2\hat{j}+6\hat{k})}{\sqrt{9+4+36}}$

$=\dfrac{1}{7}(39\hat{i}+4\hat{j}+33\hat{k})$

$\vec{d}=(4\hat{i}+3\hat{j}+\hat{k})-(2\hat{i}+2\hat{j}-\hat{k})=2\hat{i}+\hat{j}+2\hat{k}$

work done $=\vec{F}.\vec{d}=\dfrac{148}{7}$ units

12.



ΔABC be a tetrahedron

$\left|\overrightarrow{AB}\right|^2 = \left|\overrightarrow{BC}\right|^2 = \left|\overrightarrow{AC}\right|^2$

$\Rightarrow \left(\vec{b}-\vec{a}\right)^2 = \left(\vec{c}-\vec{b}\right)^2 = \left(\vec{c}-\vec{a}\right)^2$

$\Rightarrow \vec{a}.\vec{b} = \vec{b}.\vec{c} = \vec{c}.\vec{a}$ (As $|\vec{a}| = |\vec{b}| = |\vec{c}|$)

$\vec{a}.\vec{b} = \vec{b}.\vec{c}$	$\vec{b}.\vec{c} = \vec{c}.\vec{a}$
$\Rightarrow \vec{b}.(\vec{c}-\vec{a}) = 0$	$\Rightarrow \vec{c}.\left(\vec{b}-\vec{a}\right) = 0$
$\overrightarrow{OB} \perp^r \overrightarrow{AC}$	$\overrightarrow{OC} \perp^r \overrightarrow{AC}$

Smill $\overrightarrow{OA} \perp^r \overrightarrow{BC}$

Hence two opposite edges are $\perp^r$

13. $|\vec{a}| = |\vec{b}| = |\vec{c}| = \lambda$ (say)

$\vec{a}.\vec{b} = \vec{b}.\vec{c} = |\vec{c}| = \lambda$ (say)

$\vec{a}.\vec{b} = \vec{b}.\vec{c} = \vec{c}.\vec{a} = 0$

Now $|\vec{a}+\vec{b}+\vec{c}|^2 = 3\lambda^2$

$|\vec{a}+\vec{b}+\vec{c}| = \sqrt{3}\lambda$

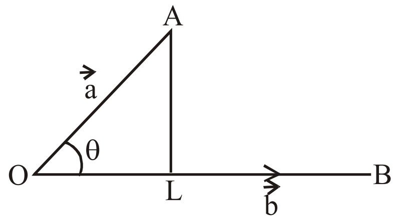
let $\theta_1, \theta_2, \theta_3$ be angles $\vec{a}+\vec{b}+\vec{c}$ makes with $\vec{a}, \vec{b}$ & $\vec{c}$.

$$\Rightarrow \cos\theta_1 = \frac{|\vec{a}|^2}{|\vec{a}|\left|\vec{a}+\vec{b}+\vec{c}\right|} = \frac{\lambda}{\sqrt{3}\lambda} = \frac{1}{\sqrt{3}}$$

Simill $\cos\theta_2 = \cos\theta_3 = \dfrac{1}{\sqrt{3}}$

$\Rightarrow \theta_1 = \theta_2 = \theta_3$

14.



Let θ be angle between $\vec{a}$ & $\vec{b}$

OL is proj. of $\vec{a}$ on $\vec{b}$

$\overrightarrow{OL}$ is proj. of $\vec{a}$ on $\vec{b}$

$OL = OA\cos\theta = |\vec{a}|\cos\theta$

$$= |\vec{a}|\frac{\left(\vec{a}.\vec{b}\right)}{|\vec{a}|\,|\vec{b}|}$$

$$\Rightarrow OL = \frac{\vec{a}.\vec{b}}{|\vec{b}|}$$

$$\overrightarrow{OL} = (OL)\hat{b} = \left(\frac{\vec{a}.\vec{b}}{|\vec{b}|}\right)\left(\frac{\vec{b}}{|\vec{b}|}\right)$$

$$= \left(\frac{\vec{a}.\vec{b}}{|\vec{b}|}\right)\vec{b}$$

15. $\left(\vec{a}\times\vec{b}\right) + \left(\vec{a}.\vec{b}\right) = \left(ab\sin\theta^2\right)^2 + \left(ab\cos\theta\right)^2$

$= a^2b^2\left(\sin^2\theta + \cos^2\theta\right)$ (As $\left(\hat{n}\right)^2 = 1$)

$= a^2b^2$

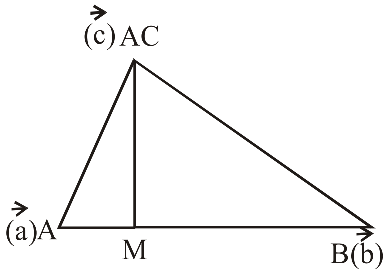
16. Area of $\Delta ABC = \dfrac{1}{2}\left|\overrightarrow{AB}\times\overrightarrow{AC}\right|$

$$= \frac{1}{2}\left|\left(\vec{b}-\vec{c}\right)\times\left(\vec{c}-\vec{a}\right)\right|$$

$$= \frac{1}{2}\left|\vec{b}\times\vec{c} - \vec{b}\times\vec{c} - \vec{a}\times\vec{c}\right|$$

$$= \frac{1}{2}\left|\vec{b}\times\vec{c} + \vec{a}\times\vec{b} + \vec{c}\times\vec{a}\right|$$

17.



Area of $\Delta = \frac{1}{2}\left|\overline{AB}\right|(CM)$

Area of $\Delta = \frac{1}{2}\left|\vec{a}\times\vec{b}+\vec{b}\times\vec{c}+\vec{c}\times\vec{a}\right|$

$\Rightarrow \frac{1}{2}\left|\overline{AB}\right|(CM) = \frac{1}{2}\left|\vec{a}\times\vec{b}+\vec{b}\times\vec{c}+\vec{c}\times\vec{a}\right|$

$\Rightarrow CM = \frac{\left|\vec{a}\times\vec{b}+\vec{b}\times\vec{c}+\vec{c}\times\vec{a}\right|}{\left|\vec{b}-\vec{a}\right|}$

18. $\vec{a}\times\left(\vec{b}+\vec{c}\right)+b\times\left(\vec{c}+\vec{a}\right)+\vec{c}\times\left(\vec{a}+\vec{b}\right)$

$=\vec{a}\times\vec{b}+\vec{a}\times\vec{c}+\vec{b}\times\vec{c}+\vec{b}\times\vec{a}+\vec{c}\times\vec{a}+\vec{c}\times\vec{b}$

$=\vec{a}\times\vec{b}-\vec{c}\times\vec{a}+\vec{b}\times\vec{c}-\vec{a}\times\vec{b}+\vec{c}\times\vec{a}+\vec{b}\times\vec{c}$

$=\vec{0}$

19. $\left(\vec{a}-\vec{d}\right)\times\left(\vec{b}-\vec{c}\right)$

$=\vec{a}\times\vec{b}-\vec{a}\times\vec{c}-\vec{d}\times\vec{b}+\vec{d}\times\vec{c}$

$=\vec{c}\times\vec{d}-\vec{b}\times\vec{d}+\vec{b}\times\vec{d}-\vec{c}\times\vec{d}$

(As $\vec{a}\times\vec{b}=\vec{c}\times\vec{d},\ \vec{a}\times\vec{c}=\vec{b}\times\vec{d}$)

$=\vec{0}$

$\Rightarrow \left(\vec{a}-\vec{d}\right) \|^{r} \left(\vec{b}-\vec{c}\right)$

20. $\vec{a}.\vec{b}=0$ & $\vec{a}.\vec{c}=0$

$\Rightarrow \vec{a}\perp^{r}\vec{b}$ & $\vec{a}\perp^{r}\vec{c}$

$\Rightarrow \vec{a}\perp^{r}$ to both $\vec{b}$ & $\vec{c}$, $\vec{a}$ is unit vector

$\Rightarrow \vec{a}=\pm\frac{\vec{b}\times\vec{c}}{\left|\vec{b}\times\vec{c}\right|}$

But $\left|\vec{b}\times\vec{c}\right|=\left|\vec{b}\right|\left|\vec{c}\right|\sin\frac{\pi}{6}=\frac{1}{2}$

As $\vec{b}$ & $\vec{c}$ are unit vector

$\Rightarrow |\vec{b}\times\vec{c}|=\frac{1}{2}$

so $\vec{a}=\pm 2\left(\vec{b}\times\vec{c}\right)$

21. $\vec{a}\times\left(\vec{a}+\vec{b}+\vec{c}\right)=\vec{a}\times\vec{0}$ As $\vec{a}\times\vec{b}\times\vec{c}=\vec{0}$

$\Rightarrow \vec{a}\times\vec{b}+\vec{a}\times\vec{c}=\vec{0}$ simill

$\Rightarrow \vec{a}\times\vec{b}-\vec{c}\times\vec{a}=\vec{0}$ $\vec{a}\times\vec{b}=\vec{b}\times\vec{c}$

$\Rightarrow \vec{a}\times\vec{b}=\vec{c}\times\vec{a}$

$\Rightarrow \vec{a}\times\vec{b}=\vec{b}\times\vec{c}=\vec{c}\times\vec{a}$

22. $\vec{a}.(\vec{b}\times\vec{c})=0$

$\Rightarrow \vec{a}=\vec{0}$ *or* $\vec{b}\times\vec{c}=\vec{0}$ *or* $\vec{a}\perp^{r}(\vec{b}\times\vec{c})$

$\vec{a}=\vec{0}$ *or* $\vec{b}\,\|^{r}\,\vec{c}$ *or* $\vec{a}\perp^{r}(\vec{b}\times\vec{c})$

But $\left(\vec{b}\times\vec{c}\right)\perp^{r}$ to both $\vec{b}$ & $\vec{c}$

$\Rightarrow \vec{a}$ lies in the plane $\vec{b}$ & $\vec{c}$

$\Rightarrow \vec{a},\vec{b},\vec{c}$ are coplaner

$\Rightarrow \vec{a}=\lambda\vec{b}+\mu\vec{c}$ for some scalars λ & μ

23. Let $\vec{a}.\vec{b}$ & $\vec{c}$ are p.v. of A, B & C

Then $\overrightarrow{AB}\times\overrightarrow{AC}$ is $\perp^{r}$ to plane containing A, B & C

Now $\overrightarrow{AB}\times\overrightarrow{AC}=\left(\vec{b}-\vec{a}\right)\times\left(\vec{c}-\vec{a}\right)$

$=\vec{b}\times\vec{c}-\vec{b}\times\vec{a}-\vec{a}\times\vec{c}+\vec{a}\times\vec{a}$

$=\vec{b}\times\vec{c}+\vec{a}\times\vec{b}+\vec{c}\times\vec{a}$

Hence $(\vec{a}\times\vec{b}+\vec{b}\times\vec{c}+\vec{c}\times\vec{a})$ is normal to

Hence containing $\vec{a}$, $\vec{b}$ & $\vec{c}$

24. $\vec{a}\times\vec{b}=\vec{a}\times\vec{c}$

$\Rightarrow \vec{a}\times\vec{b}-\vec{a}\times\vec{c}=\vec{0}$

$\Rightarrow \vec{a}\times(\vec{b}-\vec{c})=\vec{0}$

$\Rightarrow \vec{a}=\vec{0}\ or\ \vec{b}-\vec{c}=\vec{0}\ or\ \vec{a}\,||^{r|}\left(\vec{b}-\vec{c}\right)$

$\Rightarrow \vec{b}-\vec{c}=t\,\vec{a}$ for some scalar t

$\vec{b}=t\vec{a}+\vec{c}$

25. $\vec{a}.\vec{b}=\vec{a}.\vec{c}$ & $\vec{a}\neq\vec{0}$

$\Rightarrow \vec{a}.(\vec{b}-\vec{c})=0$

$\Rightarrow \vec{a}\perp^{r}(\vec{b}-\vec{c})\ or\ \vec{b}-\vec{c}=\vec{0}$

$\Rightarrow \vec{a}\perp^{r}(\vec{b}-\vec{c})\ or\ \vec{b}=\vec{0}$

Again $\vec{a}\times\vec{b}=\vec{a}\times\vec{c}$ & $\vec{a}=\vec{0}$

$\Rightarrow \vec{a}\times(\vec{b}-\vec{c})=\vec{0}$

$\Rightarrow \vec{a}\,||^{r}(\vec{b}-\vec{c})$

$\vec{a}$ can never be $\perp^{r}$ to be $\left(\vec{b}-\vec{c}\right)$ & $||^{r|}$ to $\left(\vec{b}-\vec{c}\right)$

$\Rightarrow \vec{b}=\vec{c}$

26. $\vec{a}\times\vec{b}=\vec{c}$ & $\vec{a}=\vec{b}\times\vec{c}$

$\Rightarrow \vec{c}\perp^{r}\vec{a}$ & $\vec{c}\perp^{r}\vec{b}$ & $\vec{a}\perp^{r}\vec{b}$ & $\vec{a}\perp^{r}\vec{c}$

$\Rightarrow \vec{a},\vec{b},\vec{c}$ are mutually $\perp^{r}$ vectors

$|\vec{a}\times\vec{b}|=|\vec{c}|$ & $|\vec{b}\times\vec{c}|=|\vec{a}|$

$\Rightarrow|\vec{a}|=|\vec{b}|\sin\frac{\pi}{2}=|\vec{c}|$ & $|\vec{b}||\vec{c}|\sin\frac{\pi}{2}=|\vec{a}|$

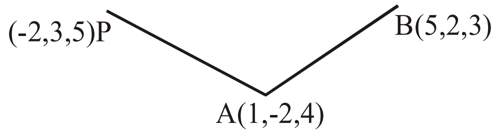
$\Rightarrow|\vec{c}||\vec{b}|^{2}=|\vec{c}|$

$\Rightarrow|\vec{b}|^{2}=1$

$\Rightarrow|\vec{b}|=1$

$\Rightarrow|\vec{a}|=|\vec{c}|$

27. $\vec{F}=\overrightarrow{AB}=4\hat{i}+4\hat{j}-\hat{k}$



Moment $=\overrightarrow{PA}\times\overrightarrow{AB}$

$=\left(3\hat{i}-5\hat{j}-\hat{k}\right)\times\left(4\hat{i}+4\hat{j}-\hat{k}\right)$

$=9\hat{i}-\hat{j}+32\hat{k}$

28. $M=\left(\overline{BA}\right)\times\overline{F}$

where $A\left(\hat{i}-\hat{j}+\hat{k}\right),\ B=\left(2\hat{i}-3\hat{j}-\hat{k}\right)$

$=-6\hat{i}+5\hat{j}-8\hat{k}$

29. $\vec{a}.\vec{b}=0$

$\Rightarrow -2+x+1=0$

$\Rightarrow X=1$

30. $\vec{u}+\vec{v}+\vec{w}=\vec{0}$

$\vec{u}.\vec{u}+\vec{u}.\vec{v}+\vec{u}.\vec{w}=0$

$9+\vec{u}.\vec{v}+\vec{u}.\vec{w}=0$ As $|\vec{u}|=3$

Similarly, $\vec{u}.\vec{v}+\vec{v}.\vec{w}+16=0$ As $|\vec{u}|=0$

$\vec{u}.\vec{w}+\vec{v}.\vec{w}+25=0$ As $|\vec{w}|=0$

Adding

$2\left(\vec{u}.\vec{v}+\vec{v}.\vec{w}+\vec{u}.\vec{w}\right)+50=0$

$\Rightarrow \vec{u}.\vec{v}+\vec{v}.\vec{w}+\vec{w}.\vec{u}=-25$

ADDENDUM

1. The no of vectors of unit length perpendicular to vectors $\vec{a} = \hat{i} + \hat{j}$ and $\vec{b} = \hat{j} + \hat{k}$ is

(a) 1 (b) 2

(c) 4 (d) Infinite

2. The values of x for which the angle between the vectors $\vec{a} = x\hat{i} - 3\hat{j} - \hat{k}$ and $\vec{b} = 2x\hat{i} + x\hat{j} - \hat{k}$ is acute and the angle between the vector $\vec{b}$ and the axis of ordinates is obtuse are

(a) 1, 2 (b) –2, –3

(c) all x < 0 (d) all x > 0

3. let $\vec{a}, \vec{b}$ and $\vec{c}$ be three vectors of same length and taken pair wise they from equal angles. If $\vec{a} = \hat{i} + \hat{j}$ and $\vec{b} = \hat{j} + \hat{k}$, then $\vec{c}$ equal to

(a) $\hat{i} + \hat{k}$

(b) $\hat{i} + 2\hat{j} + 3\hat{k}$

(c) $-\hat{i} + \hat{j} + 2\hat{k}$

(d) $\frac{1}{3}\left(-\hat{i} + 4\hat{j} - \hat{k}\right)$

4. Let $\vec{a}, \vec{b}$ and $\vec{c}$ be three non-zero vectors no two of which are collinear. If vector $\vec{a} = 2\vec{b}$ is co-linner with $\vec{c}$ and $\vec{b} + 3\vec{c}$ is co-linear with $\vec{a}$,a then $\vec{a} + 2\vec{b} + 6\vec{c}$ equal to

(a) $\lambda\vec{a}$ (b) $\lambda\vec{b}$

(c) $\lambda\vec{c}$ (d) 0

5. Let $\overrightarrow{OA} = \hat{i} + 3\hat{j} - 2\hat{k}$ and $\overrightarrow{OB} = 3\hat{i} + \hat{j} - 2\hat{k}$. The vector $\overrightarrow{OC}$ bisecting the angle AOB and C being a point on the line AB is

(a) $4(\hat{i} + \hat{j} - \hat{k})$ (b) $2(\hat{i} + \hat{j} - \hat{k})$

(c) $\hat{i} + \hat{j} - \hat{k}$ (d) None of these

6. The vector $\vec{C}$ directed along the internal bisector of the angle between the vectors $\vec{a} = 7\hat{i} - 4\hat{j} - 4\hat{k}$ and $\vec{b} = -2\hat{i} - \hat{j} + 2\hat{k}$ with $|\vec{c}| = 5\sqrt{6}$ is

(a) $\frac{5}{3}(\hat{i} - 7\hat{j} + 2\hat{k})$

(b) $\frac{5}{3}(5\hat{i} + 5\hat{j} + 2\hat{k})$

(c) $\frac{5}{3}(\hat{i} + 7\hat{j} + 2\hat{k})$

(d) $\frac{5}{3}(-5\hat{i} + 5\hat{j} + 2\hat{k})$

7. Given two vectors $\hat{i} - \hat{j}$ & $\hat{i} + 2\hat{j})$, the unit vector co-planar with two vectors and $\perp^r$ to first is

(a) $\frac{1}{\sqrt{2}}\left(\hat{i} + \hat{j}\right)$ (b) $\frac{1}{\sqrt{5}}\left(2\hat{i} + \hat{j}\right)$

(c) $\frac{1}{\sqrt{2}}\left(\hat{i} + \hat{j}\right)$ (d) None of these

8. For any vector $\vec{a}$,

$\left|\vec{a} \times \hat{i}\right|^2 + \left|\vec{a} \times \hat{j}\right|^2 + \left|\vec{a} \times \hat{k}\right|^2$ is

(a) $|\vec{a}|^2$ (b) $2|\vec{a}|^2$

(c) $3|\vec{a}|^2$ (d) None of these

9. The vectors $\vec{a} = 3\hat{i} - 2\hat{j} + 2\hat{k}$ and $\vec{b} = -\hat{i} - 2\hat{k}$ are adjacent sides of a parallelogram. Then angle between its diagonals is

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{3}$

(c) $\frac{3\pi}{4}$ (d) $\frac{2\pi}{3}$

10. If ABCD is a rhombus whose diagonals cut at the origin 0, then

$\overrightarrow{OA} + \overrightarrow{OB} + \overrightarrow{OC} + \overrightarrow{OD} =$

(a) $\overrightarrow{AB} + \overrightarrow{AC}$ (b) 3

(c) $2(\overrightarrow{AB} + \overrightarrow{AC})$ (d) $\overrightarrow{AC} + \overrightarrow{BD}$

11. If G is the centroid of a triangle ABC, then $\overrightarrow{GA} + \overrightarrow{GB} + \overrightarrow{GC} =$

(a) $\vec{0}$ (b) $3\overrightarrow{GA}$

(c) $3\overrightarrow{GB}$ (d) $3\overrightarrow{GC}$

12. If $(x, y, z) \neq (0,0,0)$ and $(\hat{i} + \hat{j} + 3\hat{k})x + (3\hat{i} - 3\hat{j} + \hat{k})y + (-4\hat{i} + 5\hat{j})z = a(x\hat{i} + y\hat{j} + z\hat{k})$ then a =

(a) 0, –2 (b) 2, 0

(c) 0, –1 (d) 1, 0

13. Two vectors $\vec{a} = 2\hat{i} + \hat{j} + 3\hat{k}, = 4\hat{i} + -\lambda\hat{j} + 6\hat{k}$ are parallel if $\lambda =$

(a) 2 (b) –3

(c) 3 (d) –2

14. If $\vec{a}, \vec{b}, \vec{c}$ are three mutually $\perp^r$ vectors of each magnitude unity then $|\vec{a} + \vec{b} + \vec{c}| =$

(a) 3 (b) 1

(c) $\sqrt{3}$ (d) None of these

15. If vectors $\vec{c}, \bar{a} = x\hat{i} + y\hat{j} + z\hat{k}$ and $\vec{b} = \hat{j}$ are such that $\vec{a}, \vec{c}$ and $\vec{b}$ form a right handed system, then $\vec{c}$ is

(a) $z\hat{i} - x\hat{k}$ (b) $\vec{0}$

(c) $y\hat{j}$ (d) $-z\hat{i} + x\hat{k}$

16. If $|\vec{a}| = |\vec{b}|$ then

(a) $(\vec{a} + \vec{b})$ is $\perp^r$ to $(\vec{a} - \vec{b})$

(b) $(\vec{a} + \vec{b})$ is $\perp^r$ $(\vec{a} - \vec{b})$

(c) $(\vec{a} + \vec{b}).(\vec{a} - \vec{b}) = 2|\vec{a}|^2$

(d) None of these

17. The projection of $(\hat{i} - 2\hat{j} + \hat{k})$ on the vector $4\hat{i} - 4\hat{j} + 7\hat{k}$ is

(a) $\frac{5\sqrt{6}}{10}$ (b) $\frac{19}{9}$

(c) $\frac{9}{19}$ (d) $\frac{\sqrt{6}}{19}$

18. If $\vec{a} + \vec{b} + \vec{c} = \vec{0}, |\vec{a}| = 3, |\vec{b}| = 5, |\vec{c}| = 7$, then angle between $\vec{a}$ and $\vec{b}$ is

(a) $\frac{\pi}{6}$ (b) $\frac{2\pi}{3}$

(c) $\frac{5\pi}{3}$ (d) $\frac{\pi}{3}$

19. Let $\vec{a} = 2\hat{i} - \hat{j} + \hat{k}, \vec{b} = \hat{i} + 2\hat{j} - \hat{k}$ and $\vec{c} = \hat{i} + \hat{j} - 2\hat{k}$ between three vectors. A vector in the plane of $\vec{b}$ & $\vec{c}$ whose projection on $\vec{a}$ is of magnitude $\sqrt{\frac{2}{3}}$ is

(a) $2\hat{i} + 3\hat{j} - 3\hat{k}$ (b) $2\hat{i} + 3\hat{j} + 3\hat{k}$

(c) $-2\hat{i} - \hat{j} + 5\hat{k}$ (d) $2\hat{i} + \hat{j} + 5\hat{k}$

20. If $\vec{a}+\vec{b}+\vec{c}=\vec{0}$, then the value of $\vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a}$ is

(a) 1 (b) 3

(c) $-\frac{3}{2}$ (d) None of these

21. If $\vec{a}=4\hat{i}+6\hat{j}$ and $\vec{b}=3\hat{i}+4\hat{k}$, then the vector form of component of $\vec{a}$ along $\vec{b}$ is

(a) $\frac{18}{10\sqrt{3}}(3\hat{j}+4\hat{k})$

(b) $\frac{18}{25}(3\hat{j}+4\hat{k})$

(c) $\frac{18}{\sqrt{3}}(3\hat{j}+4\hat{k})$

(d) $4\hat{j}+4\hat{k}$

22. If $\vec{a}$ and $\vec{b}$ are two vectors such that $\vec{a}.\vec{b}=0$ and $\vec{a}\times\vec{b}=\vec{0}$ then

(a) $\vec{a} \parallel^{r1} \vec{b}$

(b) $\vec{a} \perp^{r} \vec{b}$

(c) either $\vec{a} \sim \vec{b}$ is a null vector

(d) None of these

23. If $|\vec{a}|=7, |\vec{b}|=11, |\vec{a}+\vec{b}|=10\sqrt{3}$, then $|\vec{a}-\vec{b}|=$

(a) 10 (b) $\sqrt{10}$

(c) $2\sqrt{10}$ (d) 20

24. The unit vector $\perp^{r}$ to vectors $\hat{i}-\hat{j}$ and $\hat{i}+\hat{j}$ forming a right handed system is

(a) $\hat{k}$ (b) $-\hat{k}$

(c) $\frac{1}{\sqrt{2}}(\hat{i}-\hat{j})$ (d) $\frac{1}{\sqrt{2}}(\hat{i}+\hat{j})$

25. If $\hat{n}_1, \hat{n}_2$ be two unit vectors and θ is the angle between then $\cos\frac{\theta}{2}$ is

(a) $\frac{1}{2}|\hat{n}_1+\hat{n}_2|$ (b) $\frac{1}{2}|\hat{n}_1-\hat{n}_2|$

(c) $\frac{1}{2}(\hat{n}_1.\hat{n}_2)$ (d) $\frac{\vec{n}_1\times\vec{n}_2}{2|\hat{n}_1\times\hat{n}_2|}$

26. The position vector of A, B & C are $\hat{i}+\hat{j}+\hat{k}$, $2\hat{i}+3\hat{j}-4\hat{k}$ and $7\hat{i}+4\hat{j}+9\hat{k}$, then the unit vector perpendicular to the plane of ΔABC is

(a) $31\hat{i}-18\hat{j}-9\hat{k}$

(b) $\frac{31\hat{i}-38\hat{j}-9\hat{k}}{\sqrt{2486}}$

(c) $\frac{31\hat{i}+18\hat{j}+9\hat{k}}{\sqrt{2486}}$

(d) None of these

27. $\vec{a}$ is a vector $\perp^{r}$ to the vector $\vec{b}=\hat{i}+2\hat{j}+3\hat{k}$ and $\vec{c}=-2\hat{i}+4\hat{j}+\hat{k}$ and satisfies the condition $\vec{a}.(\hat{i}-2\hat{j}+\hat{k})=6$, then $\vec{a}=$

(a) $5\hat{i}+\frac{7}{2}\hat{j}-4\hat{k}$ (b) $10\hat{i}+7\hat{j}-8\hat{k}$

(c) $5\hat{i}-\frac{7}{2}\hat{j}+4\hat{k}$ (d) None of these

28. The projection of $\vec{a}=4\hat{i}-3\hat{j}+2\hat{k}$ on the axis making equal acute angle with co-ordinate axes is

(a) 3 (b) $\sqrt{3}$

(c) $\frac{1}{\sqrt{3}}$ (d) None of these

29. A unit vector in XY plane making an angle of 45^0 with the vector $\hat{i}+\hat{j}$ and angle 60^0 with the vector $3\hat{i}-4\hat{j}$ is

(a) $\hat{i}$ (b) $\frac{\hat{i}+\hat{j}}{\sqrt{2}}$

(c) $\frac{\hat{i}-\hat{j}}{\sqrt{2}}$ (d) None of these

30. A unit vector perpendicular to $4\hat{i}-\hat{j}+3\hat{k}$ and $-2\hat{i}+\hat{j}-2\hat{k}$ is

(a) $\frac{1}{3}(\hat{i}-2\hat{j}+2\hat{k})$

(b) $\frac{1}{3}(-\hat{i}+2\hat{j}+2\hat{k})$

(c) $\frac{1}{3}(2\hat{i}+\hat{j}+2\hat{k})$

(d) $\frac{1}{3}(2\hat{i}-\hat{j}+2\hat{k})$

31. If $\left(\vec{a}\times\vec{b}\right)+\left(\vec{a}.\vec{b}\right)^2=144$ and $|\vec{a}|=4$, then $|\vec{b}|=$

(a) 16 (b) 8

(c) 3 (d) 12

32. If $|\vec{a}|=3, |\vec{b}|=4$ and $|\vec{a}+\vec{b}|=5$ then $|\vec{a}-\vec{b}|=$

(a) 6 (b) 5

(c) 4 (d) 3

33. If $\hat{i}, \hat{j}, \hat{k}$ are unit orthonormal vectors and $\vec{a}$ is a vector, if $\vec{a}\times\vec{r}=\hat{j}$, then $\vec{a}.\vec{r}$ is

(a) 0 (b) 1

(c) –1 (d) arbitrary scalar

34. If $\vec{a}=\hat{i}+2\hat{j}-3\hat{k}$ and $\vec{b}=3\hat{i}-\hat{j}+3\hat{k}$, then the angle between $\vec{a}+\vec{b}$ & $\vec{a}-\vec{b}$ is

(a) 30^0 (b) 60^0

(c) 90^0 (d) 0^0

35. The area of parallelogram whose diagonal are the vectors $2\vec{a}-\vec{b}$ and $4\vec{a}-\vec{b}$, where $\vec{a}$ and $\vec{b}$ are unit vectors forming an angle 45^0 is

(a) $3\sqrt{2}$ (b) $\frac{1}{\sqrt{2}}$

(c) $\sqrt{2}$ (d) None of these

36. If $\vec{r}\times\vec{b}=\vec{c}\times\vec{b}$ and $\vec{r}.\vec{a}=0$ where $\vec{a}=2\hat{i}+3\hat{j}-\hat{k}$, $\vec{b}=2\hat{i}-\hat{j}+\hat{k}$ and $\vec{c}=\hat{i}+\hat{j}+3\hat{k}$, then $\vec{r}=$

(a) $\frac{1}{2}\left(\hat{i}+\hat{j}+\hat{k}\right)$ (b) $2\left(\hat{i}+\hat{j}+\hat{k}\right)$

(c) $2\left(-\hat{i}+\hat{j}+\hat{k}\right)$ (d) $\frac{1}{2}\left(\hat{i}-\hat{j}+\hat{k}\right)$

37. If $\vec{a}, \vec{b}, \vec{c}$ are non zero non-coplannar vectors then any vector $\vec{r}$ is equal to

(a) $z\vec{a}+x\vec{b}+y\vec{c}$ (b) $x\vec{a}+y\vec{b}+z\vec{c}$

(c) $y\vec{a}+z\vec{b}+x\vec{c}$ (d) None of these

38. If $|\vec{a}|=3$ and $-1\le k\le 2$, then $|k\vec{a}|$ lies in the interval

(a) [0, 6] (b) [–3, 6]

(c) [3, 6] (d) [1, 2]

39. If $\vec{a}$ and $\vec{b}$ are unit vectors then angle between $\vec{a}$ and $\vec{b}$ for which $\sqrt{3}\vec{a}-\vec{b}$ is a unit vector.

(a) 30^0 (b) 45^0

(c) 60^0 (d) 90^0

40. Two vectors $\hat{j}+\hat{k}$ and $3\hat{i}-\hat{j}+4\hat{k}$ represents two sides AB and AC respectively of ΔABC. Then the length of median through A is

(a) $\frac{\sqrt{34}}{2}$ (b) $\frac{\sqrt{48}}{2}$

(c) $\sqrt{18}$ (d) None of these

41. The position vector of a point which divides the join of points with p.v $\vec{a}+\vec{b}$ and $2\vec{a}-\vec{b}$ in the ratio 1 : 2 is

(a) $\frac{3\vec{a}+\vec{b}}{3}$ (b) $\vec{a}$

(c) $\frac{5\vec{a}-\vec{b}}{3}$ (d) $\frac{4\vec{a}+\vec{b}}{3}$

42. If $|\vec{a}|=8, |\vec{b}|=3$ and $|\vec{a}\times\vec{b}|=12$, then $\vec{a}.\vec{b}$ is

(a) $6\sqrt{3}$ (b) $8\sqrt{3}$

(c) $12\sqrt{3}$ (d) None of these

43. The projection of $\vec{a}$ on $\vec{b}$ is

(a) $\left(\frac{\vec{a}.\vec{b}}{|\vec{b}|^2}\right)\vec{b}$ (b) $\frac{\vec{a}.\vec{b}}{|\vec{b}|}$

(c) $\frac{\vec{a}.\vec{b}}{|\vec{a}|}$ (d) $\left(\frac{\vec{a}.\vec{b}}{|\vec{a}|^2}\right)\vec{b}$

44. The value of λ for which the vectors $\vec{a}=2\hat{i}+\lambda\hat{j}+\hat{k}$ and $\vec{b}=\hat{i}+2\hat{j}+3\hat{k}$ are orthogonal

(a) 0 (b) 1

(c) $\frac{3}{2}$ (d) $-\frac{5}{2}$

45. The angle between two vectors $\vec{a}$ & $\vec{b}$ with magnitude $\sqrt{3}$ and 4 and $\vec{a}.\vec{b}=2\sqrt{3}$ is

(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{3}$

(c) $\frac{\pi}{2}$ (d) $\frac{5\pi}{2}$

46. The vector in the direction of $\hat{i}-2\hat{j}+2\hat{k}$ having magnitude 9 is

(a) $\hat{i}-2\hat{j}+2\hat{k}$

(b) $\frac{\hat{i}-2\hat{j}+2\hat{k}}{3}$

(c) $3\left(\hat{i}-2\hat{j}+2\hat{k}\right)$

(d) $9\left(\hat{i}-2\hat{j}+2\hat{k}\right)$

47. If the moment of the force $\vec{f}=2\vec{i}+3\vec{j}-\vec{k}$ acting at (–1, 2, 1) about $(4, \lambda, 2)$ is $2\hat{i}-7\hat{j}-17\hat{k}$ then $\lambda=$

(a) 0 (b) 1

(c) 2 (d) 3

48. Torque (vector moment) about the point $3\hat{i}-\hat{j}+3\hat{k}$ of a force $4\hat{i}+2\hat{j}+\hat{k}$ through the point $5\hat{i}+2\hat{j}+4\hat{k}$ is

(a) $\hat{i}+2\hat{j}-8\hat{k}$ (b) $\hat{i}+2\hat{j}+8\hat{k}$

(c) $\hat{i}-2\hat{j}-8\hat{k}$ (d) None of these

49. Position vector of A, B & C are (1, 1, 1), (4, 5, 1) and (5, –2, 1). Then area of ΔABC is

(a) 5 sq. units (b) $\frac{25}{2}$ sq. units

(c) 25 sq. units (d) 50 sq. units

50. Angle between $\vec{a}\times\vec{b}$ & $\vec{b}\times\vec{a}$ is ____

(a) 180^0 (b) 90^0

(c) 0^0 (d) 45^0

51. If θ is angle between two unit vectors $\hat{e}_1$ & $\hat{e}_2$ then $\frac{1}{2}\left|\hat{e}_1-\hat{e}_2\right| =$

(a) $\cos\frac{\theta}{2}$ (b) $\sin\frac{\theta}{2}$

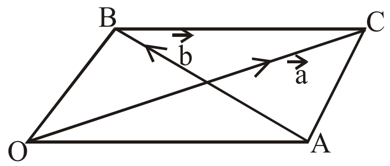
(c) $\cos\theta$ (d) $\sin\theta$

52. If $\vec{a}, \vec{b}, \vec{c}$ are non-coplaner vectors then the vectors $5\vec{a}+6\vec{b}+7\vec{c}$, $7\vec{a}-8\vec{b}+9\vec{c}$ and $3\vec{a}+20\vec{b}+5\vec{c}$ are

(a) collinear (b) coplaner

(c) non coplaner (d) None of these

53. If OACB is a parallelogram with $\overline{OC}=\vec{a}$ and $\overline{AB}=\vec{b}$ then $\overline{OA}$



(a) $\vec{a}+\vec{b}$ (b) $\vec{a}-\vec{b}$

(c) $\frac{1}{2}\left(\vec{b}-\vec{a}\right)$ (d) $\frac{1}{2}\left(\vec{a}-\vec{b}\right)$

54. The resolved part of force $\vec{F}=\hat{i}+2\hat{j}-4\hat{k}$ in the direction of $\vec{a}=2\hat{i}+4\hat{j}-4\hat{k}$ is

(a) $\frac{13}{9}\left(-\hat{i}+2\hat{j}-2\hat{k}\right)$

(b) $\frac{13}{9}\left(\hat{i}+2\hat{j}-2\hat{k}\right)$

(c) $\frac{13}{9}\left(-\hat{i}-2\hat{j}-2\hat{k}\right)$

(d) None of these

55. If unit vector $\vec{a}$ & $\vec{b}$ are inclined at an angle 2θ such that $|\vec{a}-\vec{b}|<1$ and $0\le 0\le\pi$, then θ lies in the interval

(a) $\left[0,\frac{\pi}{6}\right]$ *or* $\left[\frac{5\pi}{6},\pi\right]$

(b) $\left[\frac{\pi}{6},\pi\right]$

(c) $\left[\frac{\pi}{6},\frac{\pi}{2}\right]$

(d) $\left[\frac{\pi}{2},\frac{5\pi}{6}\right]$

56. Let $\vec{a}=2\hat{i}+\hat{j}+\hat{k}$, $\vec{b}=\hat{i}+2\hat{j}-\hat{k}$ and $\vec{c}$ be co-planner. If $\vec{c}$ is $\perp^r$ to $\vec{a}$ then $\vec{c}=$

(a) $\frac{1}{2}\left(-\hat{j}+\hat{k}\right)$

(b) $\frac{1}{\sqrt{3}}\left(-\hat{i}-\hat{j}+\hat{k}\right)$

(c) $\frac{1}{\sqrt{5}}\left(\hat{i}-2\hat{j}\right)$

(d) $\frac{1}{\sqrt{3}}\left(\hat{i}-\hat{j}-\hat{k}\right)$

57. If $\vec{a}, \vec{b}, \vec{c}$ 3 vectors such that $\vec{a}.(\vec{b}+\vec{c}) +\vec{b}.(\vec{c}+\vec{a})+\vec{c}.(\vec{a}+\vec{b})=0$ and $|\vec{a}|=1$, $|\vec{b}|=4$, $|\vec{c}|=8$ then $|\vec{a}+\vec{b}+\vec{c}|=$

(a) 13 (b) 81

(c) 9 (d) 5

58. A vector $\vec{r}$ satisfies $\vec{r}\times\vec{a}=\vec{b}$ and $\vec{r}.\vec{a}=0$. then

(a) $\vec{r}=\frac{\vec{a}\times\vec{b}}{\vec{a}.\vec{b}}$ (b) $\frac{\vec{a}\times\vec{b}}{\vec{a}.\vec{a}}$

(c) $\frac{\vec{a}\times\vec{b}}{\vec{b}.\vec{b}}$ (d) None of these

Answer Keys

1.	(b)	15.	(a)	29.	(d)	44.	(d)
2.	(b)	16.	(b)	30.	(b)	45.	(b)
3.	(d)	17.	(b)	31.	(c)	46.	(c)
4.	(d)	18.	(d)	32.	(b)	47.	(c)
5.	(b)	19.	(c)	33.	(d)	48.	(a)
6.	(a)	20.	(c)	34.	(c)	49.	(b)
7.	(b)	21.	(b)	35.	(b)	50.	(a)
8.	(a)	22.	(c)	36.	(c)	51.	(b)
9.	(a)	23.	(c)	37.	(b)	52.	(b)
10.	(b)	24.	(a)	38.	(a)	53.	(d)
11.	(a)	25.	(a)	39.	(a)	54.	(b)
12.	(c)	26.	(b)	40.	(a)	55.	(a)
13.	(d)	27.	(a)	41.	(d)	56.	(a)
14.	(c)	28.	(b)	42.	(c)	57.	(c)
				43.	(a)	58.	(b)

CHAPTER - 2

THREE DIMENSIONAL GEOMETRY

A. Multiple Choice Questions (MCQ)

1. Equation of the plane perpendicular to y-axis at the point (0, –2, 0) is ____

   (a) $x+2=0$ (b) $y+2=0$

   (c) $z=0$ (d) None of these

2. If $(0, 1, \alpha)$ be the dcs of a straight line then write the value of α.

   (a) 1 (b) 0

   (c) 2 (d) 3

3. If $A(6, 3, 2), B(5,1,4), C(3, 4, 7)$ and $D(0, 2, 5)$ are 4 points then what is the projection of CD on line AB ?

   (a) 2 (b) 1

   (c) 3 (d) 4

4. How many straight lines in space through the origin are equally inclined to the coordinate axes ?

   (a) 7 (b) 6

   (c) 8 (d) 9

5. What is the projection of the line segment joining (1, 3, –1), and (3, 2, 4) on z-axis. ?

   (a) 5 (b) 4

   (c) 6 (d) 3

6. If α, γ are the angles which a directed line makes with the positive direction of the coordinate axes then value of $\sin^2\alpha+\sin^2\beta+\sin^2\gamma$ = ____

   (a) 3 (b) 4

   (c) 2 (d) 1

7. A line makes angles α, β, γ with the coordinate axes. If $\alpha+\beta=\frac{\pi}{2}$ then what is the value of $(\cos\alpha+\cos\beta+\cos\gamma)^2$.

   (a) $1+\sin 2\alpha$ (b) $1+\sin 2\beta$

   (c) $1+\sin 2\gamma$ (d) None of these

8. The equation of the line passing through (4, –6, 1) and parallel to the line $\frac{x-1}{1}=\frac{y+2}{3}=\frac{z-1}{-1}$ is ____

   (a) $\frac{x-4}{1}=\frac{y-6}{3}=\frac{z-1}{-1}$

   (b) $\frac{x-4}{1}=\frac{y+6}{3}=\frac{z-1}{-1}$

   (c) $\frac{x-4}{1}=\frac{y-6}{3}=\frac{z+1}{-1}$

   (d) None of these

9. The image of the point (–2, 3, –5) w.r.t. zx-plane is ____

   (a) (2, –3, 5) (b) (–2, –3, –5)

   (c) (–2, –3, 5) (d) None of these

10. What is the distance of the point (1, 2, –3) from xy-plane.

    (a) 4 (b) 3

    (c) 5 (d) None of these

11. If the equation of z-axis is $\frac{x}{a}=\frac{y}{b}=\frac{z-d}{c}$ then the values of a, b, c, d are ____

    (a) 1, 0, 0, 0 (b) 0, 1, 0, 0

    (c) 0, 0, 1, 0 (d) None of these

12. How many independent constants are there in the general equation of a plane $ax+by+cz+d=0$?

(a) 4 (b) 3

(c) 2 (d) 1

13. If $|x|=1, |y|=2, |z|=3$ then how many points in R^3 are there having coordiantes (x, y, z)?

(a) 6 (b) 7

(c) 8 (d) 4

14. The equation of the plane passing through the point (1, –2, 3) and $\perp^{r}$ to y-axis is ____

(a) y = –2 (b) x = 1

(c) z = 3 (d) None of these

15. The value of k for which the line $\frac{x-2}{3}=\frac{1-y}{k}=\frac{z-1}{4}$ is parallel to the plane $2x+6y+3z-4=0$ is ____

(a) 2 (b) 4

(c) 3 (d) –3

16. If the dcs of a st. line is <2/7, 3/7, k/7 > then what is the value of k ?

(a) ±6 (b) ±7

(c) ±8 (d) None of these

17. If a line makes an angle 90^0 with x-axis and 60^0 with y-axis then what is the angle it makes with z-axis ?

(a) $\frac{\pi}{3}$ (b) $\frac{2\pi}{3}$

(c) $\frac{\pi}{6}$ (d) $\frac{5\pi}{6}$

18. The distance between the planes $x-xy+3z+1=0$ and $2x-4y+6z+3=0$ is ____

(a) $\frac{1}{\sqrt{55}}$ (b) $\frac{1}{\sqrt{56}}$

(c) $\frac{1}{\sqrt{57}}$ (d) $\frac{1}{\sqrt{58}}$

19. The equation of the plane which makes intercepts 1, 2, 3 on x, y, z axes respectively.

(a) $x+2y+3z=0$

(b) $\frac{x}{1}+\frac{y}{2}+\frac{z}{3}=1$

(c) $(x-1)+(y-2)+(z-3)=0$

(d) None of these

20. What are the dcs of the line $\perp^{r}$ to the plane $3x-2y-2z+1=0$?

(a) 3, –2, –2

(b) $\frac{3}{\sqrt{17}}, \frac{-2}{\sqrt{17}}, \frac{-2}{\sqrt{17}}$

(c) $\frac{3}{\sqrt{13}}, \frac{-2}{\sqrt{13}}, \frac{-2}{\sqrt{13}}$

(d) None of these

21. The ratio in which the line segment joining the points (1, 2, –2) (4, 3, 4) is divided by the xy-plane is ____

(a) 2 : 1 (b) 1 : 2

(c) –1 : 2 (d) 2 : –1

22. The equation of the line passing through (–3, 1, 2) and perpendicular to the plane 2y – z = 3 is ____

(a) $\frac{x+3}{0}=\frac{y-1}{2}=\frac{z-2}{-1}$

(b) $\frac{x+3}{1}=\frac{y-1}{2}=\frac{z-2}{-1}$

(c) $\frac{x}{3}=\frac{y}{1}=\frac{z}{2}$

(d) None of these

23. The distance of the point (4, 5, –3) from y-axis is ____

(a) 4 (b) 3

(c) 5 (d) 6

24. Projection of the line segment joining (1, 3,–1) and (3, 2, 4) on z-axis is ____

(a) 4 (b) 7

(c) 5 (d) 6

25. Equation of the plane that passes through y-axis and z-axis is ____

(a) y = 0 (b) z = 0

(c) x = 0 (d) None of these

26. What are the dcs of the line passing through (1, –1, 1) and (2, –5, –3) ?

(a) $\langle 1,-4,-4 \rangle$

(b) $\left\langle \frac{1}{\sqrt{33}},\frac{-4}{\sqrt{33}},\frac{-4}{\sqrt{33}} \right\rangle$

(c) $\left\langle \frac{1}{2},\frac{-1}{5},\frac{-1}{3} \right\rangle$

(d) None of these

27. What are the dcs of the line x = y = z

(a) $\langle 1,1,1 \rangle$

(b) $\left\langle \frac{1}{\sqrt{3}},\frac{1}{\sqrt{3}},\frac{1}{\sqrt{3}} \right\rangle$

(c) $\left\langle \frac{-1}{\sqrt{3}},\frac{-1}{\sqrt{3}},\frac{-1}{\sqrt{33}} \right\rangle$

(d) None of these

28. The planes $2x+y+kz-1=0$ and $kx-y+z+2=0$ are perpendicular if k = ____

(a) $-\frac{1}{4}$ (b) $\frac{1}{3}$

(c) $\frac{1}{4}$ (d) None of these

29. The angle between the planes $x+y=0$ and $y+z=1$ is ____

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{6}$

(c) $\frac{\pi}{3}$ (d) None of these

30. If the lines $\frac{x-1}{4}=\frac{y-2}{1}=\frac{z-3}{1}$ and $\frac{x-3}{k}=\frac{y}{1}=\frac{z}{1}$ are parallel then value of k is ____

(a) 1 (b) 2

(c) 3 (d) 4

31. If the line $\frac{x-1}{1}=\frac{y}{1}=\frac{z-k}{1}$ lies on the planes $x+y-z=0$ then values of k is ____

(a) 1 (b) 2

(c) 3 (d) None of these

32. If the points (1, 2, 1), (–1, 0, 1), (k, 1, 1) are collinear then value of k is ____

(a) –1 (b) 0

(c) 2 (d) None of these

33. The distance between the planes $2x-3y+6z+1=0$ and $4x-6y+12z-5=0$ is ____

(a) $\frac{1}{2}$ (b) $\frac{1}{7}$

(c) $\frac{2}{7}$ (d) $\frac{6}{7}$

Answer Keys

1.	(b)	9.	(b)	17.	(c)	25.	(c)
2.	(b)	10.	(b)	18.	(b)	26.	(b)
3.	(b)	11.	(c)	19.	(b)	27.	(b)
4.	(c)	12.	(b)	20.	(b)	28.	(b)
5.	(a)	13.	(c)	21.	(b)	29.	(c)
6.	(c)	14.	(a)	22.	(a)	30.	(d)
7.	(a)	15.	(c)	23.	(c)	31.	(a)
8.	(b)	16.	(a)	24.	(c)	32.	(b)
						33.	(a)

B. Long Answer Type Questions

1. Prove that the lines $\frac{x+3}{2}=\frac{y+5}{3}=\frac{z-7}{-3}$ and $\frac{x+1}{4}=\frac{y+1}{5}=\frac{z+1}{-1}$ are coplanar. Find the equation of the plane containing them.

2. Show that the lines $\frac{x-4}{1}=\frac{y+3}{-4}=\frac{z+1}{7}$ and $\frac{x-1}{2}=\frac{y+1}{-3}=\frac{z+10}{8}$ are coplanar. Find their point of intersection and equation of the plane in which they lie.

3. Find the distance of the point (4, 5, 2) from the line $\frac{x}{2}=\frac{y}{3}=\frac{z}{1}$.

4. If the edges of a rectangular parallelo piped are of length a, b, c then the angle between 4 diagonals are $\cos^{-1}\left(\frac{\pm a^2 \pm b^2 \pm c^2}{a^2+b^2+c^2}\right)$

5. If $\langle l_1, m_1, n_1\rangle$ and $\langle l_2, m_2, n_2\rangle$ are dcs of 2 mutually perpendicular lines then show that the dcs of the line perpendicular to both of them are $m_1n_2 - m_2n_1, n_1l_2 - n_2l_1$, $l_1m_2 - l_2m_1$.

6. Find the image of the part (2, 3, 4) w.r.t. the plane $x - y + 2z = 4$. Obtain the foot of the perpendicular from P on the plane and the corresponding perpendicular distance.

7. Prove that the straight lines whose dcs are connected by the relations $l + 2m + 3n = 0$ and $3lm - 4\ln + mn = 0$ are perpendicular to each other.

8. Prove that the lines $\frac{x+4}{1}=\frac{y+5}{1}=\frac{z-7}{-2}$ and $2x+3y+z-1=0,\ 5x+y+2z+3$ are coplanar.

9. Find the equation of the plane through the points (2, 2, 1) and (9, 3, 6) and perpendicualr to the plane $2x+6y+6z-1=0$.

10. How far is the point (4, 1, 1) from the line of intersection of the planes $x+y+z=4$, $x-2y-z=4$.

11. Find the equation of the plane passing through the point (–1, 3, 0) is perpendi cular to both the planes $x+2y+2z-5=0$ and $3x+3y+2z-8=0$.

12. Find the point of intersection of the line $\frac{x-2}{3}=\frac{y+1}{4}=\frac{z-2}{12}$ and the plane $x-y+z=5$.

13. Find the symmetric form of the equation to the line of intersection of the plane $3x-2y+z=1$ and $5x+4y-6z=2$.

14. Find the angle between the plane $x+y+4=0$ and the line $\frac{x+3}{2}=\frac{y-1}{1}=\frac{z+4}{-2}$.

15. Prove that the line joining (1, 2, 3) and (2, 1, –1) intersect the line joining (–1, 3, 1) and (3, 1, 5).

16. Find the equation of the line through the point (1, –2, 1) and parallel to the line $\frac{x}{2}=\frac{y-1}{-1}=\frac{z+2}{3}$.

17. Find the equation of the plane through the points (1, 2, –3), (2, 3, –4) and $\perp^r$ to the plane $x+y+z+1=0$.

18. Find the perpendicular distance of the point (–1, 3, 9) from the line $\frac{x-13}{5}=\frac{y+8}{-8}=\frac{z-31}{1}$.

19. Prove that the measure of the angle between two main diagonals of a cube is $\cos^{-1}\frac{1}{3}$.

20. Find the equation of the plane passing through the intersection of the planes $x+2y+3z-4=0$ and $2x+y-z+5=0$ and also perpendicular to the plane $2x-y+2z+3=0$.

21. Find the equation of the plane passing through the foot of the perpendicular drawn from the part (a, b, c) on co ordinate axes.

22. If A(1, 0, –2), B(–2, 4, –2) and C(1,5,10) be the vertices of a triangle and the bisector of the angle $\angle BAC$ meets BC at D. Find the coordinates of D.

ANSWER HINTS

1. The lines are $\frac{x+3}{2}=\frac{y+5}{3}=\frac{z-7}{-3}=r_1$ (say) (1) $\frac{x+1}{4}=\frac{y+1}{5}=\frac{z+1}{-1}=r_2$ (2)

A point on line (1) is $(2r_1-3, 3r_1-5, 7-3r_1)$

A point on line (2) is $(4r_2-1, 5r_2-1, -r_2-1)$

If the lines are coplanar then they must intersect. At the point of intersection $2r_1-3=4r_2-1 \Rightarrow 2r_1-4r_2-2=0$.

$\Rightarrow r_1-2r_2-1=0$ (3)

Taking $3r_1-5=5r_2-1 \Rightarrow 3r_1-5r_2-4=0$ (4)

on solving $r_1=3, r_2=1$. Values of r_1 & r_2 satisfy eqn. (4) so the lines are coplanar. Eqn. of plane containing them is

$$\begin{vmatrix} x+3 & y+5 & z-7 \\ 2 & 3 & -3 \\ 4 & 5 & -1 \end{vmatrix}=0$$

2. Let the lines are $\frac{x-4}{1}=\frac{y+3}{-4}=\frac{z+1}{7}$ (1)

$\frac{x-1}{2}=\frac{y+1}{-3}=\frac{z+10}{8}$ (2)

Two parts on the line are (4, –3, –1) and (1, –1, –10)

As $\begin{vmatrix} 1-4 & -1+3 & -10+1 \\ 1 & -4 & 7 \\ 2 & -3 & 8 \end{vmatrix}=0$

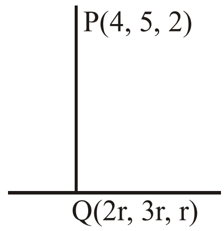
$\Rightarrow$ (1) and (2) are coplanar.

Find the general points on (1) and (2) as $(r_1+4, -4r_1-3, 7r_1-1)$ and

$(2r_2+1, -3r_2-1, 8r_2-10)$

Take $r_1+4=2r_2+1, -4r_1-3=-3r_2-1$ on solving find r_1 and r_2. Then find point of intersection. Then find the equation of the plane containing (1) and (2).

3. Let $\frac{x}{2}=\frac{y}{3}=\frac{z}{1}=r \Rightarrow x=2r, y=3r, z=r$



Drs of PQ are $\langle 2r-4, 3r-5, r-2\rangle$ PQ $\perp^r$ to the given line

$\Rightarrow 2(2r-4)+3(3r-5)+1(r-2)=0$

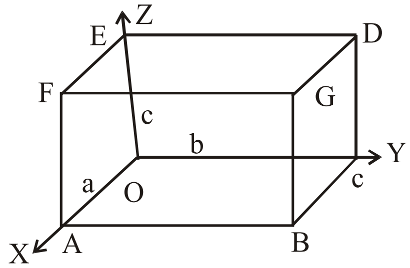
$\Rightarrow r=\frac{25}{14}$

Then find coordinates of Q. Length of PQ is the required distance.

4. Let OA = a, OC = b, OE = c

Here O(0, 0, 0), A(a, 0, 0), B(a, b, 0)

C(0, b, 0), D(0, b, c), E(0, 0, c), F(a, 0, c), G(a, b, c)



drs of OG are are (a, b, c)

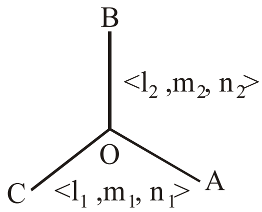
drs of EB are (a, b, –c). Let θ is the angle between OG and EB.

$$\cos\theta = \frac{a^2+b^2-c^2}{\sqrt{a^2+b^2+c^2}\sqrt{a^2+b^2+c^2}}$$

$$\Rightarrow \theta = \cos^{-1}\left(\frac{a^2+b^2-c^2}{a^2+b^2+c^2}\right)$$

Similarly we can find the angle between the other diagonals.

5. Let OA and OB are two mutually $\perp^r$ lines whose dcs are $\langle l_1, m_1, n_1\rangle$ and $\langle l_2, m_2, n_2\rangle$ respectively.



Let OC be a line $\perp^r$ to OA and OB. Let dcs of OC are $\langle l, m, n\rangle$

Then by $\perp^r$ condition

$ll_1 + mm_1 + nn_1 = 0$, $ll_2 + mm_2 + nn_2 = 0$

By cross multiplication

$$\frac{l}{m_1n_2 - m_2n_1} = \frac{m}{n_1l_2 - n_2l_1} = \frac{n}{l_1m_2 - l_2m_1}$$

$$= \frac{l^2+m^2+n^2}{\sqrt{(m_1n_2-m_2n_1)^2 + (n_1l_2-n_2l_1)^2 + (l_1m_2-l_2m_1)^2}}$$

Values of l, m, n will give the required dcs.

6. The equation of the plane

is x – y + 2z = 4(1)

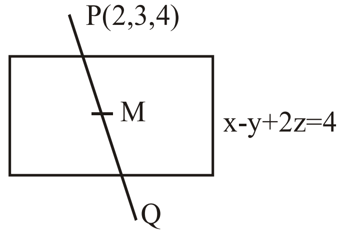
The given part is P(2, 3, 4)

Here PM = MQ and Q is the image part of P.

Drs of P M Q are <1, –1, 2>

Equation of PMQ is

$$\frac{x-2}{1} = \frac{y-3}{-1} = \frac{z-4}{2} = r \quad (2)$$



Let $Q(r+2, 3-3, 2r+4)$. As M is the mid point of P and Q

$$\Rightarrow M\left(\frac{r+2+2}{2}, \frac{3-r+3}{2}, \frac{2r+4+4}{2}\right)$$

Using coordinates of M in (1) we get r. Then we can find the coordinates of Q.

7. The given equations are $l+2m+3n=0$ (1)

$3lm - 4\ln + nm = 0$ (2)

From (1) $l = -2m - 3n$. Using the value of l in (2) and simplifying we get $l - (3 \pm 2\sqrt{2})n, m = \pm\sqrt{2}n$

Drs of the lines are $\left(-(3+2\sqrt{2}), \sqrt{2}, 1\right)$ and $\left(-3+2\sqrt{2}, -\sqrt{2}, 1\right)$ then verify the $\perp^r$ condition.

8. Equation of 1st line is $\frac{x+4}{1} = \frac{y+5}{3} = \frac{z-7}{2}$ (1)

Equation of 2nd line is

$$\left.\begin{array}{l} 2x+3y+z-1=0 \\ 5x+y+2z+3=0 \end{array}\right\} (2)$$

Symmetrical form of (2) is

$$\frac{x}{10} = \frac{y}{-11} = \frac{1}{-13}$$

$$\Rightarrow x = -\frac{10}{13},\ y = \frac{11}{13},\ z = 0$$

So a part on line (2) is $\left(-\frac{10}{13}, \frac{11}{13}, 0\right)$

Then verify coplanarity condition.

9. Let A(2, 2, 1) and B(9, 3, 6) are 2 given parts. Eqn. of the plane passing through the (2, 2, 1) is

$$a(x-2)+b(y-2)+c(z-1)=0 \quad (1)$$

when line (1) will pass through (9,3,6) then a(9 –2) +b(3–2) + c(6–1) = 0

$\Rightarrow 7a+b+5c=0$ (2)

The given plane is $2x+6y+6z-1=0$ (3)

since this plane is $\perp^r$ to (3)

$\Rightarrow 2a+6b+6c=0$ (4)

By cross multiplication from (2) and (4) we have

$$\frac{a}{3}=\frac{b}{4}=\frac{c}{-5}=k \Rightarrow a=3k, b=4k, c=-5k$$

putting the values of a, b, c in eqn. (1) we get the required plane.

10. Two given planes are x+y+z=4(1) x–2y–z=4(2) Adding (1) and (2) 2x–y=8

$\Rightarrow y=2x-8=2(x-4)$

Subtracting (2) from eqn.(1)

$y=\frac{-2}{3}z$ (4)

Now $\frac{x-4}{1}=\frac{y}{2}=\frac{z}{-3}=r$ (5)

Any part on the line is (r+4, 3r, –3r)

Drs of PQ are <r, 2r–1, –3r–1>

Let P(4, 1, 1). PQ is $\perp^r$ to the line (5) using $\perp^r$ condition we get r. Then we can find Q.

Required distance = PQ

11. The given planes are x+2y+2z–5=0(1) 3x+3y+2z-8 = 0 (2) Given point is (–1,3,0)

Equation of plane passing through (–1, 3, 0) is a(x+1) + b(y–3) +c(z-0)=0(3)

Plane (3) is $\perp^r$ to (1) and (2) then

a + 2b + 2c = 0 (4) 3a + 3b + 2c = 0(5)

By cross multiplication from (4) and (5) we get a, b, c.

Putting the values of a, b, c in eqn.(3) we get the required plane.

12. Let $\frac{x-2}{3}=\frac{y+1}{4}=\frac{z-2}{12}=r$ (1)

$x=2+3r, y=4r-1, z=2+12r$

Let $P(2+3r, 4r-1, 2+12r)$. P is a point on the plane $x-y+z=5$.

Then we can find r and coordinates of P.

14. The given plane is x + y + 4 = 0 (1)

Drs of normal to plane (1) are $\langle 1,1,0\rangle$. Let θ is the angle between plane (1) and the line

$\frac{x+3}{2}=\frac{y-1}{1}=\frac{z+4}{-2}$ (2)

$\Rightarrow$ Angle between line (2) and normal to the plane is $\frac{\pi}{2}-\theta$.

$$\cos\left(\frac{\pi}{2}-\theta\right)=\frac{2.1+1.1-2.0}{\sqrt{2^2+1^2+2^2}\sqrt{1^2+1^2}}$$

Then we can find θ

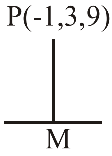
15. Find the equations of lines in two points form.

Then show that the lines are coplanar.

Find the drs of both the lines and show that are not parallel.

Therefore both the lines will intersect

18. The given line is



$$\frac{x-13}{5}=\frac{y+8}{-8}=\frac{z-31}{1}=r \qquad (1)$$

$x=13+5r,\ y=-8-8r,\ z=31+r$

Let $M(13+5r,\ -8-8r,\ 31+r)$

The find drs of PM. PM is $\perp^r$ to line (1)

Using $\perp^r$ condition we get r. Then we can find the coordinates of M.

20. The intersection of planes x+2y+3z–4 = 0 (1) and 2x+y-z+5=0 (2) is given as $(x+2y+3z-4)+k(2x+y-z+5)=0$ (3)

Plane (3) is $\perp^r$ to the plane $2x-y+2z+3=0$ (4)

Using $\perp^v$ condition find k. Putting the value of k in eqn. (3) we get the required plane.

21. Let P be the part (a, b, c). Let PA, PB, PC are $\perp^r s$ drawn from the part P on xy, yz and zx planes respectively. Let A(a, b, 0), B(0, b, c), C(a, 0, c).

Then find the eqn. of the plane passing through the parts A, B, C.

Unit - V

LINEAR PROGRAMMING

CHAPTER - 1

LINEAR PROGRAMMING

A. Multiple Choice Questions (MCQ)

1 In an LPP maximize z = 8000x + 12000y subject to $3x+4y \le 60$, $x+3y \le 30$, $x \le 0$, $y \ge 0$ then maximum value of z is ____

(a) 164000 (b) 166000

(c) 168000 (d) 170000

2. In an LPP, max z = x + y subject to $2x+y \le 50$, $x+2y \le 40$, $x \ge 0$, $y \ge 0$, then maximum value of z is ____

(a) 20 (b) 30

(c) 40 (d) 50

3. For an APP, max z = 2.5x + y Subject to $x+3y \le 12$, $3x+y = 12$, $x \ge 0, y \ge 0$ then

Maximum value of z is ____

(a) 10 (b) 10.5

(c) 9 (d) 9.5

4. For an LPP, maximize z = 60x + 8y Subject to $x+y \le 500$, $x \le 400$, $y \le 200$, $x \ge 0$, $y \ge 0$ then maximum value of z is ____

(a) 25,000 (b) 30,000

(c) 35,000 (d) 40,000

5. For an LPP, minimize z = 60x + 80y Subject to $3x+4y \ge 8$, $5x+2y \ge 11$, $x \ge 0$, $y \ge 0$ then minimum value of z is ____

(a) 190 (b) 180

(c) 170 (d) 160

6. For an LPP maximize z = 20x + 10y Subject to $1.5x+3y \le 42$, $3x+y \le 24$, $x \ge 0, y \ge 0$ then maximum value of z is ____

(a) 100 (b) 200

(c) 300 (d) 400

7. For an LPP, maximize z = 5x + 3y Subject to $2x+y \le 12$, $3x+2y \le 20$, $x \ge 0$, $y \ge 0$ then maximum value of z is ____

(a) 25 (b) 30

(c) 32 (d) 40

8. For an LPP, maximize z = 22x + 18y Subject to $x+y \le 20$, $3x+2y \le 48$, $x \ge 0$, $y \ge 0$ then maximum value of z is ______

(a) 392 (b) 390

(c) 388 (d) 386

9. For an LPP, Maximize $z = 5x_1 + 7x_2$

Subject to $x_1 + x_2 \le 4$, $3x_1 + 8x_2 \le 24$, $10x_1 + 7x_2 \le 35$ $x_1 \ge 0, x_2 \ge 0$ then maximum value of z is ____

(a) $\frac{118}{5}$ (b) $\frac{124}{5}$

(c) $\frac{128}{5}$ (d) $\frac{133}{5}$

10. The solution of LPP, maximize z = x+y

Subject to $3x + 4y \le 12$, $x \ge 0, y \ge 0$ is

(a) 3 (b) 4

(c) 5 (d) 6

11. The solution of LPP maximize z = 2x+3y

Subject to $x + y \le 1$, $x \ge 0$, $y \ge 0$ is

(a) 3 (b) 4

(c) 5 (d) 6

12. Maximum value of $z = x + y$

Subject to $2x + 3y \le 6$, $x \ge 0$, $y \ge 0$ is

(a) 2 (b) 3

(c) 4 (d) 5

13. The solution of maximize z = 20x+30y

Subject to $3x + 5y \le 15$, $x \ge 0$, $y \ge 0$ is

(a) 80 (b) 90

(c) 100 (d) 110

14. Solution of LPP, minimize $z = 6x_1 + 7x_2$

Subject to $x_1 + 2x_2 \ge 2$, $x_1, x_2 \ge 0$ is

(a) 4 (b) 5

(c) 6 (d) 7

15. Solution of minimize $z = 5x + 7y$

Subject to $2x + y \ge 8, x + 2y \ge 10$, $x \ge 0, y \ge 0$ is

(a) 15 (b) 16

(c) 17 (d) 18

16. Solution of LPP maximize z = 5x + 3y

Subject to $3x + 5y \le 15, 5x + 2y \le 10$, $x \ge 0$, $y \ge 0$

(a) $\frac{240}{19}$ (b) $\frac{235}{19}$

(c) $\frac{243}{19}$ (d) $\frac{244}{19}$

17. Solution of LPP, maximize z = 3x + 2y

Subject to $x + y \le 400$, $2x + y \le 500$, $x \ge 0$, $y \ge 0$ is

(a) 700 (b) 800

(c) 900 (d) 1000

18. The solution of LPP maximize z = x+2y

Subject to $2x + y \le 4, x \ge 0, y \ge 0$ is

(a) 6 (b) 7

(c) 8 (d) 9

19. The solution of minimize z = 3x + 2y

Subject to $x + 3y \ge 3$, $x + y \ge 2$, $x \ge 0$, $y \ge 0$ is

(a) 6 (b) 7

(c) 8 (d) 9

20. The solution of minimize z = 3x + 2y

Subject to $5x + y \ge 10$, $x + y \ge 6$, $x \ge 0$, $y \ge 0$ is

(a) 13 (b) 14

(c) 15 (d) 16

21. The solution of LPP maximize $z = 20x + 10y$

Subject to $x + 2y \le 40$, $3x + y \le 30$, $x \ge 0$, $y \ge 0$ is ____

(a) 250 (b) 260

(c) 270 (d) 280

Answer Keys

1.	(c)	6.	(b)	11.	(a)	16.	(b)
2.	(b)	7.	(c)	12.	(b)	17.	(c)
3.	(b)	8.	(a)	13.	(c)	18.	(a)
4.	(d)	9.	(b)	14.	(d)	19.	(b)
5.	(d)	10.	(b)	15.	(a)	20.	(a)
						21.	(b)

B. Long Answer Type Questions

1. The kind of cake requires 200gm of flour and 25 gm of fat and another kind of cake requires 100gm of flour and 50 gm of fat. Find the maximum number of cakes which can be made from 5kg of flour and 1kg of fat assuming that there is no shortage of the other ingredients used in making the cakes.

2. For an LPP, maximize $z = 2.5x + y$

   Subject to $x + 3y \le 12, 3x + y \le 12, x, y \ge 0$

3. Solve the LPP maximize $z = 3x_1 + 5x_2$

   Subject to $5x_1 + 3x_2 \le 30$, $x_1 + 2x_2 \le 12$, $2x_1 + 5x_2 \le 20$ and $x_1, x_2 \ge 0$.

4. Solve the LPP $z = 5x + 7y$

   Subject to $2x + y \ge 8$, $x + 2y \ge 10$

   and $x \ge 0$, $y \ge 0$

5. Solve the LPP maximize $z = 5x + 3y$

   Subject to $3x + 5y \le 15, 5x + 2y \le 10$, $x \ge 0$, $y \ge 0$

6. Solve the LPP, maximize z = 3x+2y

   Subject to $x + y \le 400$, $2x + y \le 500$

   and $x \ge 0$, $y \ge 0$.

7. Maximize z = 5x + 7y

   Subject to $x + y \le 4, 3x + 8y \le 24$,

   $10x + 7y \le 35$ and $x \ge 0, y \ge 0$

8. Maximize $z = -10x + 2y$

   Subject to $-x + y \ge -1$, $x + y \le 6$,

   $y \le 5, x, y \ge 0$

ANSWER HINTS

1. Let 1st kind of cake be A and 2nd kind of cake is B. Let the maximum number of cakes A and cakes B are x and y respectively.

Ingredients	Cake A	Cake B	
Flours	200	100	5000
Fats	25	50	1000
max no. of cakes	x	y	

$z = x + y \quad 200x + 100y \leq 5000$

$25x + 50y \leq 1000, \; x \geq 0, \; y \geq 0$

i.e. Maximize $z = x + y$ (1)

Subject to $2x + y \leq 50$ (2)

$x + 2y \leq 40$ (3)

$x \geq 0, \; y \geq 0$ (4)

Converting the inequalities to equalities

$2x + y = 50$ (5)

$x + y = 40$ (6)

$x = 0, \; y = 0$ (7)

First we draw the graph of (5)

when $x = 0 \Rightarrow y = 50$

when $y = 0 \Rightarrow x = 25$

x	0	25
y	50	0

Line (5) passes through (0, 50) and (25, 0).

For equation (6) when $x = 0 \Rightarrow y = 20$

when $y = 0 \Rightarrow x = 40$

x	0	40
y	20	0

The line passes through (0, 20), (40,0)

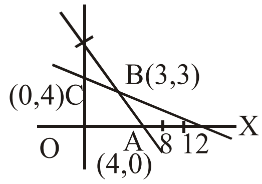
A is (25,0) B is (20,10), C(0, 20)

	x	y	$z = x + y$
A	25	0	$z = 25$
B	20	10	$z = 30$
C	0	20	$z = 20$

Maximum of total cakes out of which 20 be x and 10 be y.

2. $x + 3y = 12$ (1)

$3x + y = 12$ (2)



x	0	12
y	4	0

Line (1) intersect x-axis at (12,0) and y-axis at (0,4)

For (2)

x	0	4
y	12	0

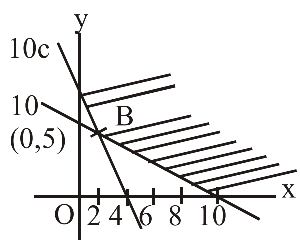
Line (2) passes through (0, 2), (4,0).

A is (4,0), B is (3,3), C(0,4)

point	x	y	$z = 2.5x + y$
0	0	0	$z = 0$
A	4	0	$z = 10$
D	3	3	$z = 10.5$
C	0	4	$z = 4$

Maximum of z is 10.5

4. Given LPP is min z = 5x + 7y



Subject to $2x + y \geq 8, x + 2y \geq 10$ and $x \geq 0, \; y \geq 0$.

Converting the given inequations into equations we get the following equations.

$2x + y = 8(1) x + 2y = 10(2)$

$x = 0(3)\ \ y = 0\ (4)$

Line (1) meets the coordinate axes at (4,0) and (0,8) The point (0,0) does not satisfy the inequation $2x + y \geq 8$ so the region is on the side of the line (1) where (0, 0) is not satisfied.

The shaded portion are A(10,0), B(2,4), C(0,8)

The value of the objective function at these points are given in the following table. z = 5x + 7y

$A(10,0) \Rightarrow z = 5.10 + 7.0 = 50$

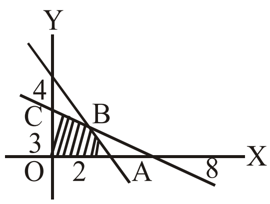
$B(2,4) \Rightarrow z = 5.2 + 7.4 = 38$

$C(0,8) \Rightarrow z = 5.0 + 7.8 = 56$

so minimum value of the objective function is 38.

5. Given LPP is maximize z = 5x + 3y

Subject to $3x + 5y \leq 15,\ 5x + 2y \leq 10$ and $x \geq 0, y \geq 0$.



Converting the inequations to equations we have

$3x + 5y = 15(1)\ 5x + 2y = 10(2) x = 0(3)$

$y = 0(4)$

line (1) meets the coordinate axis at (5,0) and (0,3). line (2) meets the coordinate axes at (2,0) and (0,5)

OABC is the feasible region.

Coordinates of the corner points of the feasible region are 0(0,0), A(2,0), $B\left(\frac{20}{19} \times \frac{45}{19}\right)$ and C(0,3)

Values of the objective function

z = 5x + 3y

$0(0,0) \Rightarrow z = 0$

$A(2,0) \Rightarrow z = 5.2 + 3.0 = 10$

$B\left(\frac{20}{19}, \frac{45}{19}\right) \Rightarrow z = 5.\frac{20}{19} + 3.\frac{45}{19} = \frac{235}{19}$

$C(0,3) \Rightarrow z = 5.0 + 3.3 = 9$

Maximum value of z is $\frac{235}{19}$ which occurs at $B\left(\frac{20}{19}, \frac{45}{19}\right)$

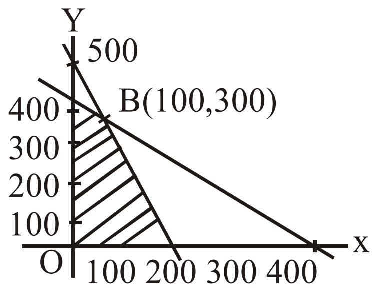
6. Given LPP is maximize z = 3x + 2y

Subject to $x + y \leq 400,\ 2x + y \leq 500$ and $x \geq 0,\ y \geq 0$.

converting the inequations to equations we have

$x + y = 400(1) 2x + y = 500(2), x = 0(3)$

$y = 0(4)$



Line (1) meets the coordinate axes at (400,0) and (0, 400). Joining these two points we get line (1). Here (0,0) satisfies the inequation $x+y\le 400$.

So the region on the side of line (1) containing the origin represents the solution of the inequation.

Line (2) meets the coordinate axes at (250,0) and (0, 500). Joining these two points we get line (2). Here (0,0) satisfies the inequation $2x+y\le 500$. So the region on the side of line (2) containing the origin represents the solution of this inequation. The shaded portion is the feasible region.

The vertices of the feasible region OABC are 0(0,0). A(250,0), B(100,300) and C(0, 400)

$A(250,0), B=(100,300)$ and $C(0,400)$

At $0(0,0)\Rightarrow z=3.0+2.0=0$

At $A(250,0)\Rightarrow z=3.250+2.0=750$

At $B(100,300)\Rightarrow z=3.100+2.300=900$

At $C(0,400)\Rightarrow z=3.0+2.400=800$

Maximum value of z = 900 which occurs at B(100, 300)

8. The given LPP is maximize z = –10x + 2y

Subject to $-x+y\ge -1$, $x+y\le 6$,

$y\le 5,\ x,y\ge 0$

Changing the inequations to equations

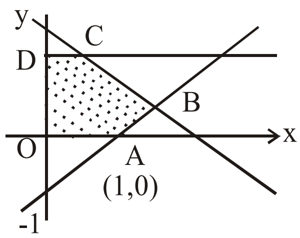
$-x+y=-1(1)\ x+y=6(2)\, y=5(3)$

$x=0(4)\, y=0(5)$

Line (1) intersects x-axis at (1,0) and y-axis at (0,–1). Joining these two points we get line (1). Line (2) intersects x-axis at (6,0) and y-axis at (0,6). Joining these two points we get line (2).

The equation (3) is a line parallel to x-axis and at a distance 5 from it.

Equation (4) and (5) represent y-axis and x-axis respectively.



The feasible region is OABCD, 0(0,0), A(1,0) B(7/2,5/2), C(1,5), D(0,5)

At $0(0,0)\Rightarrow z=0$

At $A(1,0)\Rightarrow z=-10.1+2.0=-10$

At $B(7/2,5/2)\Rightarrow z=-10.7/2+2.5/2=-30$

At $C(1,5)\Rightarrow z=-10.1+2.5=0$

At $D(0,5)\Rightarrow z=-10.0+2.5=10$

Maximum value of z is 10 occuring at D(0,5)

Unit - VI

PROPABILITY

CHAPTER - 1

PROBABILITY

A. Multiple Choice Questions (MCQ)

1. A die is thrown. What is the probability of getting a number greater than or equal to 3?
   (a) 1/3 (b) 2/3
   (c) 4/3 (d) None of these

2. A die is thrown twice. What is the probability that the sum of points is atleast 10 ?
   (a) 4/6 (b) 5/6
   (c) 1/6 (d) 7/6

3. Two dice are thrown. What is the probability that the sum is 7 or 11 ?
   (a) 1/9 (b) 4/9
   (c) 2/9 (d) 5/9

4. If 4 unbiased coins are tossed then what is the probability of getting 4 heads?
   (a) 3/16 (b) 1/16
   (c) 5/16 (d) 7/16

5. A coin is weighted so that the head is 4 times as that appear as tail. What is P(H) ?
   (a) 2/5 (b) 3/5
   (c) 4/5 (d) None of these

6. $P(A)=\frac{1}{2}, P(A\cap B)=\frac{1}{3}$ then what is the value of $P(A-B)^C$ is ___
   (a) 5/6 (b) 1/6
   (c) 7/6 (d) 9/6

7. The probability of getting exactly 2 heads in a single throw of two unbiased coins.
   (a) 3/4 (b) 5/4
   (c) 1/4 (d) None of these

8. 3 dice are rolled. What is the probability that the same number will appear on all the dice ?
   (a) 1/36 (b) 5/36
   (c) 7/36 (d) 11/36

9. Two cards are drawn from a pack of 52 cards what is the probability that both are spades ?
   (a) 1/16 (b) 1/17
   (c) 1/18 (d) 1/19

10. What is the probability that one digit positive integer is even ?
    (a) 4/9 (b) 5/9
    (c) 7/9 (d) None of these

11. What is the probability of getting a total of 11 from a throw of two dice ?
    (a) 1/18 (b) 1/16
    (c) 1/36 (d) None of these

12. A coin is tossed 3 times. The probability of getting 3 heads is ____
    (a) 1/8 (b) 3/8
    (c) 2/8 (d) None of these

13. What is the probability of getting a red card from a pack of 52 cards ?

(a) 1/3

(b) 1/2

(c) 1/13

(d) None of these

14. What is the probability of getting at least one tail when 4 coins are tossed?

(a) 13/16 (b) 11/16

(c) 3/4 (d) None of these

15. What is the probability of not getting 2 or 3 in a single toss of dice ?

(a) 1/3 (b) 2/3

(c) 5/3 (d) 4/3

16. In a simultaneous toss of two coins what is the probability of getting one tail?

(a) 1/2 (b) 3/2

(c) 5/6 (d) None of these

17. A coin is tossed twice. Find the probability of getting atleast one head.

(a) 1/4

(b) 3/4

(c) 5/4

(d) None of these

18. A couple have 2 daughters. What is the probability that their next child will be a daughter ?

(a) 2/3 (b) 1/3

(c) 1/2 (d) 1/4

19. A bag contians 7 white and 9 black balls. If a ball is drawn at random, what is the probability that it is white ?

(a) 5/16 (b) 3/16

(c) 7/16 (d) 9/16

20. If $P(A)=0.6$, $P(B)=0.4$, $P(A\cap B)=0.2$ then $P(B\,|\,A)=$ ____

(a) 2/3 (b) 1/5

(c) 1/3 (d) 1/6

21. When a pair of dice is thrown then the probability of obtaining an even prime number on each die is ____

(a) 1/36 (b) 4/36

(c) 5/36 (d) 7/36

22. If A and B are independent events and P(A) = 3/5 and P(B) = 1/5 then $P(A\cap B)=$____

(a) 3/5 (b) 3/10

(c) 3/15 (d) 3/25

23. If A and B are independent events with P(A) = 0.3 P(B) = 0.4 then $P(A\cap B)$ is ____

(a) 0.38 (b) 0.48

(c) 0.58 (d) 0.68

24. If A and B are independent events and P(A) = 0.3 and P(B) = 0.4 then $P(A\,|\,B)$ is ____

(a) 0.2 (b) 0.3

(c) 0.4 (d) 0.5

25. If A and B are two events such that $P(A)=\frac{1}{4}$, $P(B)=\frac{1}{2}$ and $P(A\cap B)=\frac{1}{8}$ then P(not A and not B) =

(a) 3/4 (b) 3/5

(c) 3/8 (d) 3/11

26. If A and B are two independent events and P(A) = 0.3 P(B) = 0.6 then P(A and B) is ____

(a) 0.11 (b) 0.12

(c) 0.15 (d) 0.18

27. If A and B are two independent events such that P(A) = 0.3, P(B) = 0.6 then P(A and not B) = ____

(a) 0.9 (b) 0.12

(c) 0.13 (d) 0.15

28. If A and B are independent events and P(A) = 0.3 P(B) = 0.6 and P(neither A nor B) is ____

(a) 0.28 (b) 0.38

(c) 0.48 (d) 0.58

29. If E and F are events such that P(E) = 0.6, P(F) = 0.3 and $P(E \cap F) = 0.2$ then P(E | F) = ____

(a) 1/3 (b) 2/3

(c) 3/4 (d) 4/5

30. If P(B) = 0.5 and $P(A \cap B) = 0.32$ then P(A|B) is ____

(a) $\frac{12}{25}$ (b) $\frac{14}{25}$

(c) $\frac{16}{25}$ (d) $\frac{18}{25}$

31. If P(A) = 0.8, P(B) = 0.5 and P(B|A) = 0.4 then $P(A \cap B) =$

(a) 0.31 (b) 0.32

(c) 0.33 (d) 0.34

32. If P(A) = 0.8, P(B) = 0.5 and P(B|A) = 0.4 then P(A|B) =

(a) 0.64 (b) 0.65

(c) 0.66 (d) 0.67

33. P(A) = 0.8, P(B) = 0.5 and P(B|A) = 0.4 then $P(A \cup B) =$

(a) 0.95 (b) 0.96

(c) 0.97 (d) 0.98

34. If 2P(A) = P(B) = 5/13 and P(A|B) =2/5 then $P(A \cup B) =$

(a) 9/26 (b) 11/26

(c) 13/26 (d) 15/26

35. If P(A) = 6/11, P(B) = 5/11, $P(A \cup B) =$ 2/11 then P(A|B) =

(a) $\frac{4}{5}$ (b) $\frac{3}{5}$

(c) $\frac{2}{5}$ (d) $\frac{1}{5}$

36. If P(A) = 6/11, P(B) = 5/11, $P(A \cup B) =$ 7/11 then P(B|A) =

(a) 1/3 (b) 2/3

(c) 3/4 (d) 4/5

37. If P(A) = 1/2, P(B) = 0 then P(A|B) is

(a) 0 (b) 1/2

(c) 1 (d) not defined

38. A fair dice is rolled. If E = {1, 3, 5}, F = {2, 3} G = {2, 3, 4, 5} then P(E|F) =

(a) 1/2 (b) 1/3

(c) 1/4 (d) 1/5

39. If A and B are events such that P(A|B) = P(B|A) then

(a) $A \subset B$ but $A \neq B$

(b) A = B

(c) $A \cap B = \phi$

(d) P(A) = P(B)

40. If A and B are independent events such that $P(A \cup B) = 0.6$, P(A) = 0.2 then P(B) =

(a) 1/4 (b) 1/3

(c) 1/2 (d) 2/3

41. If A and B are two events such that P(A) = 0.3, P(B) = 0.4 and $P(A \cup B)$ = 0.6 then P(A|B) is

(a) 1/2 (b) 1/3

(c) 1/4 (d) 1/5

42. If A and B are events such that P(A) = 0.16 P(B) = 0.4, $P(A \cup B)$ = 0.2 then P(B|A) is

(a) 1/2 (b) 1/3

(c) 1/4 (d) 1/5

ANSWER KEYS

1. (b)	2.(c)	3.(c)	4.(b)
5.(c)	6.(a)	7.(c)	8.(a)
9.(a)	10.(a)	11.(a)	12.(a)
13.(b)	14.(c)	15.(b)	16.(a)
17.(b)	18.(c)	19.(c)	20.(c)
21.(a)	22.(d)	23.(c)	24.(b)
25.(c)	26.(d)	27.(b)	28.(a)
29.(b)	30.(c)	31.(b)	32.(a)
33.(d)	34.(b)	35.(a)	36.(b)
37.(d)	38.(b)	39.(d)	40.(c)
41.(c)	42.(b)		

B. Long Answer Type Questions

1. The probability of a shooter hitting a target is 4/5. Find the minimum number of times he must fire so that the probability of hitting the target atleast once is greater than 0.999.

2. If P(A) = 0.4, P(B|A) = 0.3 and $P\left(B^C \mid A^C\right)=0.2$ then find P(B)

3. If A and B are two events with P(A) = 3/8, P(B) = 1/2 and $P(A \cap B)=1/4$ then find $P\left(A^C \cap B^C\right)$ and $P\left(A \cap B^C\right)$.

4. 3 cards are drawn from a pack of 52 cards. Find the probability that they are of different suits.

5. If A and B are independent events then prove that A^C and B are independent.

6. From a bag containing 3 black and 4 white balls, 2 balls are drawn at random one after another. Find the probability that the 2nd ball selected is white.

7. From a pack of cards containing 5 black and 4 red cards, 2 cards are drawn one after the other. Find the probability that the 1st card drawn is black if the second card is known to be white.

8. 4 balls are drawn successively (and not replaced) from a bag containing 6 white and 4 black balls. Find the probability that they are alternately of different colours.

9. A pair of dice is thrown. If 2 numbers appearing are different. Find the probability that the sum of points is 8.

10. What is the chance that a leap year selected at random will contain 53 sundays.

11. If A and B are independent events such that $P(A \cap B)$ = 3/50 $P(A \cup B)$ = 11/25 then find P(A) and P(B).

12. Seeds in a certain batch have an 80% germination rate. If one plant 2 seeds from this batch in the same pot then what is the probability that

    (i) at least one will germinate ?

    (ii) exactly one will germinate ?

13. If $A_1, A_2, \ldots, A_n$ are events then prove that $P\left(A_1 \cup A_2 \cup \ldots \cup A_2\right) \leq P\left(A_1\right)+P\left(A_2\right)$ $+\ldots+P\left(A_n\right)$.

14. A box contains 25 tickets numbered from 1 to 25. 2 tickets are drawn at random. What is the probability that the product of numbers is even ?

15. Prove that for any two events A and B, $P(A \cap B) \geq P(A)+P(B)-1$.

16. A bag contains 8 white and 6 red balls. If 5 balls are drawn at random find the probability that 3 are white balls.

17. A bag contains 5 white and 3 black marbles and a 2nd bag contains 3 white and 4 black marbles. A bag is selected at random and a marble is drawn from it. Find the probability that it is white assuming each bag can be chosen with the same probability.

18. An urn contains 5 red and 5 black balls. A ball is drawn at random, its colour is noted and return to the urn. 2 additional balls of the colour drawn are put in the urn and then a ball is drawn at random. What is the probability that the 2nd ball is red ?

19. Two balls are drawn from a bag containing 5 white and 7 black balls. Find the probability of selecting 2 white balls if the 1st ball is replaced before drawing the second.

20. A bag 1 contains 2 white and 3 red balls. A bag 2 contains 4 white and 5 red balls. One ball is drawn at random from one of the bags and it is found to be red. Find the probability that it was drawn from the bag 2.

21. Find the probability distribution of number of doublets in 3 tosses of a pair of dice.

22. A random variable X has the following probability distribution

X	0	1	2	3	4	5	6	7
$P(X)$	0	k	$2k$	$2k$	$3k$	k^2	$2k^2$	$7k^2+k$

Find (i) P(X < 3) (ii) P(X >76)
(iii) P(0 < x < 3)

23. Two cards are drawn successively with replacement from a well-shuttled deck of 52 cards. Find the probability distribution of number of a ces.

24. A random variable has the following distribution

X	0	1	2	3	4	5	6	7
$P(X)$	0	$2p$	$2p$	$3p$	p^2	$2p^2$	$7p^2$	$2p$

Find the value of P.

Answer Hints

1. Probability of hitting the target = 4/5

The probability of not hitting the target = 1–4/5 = 1/5.

Let x denotes the number of shorts in which a shooter hit the target in n shots.

$P(x \geq 1) = 1 - p(x=0) = 1 - {}^nC_0 (4/5)^0 (1/5)^n$

$= 1 - (1/5)^n$

$1 - (1/5)^9 = 0.999$ then find n.

2. P(A) = 0.4. P(B|A) = 0.3

$\Rightarrow \frac{P(B \cap A)}{P(A)} = 0.3 \Rightarrow P(B \cap A) = 0.12$

$P(B^C \mid A^C) = 0.2 \Rightarrow \frac{P(A^C \cap B^C)}{P(A^C)} = 0.2$

$\Rightarrow \frac{1 - P(A^C \cap B^C)^C}{1 - P(A)} = 0.2$

$\Rightarrow \frac{1 - P(A \cup B)}{1 - 0.4} = 0.2$

Use $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

3. $P(A) = 3/8, P(B) = 1/2, P(A \cap B) = 1/4$

Then find $P(A \cup B)$

$P(A^C \cap B^C) = P(A \cup B)^C 1 - P(A \cup B)$

$P(A \cap B^C) = P(A) - P(A \cap B)$

4. From pack of 52 cards, 3 cards are to be drawn. Let A be the event that 1st draw gives a card belonging to any one of the 4 suits.

$P(A) = \frac{52}{52} = 1$

After drawing the 1st card we have left with 51 cards, for 2nd draw. Let B is the event that the 2nd draw gives a card from remaining 3 suits with 39 cards.

$P(B \mid A) = \frac{39}{51} = \frac{3}{17}$

At the 3rd draw there are 50 cards left. Let C is the event that the 3rd draw gives a card from remaining 2 suits with 26 cards.

$P(C \mid A \cap B) = \frac{26}{50}$

Required probability =

$P(A \cap B \cap C) = P(A).P(B|A).P(C|A \cap B)$

Here $P(A \cap B) = P(A).P(B)$

$P\left[\left(A^C \cap B\right) \cup \left(A \cap B\right)\right] = P(B)$

$\Rightarrow P\left(A^C \cap B\right) + P(A \cap B) = P(B)$

$\Rightarrow P\left(A^C \cap B\right) = P(B) - P(A \cap B)$

$= P(B) - P(A).P(B)$

$= P(B)\left[1 - P(A)\right] = P(B).P\left(A^C\right)$

So A^C and B are independent.

6. Here $|S| = P(7,2)$

Let E is the event of drawing 2 balls where 2nd ball is white

$|E| = P(7,1) \times P(4,1)$.

Then find $P(E) = \frac{|E|}{|S|}$

8. There are 6 white and 4 black balls in a bag. Four balls are drawn successively.

There are 2 possible mutually exclusive ways

(i) starting with white, WBWB

(ii) starting with Black, BWBW

Required probability

= P(WBWB) + P(BWBW)

9. $|S| = 6 \times 6 = 36$. Let A is the event that sum of the points is 8. A = {26, 62, 35, 53, 44}.

Let B is the event that the numbers appearing are different $\Rightarrow |B| = 30$

$A \cap B = \{26, 62, 35, 53\}$

Required Probability = P(A|B) =

10. A leap year has 366 days. B week contains 7 days. 366 days contains 52 weeks and 2nd days. These 2 days may be SM, MT, TW, with Th F, FS, SS. Required probability = 2/7

11. $A(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\Rightarrow \frac{11}{25} = P(A) + P(B) - \frac{3}{50}$

$\Rightarrow P(A) + P(B) = \frac{11}{25} + \frac{3}{50} = \frac{25}{50} = \frac{1}{2}$

$\left[P(A) - P(B)\right]^2 = \left[P(A) + P(B)\right]^2 - 4P(A)P(B)$

$\Rightarrow P(A) - P(B) = \frac{1}{10}$

On solving we can find P(A) and P(B)

12. P(a seed will germinate) = 0.8

P(a seed will not germinate) =1–0.8=0.2

P(both seeds will not germinate) $= 0.2 \times 0.2 = 0.04$

P(at least one will germinate)

= 1 – P(both seeds do not germinate)

P(Exactly one will germinate)

= P(1st germinate and 2nd will not germinate) + P(1st will not germinate and 2nd will germinate)

13. $P(A_1 \cup A_2) = \frac{|A_1 \cup A_2|}{|S|} = \frac{|A_1| + |A_2| - |A_1 \cap A_2|}{|S|}$

Then apply method of induction

14. Out of 25 tickets, 2 tickets can be drawn in C(25, 2) ways. |S| = C(25,2). Let A is the event that product of the numbers is even. One even number can be choosen from 12 even numbers in C(12, 1) ways.

Another number is selected from 24 cards.

$|A| = C(12,1).C(24,1)$

$P(A) = \frac{|A|}{|S|}$

15. $P(A\cup B)=P(A)+P(B)-P(A\cap B)$

$\Rightarrow P(A\cap B)=P(A)+P(B)-P(A\cup B)$

$P(A\cup B)\le 1\Rightarrow -P(A\cup B)\ge -1$

$P(A)+P(B)-P(A\cup B)\ge P(A)+P(B)-1$

Then use condition on LHS.

17. Total marbles in 1st bag = 5 + 3 = 8

Total marbles in 2nd bag = 3 + 4 = 7

Probability of selection of each bat = 1/2

P(A white marble is drawn)

= P(1st bag is selected with a white marble) +P(2nd bag is selected with a white marble)

$=P=\frac{1}{2}.\frac{5}{8}+\frac{1}{2}.\frac{3}{7}$

18. R = set of red balls $\Rightarrow |R|=5$

B = set of black balls $\Rightarrow |B|=5$

$|S|=10$. P(drawing a red ball)

$=\frac{5}{10}=\frac{1}{2}$

If 2 red balls are added to the 4rn then the 4rn contains 7 red balls and 5 black balls.

$|R|=7, |B|=5, |S|=12$

P(drawing a red ball) $=\frac{|R|}{|S|}=\frac{7}{12}$

P(drawing a black ball)

$=\frac{|B|}{|S|}=\frac{5}{10}=\frac{1}{2}$

If 2 black balls are added then $|S|=12|B|=7$.

P(drawing a red ball) = 5/12

P (drawing that 2nd ball is red)

$=\frac{1}{2}.\frac{7}{12}+\frac{1}{2}\times\frac{5}{12}$

22. Here sum of all probability = 1

$\Rightarrow 0+k+2k+2k+3k+k^2+2k^2+7k^2+k=1$

$\Rightarrow 10k^2+9k-1=0\Rightarrow k=-1,1/10$ (rejecting –1)

$P(x<3)=P(x=0)+p(x=1)+p(x,2)$

$P(x>6)=P(x=7)=7k^2+k$

23. Let X = number of aces

P(getting an ace) $=\frac{4}{52}=\frac{1}{13}$

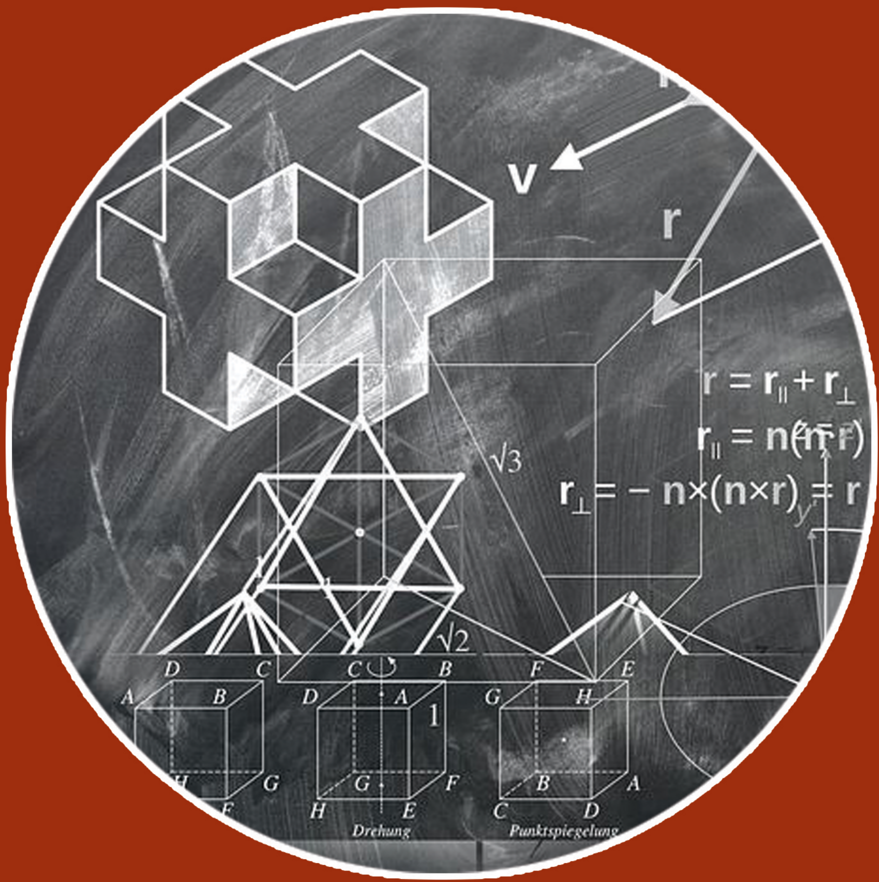
P(not getting an ace) $=1-\frac{1}{13}=\frac{12}{13}$

$P(X=0)\Rightarrow\frac{12}{13}.\frac{12}{13}$

$P(X=1)=$ P(getting an ace)

$=\frac{1}{13}\times\frac{12}{13}+\frac{12}{13}\times\frac{1}{13}$

$P(X=2)=$P(getting 2aces) $=\frac{1}{13}\times\frac{1}{13}$


v
r
r = r∥ + r⊥
√3
√2
D C
A B
H G
F
C B
D A
1
G F
H E
F E
G H
B A
C D
Drehung
Punktspiegelung